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A

MANUAL OF LOGIC

VOL. I.

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A

MANUAL OF LOGIC

BY

J. WELTON, M.A. LOND. AND CAMB.

LATE SCHOLAR OF GONVILLE AND CAIUS COLLEGE, CAMBRIDGE
PROFESSOR OF EDUCATION IN THE UNIVERSITY OF LEEDS

VOLUME I.

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PREFACE TO THE FIRST EDITION.

THIS treatise aims at giving a fairly complete view of logical doctrine, as generally accepted at the present time. Of necessity, in a subject where so many conflicting opinions are advanced, some parts of the treatment are somewhat controversial; but it is hoped that every view which is criticized is fairly stated.

The discussions of disputed points have been mainly confined to the consideration of the theories of logical doctrine advanced in well-known works. It was felt that such a consideration would help in making clear those general principles which can be philosophically justified, and which form the ultimate foundation of the science.

A new method of diagrammatically representing categorical propositions is suggested in § 106, and it is hoped that some freshness of treatment will be found in other parts of the book.

The work is primarily intended for the use of students preparing for the Examinations of the University of London; but it is hoped that it will be of service to those who are reading for any examination of which Logic forms a part.

The paragraphs are of three classes. First, those entirely unmarked, which contain the outlines of the subject; Second, those marked with an asterisk, whose contents are somewhat less elementary; Third, those printed in smaller type, which contain fuller discussions of particular points and the treatment of more difficult topics. It is recommended that these last be omitted on a first reading by all to whom the subject is a new one. On a second reading, it is hoped they may be found both of use and of interest.

The marginal analysis has been made sufficiently full to serve for a final rapid revision immediately before an examination. Such an analysis is, probably most useful when it accompanies the text, as any point which it leaves obscure, or which has been forgotten, can then be immediately, and without trouble, referred to.

I wish to acknowledge the help I have derived from the courses of lectures by Dr. Venn, and Mr. W. E. Johnson, which I have had the privilege of attending at Cambridge. I have also received most valuable

assistance from the works of Dr. Venn, Dr. Keynes, Lotze, Ueberweg, Mr. Bosanquet, Mill, Whewell, Mr. Bradley, Miss Jones, Dr. Ray, Professor Bowen, Jevons, and Professor Fowler. In addition to these, the works of other logicians—especially those of Mr. Stock, Professor Bain, Mr. Abbott, the late Archbishop Thomson, Mansel, Mr. A. Sidgwick, Hamilton, and Whately, as well as the *Port Royal Logic*, have been frequently consulted.

My best thanks are due to my friend, Mr. H. Holman, B.A., late Scholar of Gonville and Caius College, Cambridge, for his kindness in reading the proof sheets and suggesting improvements. Mr. Holman has also prepared a book of exercises and illustrative examples, with a Key containing references to this work, which I strongly urge all students of this Manual to use as a companion to it; for the working of exercises—especially on the more formal parts of the subject—is essential to a thorough mastery of Logic.

J. W.

UNIVERSITY CORRESPONDENCE COLLEGE,
August, 1891.

PREFACE TO THE SECOND EDITION.

Though the general character, and much of the matter, of the volume remains unchanged, yet this Edition differs from the First in two important points.

In the first place, the book has been considerably shortened by the omission of much of the controversial and historical matter. It is hoped that this curtailment—which appears to be generally desired by those who have used the book—will render the work more manageable by beginners in the study of Logic, and will make the task of obtaining a connected and consistent view of logical doctrine an easier one. At the same time it is believed that nothing of real importance to the students for whom the book is primarily intended has been omitted.

In the second place, an endeavour has been made to give greater prominence to the distinctions of thought which underlie those distinctions of language

to which the traditional logic so largely confines itself This endeavour has led to considerable alterations in the Book on Propositions, especially in those sections which deal with Hypotheticals. It is hoped that these alterations will make clearer the true character of the mental act of judgment, and will thus lead the reader to a more thorough grasp of the fundamentals of the subject. In making these alterations I have received material assistance from my friend Professor Mackenzie, of the University College of South Wales, who kindly read them through in manuscript, and made some valuable suggestions.

I would also acknowledge my indebtedness to my critics, especially to Dr. Keynes, who in the third edition of his valuable work on *Formal Logic* has drawn attention to several points on which I had failed to make my position clear, and also to several errors. In all these cases, I have endeavoured to profit by the criticism, and, in particular, I have entirely altered my treatment of Eductions of Disjunctive Propositions (§ 106 in this Edition, § 120 in the First Edition). The omissions in this Edition will account for any failure to notice criticisms directed towards those parts.

J. W.

LEEDS,
January, 1896.

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INTRODUCTION.

CHAPTER I.

THOUGHT AND LANGUAGE

1. Relation of Language to Logic.

Logic treats of the processes of thought by means of which knowledge in its most general aspect is attained. Now when we examine into what we mean by knowledge we find we can analyse it into the matter known and the activity of mind by which it is known. We do not mean that these can be separated, so that either can exist apart from the other, but that we can distinguish these two aspects in thought. The matter known we speak of in general as the objective or external world, which we can perceive, but whose existence is independent of this or that act of perception. Moreover, when a portion of it is perceived by any individual it must be perceived in one particular way, and is so perceived under like conditions by every normally-constituted mind. This then we regard as *reality*, and we may say that this constraining power is the characteristic of the real. But to say this is to say that reality can only be conceived by us as in essential relation to consciousness. When we say, for instance, that we regard our houses as continuing to exist unmodified by our absence, we mean that we believe that if we could then perceive them, they would appear to us in their ordinary aspect; for in this way only can we make our experiences consistent. But we mean no more

Logic treats of knowledge in general.

We can analyse knowledge into (1) object known (2) the constitutive activity of thought.

Reality is that which constrains an activity

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and cannot be conceived out of relation to thought.

than this : existence out of all possible relation to consciousness is meaningless to us. Hence we see that reality is not independent of thought, but is, on the contrary, constituted for each one of us by that synthetic activity of thought which alone makes sensuous experience intelligible. It is thought which by synthesizing impressions received through different senses gives us ideas of material 'things'; it is thought which grasps the relations of substance, identity, time, space, mutual interaction, etc., in which these things exist, and so constitutes for each one of us an idea of a single and self-consistent world. The world for each one of us exists only as thought.

Logic deals with thought *in general*,

But the world as thought exists in the same way for all. In so far as the concept of the world held by different persons varies, the variation is due to differences in the thoroughness with which it is thought. It is in general agreement, indeed, that we find the test of accuracy in the interpretation by individuals of the impressions they receive. If the distinction between red and blue cannot be perceived by a particular individual, we say he is 'colour-blind,' *i.e.*, we assert that though the two impressions to him are but one, yet they are two in reality, because that is the common testimony of mankind. It is not with the knowledge or the thought of individuals that Logic is concerned but with knowledge and accurate thought in general. But to deal with this would be impossible were there no commonly received and understood means of expressing and communicating thought. Such a means is language.

and is, therefore, closely connected with language ;

Language we may define as *a system of bodily actions, with a sensible effect at every moment to guide it, which is used for the purpose of carrying on and of expressing thought.* The simplest form of language is purely imitative as exemplified in those natural and expressive gestures to which man is sometimes compelled to resort when desiring to communicate with persons between whom and himself there is no common speech. But interesting as this is from a psychological point of view it does not here concern us. The only language with which Logic is concerned is the ordinary conventional language

of spoken or written words; and it is concerned with that only in so far as the distinctions of language correspond with distinctions of thought. But here we must acknowledge a temptation from which Logic has by no means always kept free—the temptation to substitute the symbol for the substance, to deal with language and the distinctions of language rather than with the underlying reality of thought.

but only as far as distinctions of language correspond with distinctions of thought.

*2. Chief Functions of Language.

Language has five main functions.

Language we have seen to be both an instrument for thinking and a means of expressing and communicating thought (*see* § 1). We may analyse its functions under the former of these heads into three, and under the latter into two, main classes. We will treat each of these functions separately.

(i.) Language gives the power of analysing complex wholes.

(i.) It makes possible analysis of complex wholes.

Unless we could mark any particular element of a complex impression by some sign—as a name—we should find it impossible to fix attention upon that element to the practical exclusion of others from consideration. We receive an impression as a whole; for instance, we see a man writing. But, with the aid of language, we can separate this into two ideas—the man and the act of writing—and we connect these two under the relation of agent and action. In other cases the relation may be that of subject and attribute, as when we separately dwell upon an action and its moral character.

(ii.) Language makes possible the formation of Concepts.

(ii.) It gives the power of forming Concepts.

Bound up with this power of analysis is the ability to form concepts, *i.e.*, intelligible syntheses of the attributes and relations which constitute the essential nature of a class of things. Such concepts involve no sensible mental images; it is by being named that their elements gain that definiteness and independence which is necessary to enable us so to group them. Nor is it only in the *formation* of concepts

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that names are necessary. The concept, when formed, would soon become vague, and tend to disintegration, if it were not held together, and made definite, by a name. The name fits it to be an object of thought, and preserves it for future use without the necessity of repeating the whole process of its formation.

(iii.) Language shortens the process of thinking.

(iii.) It shortens the processes of thought.

Even without language all thinking is more or less symbolic; for the ideas present to full consciousness derive their full import from their relation to other ideas to which no direct attention is paid. With thought aided by language, especially conventional language, this process is carried much further. We scarcely ever make the meanings of the words we use explicit to our minds, though they are implicitly present, as is shown by the mental shock consequent on hearing, or reading, a proposition connecting incongruous ideas, as 'The victors sued for peace.' In the case of many concepts, indeed, our knowledge of what they involve is always more or less hazy. This may either be because of their great complexity, as, for instance 'The British Constitution'; or, more frequently, because we learn to use the name through its application to individual objects whose special qualities we have no need to investigate; this is the case with the names of very common objects, as 'dog' or 'horse.' In all such cases the name necessarily stands for a number of attributes of which our idea is more or less shadowy. It is, in fact, only in the case of scientific terms well known to us, as 'square,' 'triangle,' that our concepts are, as a rule, perfectly *distinct*, and, in such cases, the attributes implied are generally more or less consciously apprehended whenever the word is used. But in ordinary speech it is not so, and this symbolic use of words for the ideas, or groups of ideas, they imply is an abbreviation of thought very similar to the shortening of mathematical operations by the aid of symbols. This abbreviation makes it possible to carry on trains of thought infinitely more complex than would otherwise be possible.

(iv.) Language is a direct means of communicating thought.

(iv.) It is a direct means of communicating thought.

This is the most obvious function of language. We always think of it as chiefly useful for this purpose, though, of course, it must be primarily an instrument for conducting thought, as, otherwise, we should have nothing to communicate. It is this function of language which makes all social intercourse possible, and enables each person to profit by the knowledge acquired by others with whom he is brought in contact. Thus, mental development is facilitated and made infinitely more speedy, and of greater breadth and richness, than would be possible if each mind were condemned to exist in isolation.

(v.) Language is a means of recording thought.

(v.) It enables us to record thought.

This is the great use of writing and printing, and is, evidently, an extension of the function last described. By this means we can benefit by the experience and share in the knowledge and thoughts, not only of those few persons whom we may chance to meet, but of men of all times and all places who have given us, in their writings, a record of their intellectual work. It is often an advantage, too, to record our own thoughts and discoveries for future reference; and, thus, written language is an aid to the development of our own thought as well as a means for communicating it to others.

*3. Ambiguities of Language.

Indeterminateness in the meaning of words leads to confusion of thought and misunderstanding.

It has often been pointed out that the exact signification of a word depends on the context, and that this flexibility of meaning is necessary if words are to express ideas equally fluctuating. But it is evident that this indeterminateness may be a frequent cause of confusion. For, not only may a word be understood in a slightly different sense from that in which it is employed, but the identity of the word may cause the person using it to think his idea remains the same when it may have undergone even considerable modification. It is not meant that words have *no* fixed meaning—they have a

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definite kernel of meaning, (which in names is called their connotation, *see* § 28) which is fixed for the time, though even this may change gradually—but that the full force of the word is only grasped when the context is known.

But, in addition to this necessary modification of the fixed kernel of meaning which the shades of thought render necessary, there are, in all languages, other causes of ambiguity.

The same word used to express entirely different ideas is not likely to lead to error.

The use of the same verbal symbol to express entirely different ideas, (often due to derivation from quite different roots), though it may give rise to puns, probably never causes any real misconception. No one is likely to confound *rein* with *reign*, because they have the same sound; nor the noun *tear* with the verb spelled in the same way but meaning to rend. Not even when spelling and pronunciation both agree is confusion likely to be caused. No hesitation could possibly be felt on reading the word *vice* in a passage as to whether a moral fault, or an instrument for holding things firmly, was referred to. These are only apparent ambiguities, but they show plainly how much the meaning may, in extreme cases, depend on the context.

Synonyms may cause confusion.

The presence of synonyms in a language often leads to delicate shades of meaning being overlooked or confused, so that the idea conveyed is not exactly the same as that intended. This richness of vocabulary frequently leads to confusion of thought in another way. People often think they understand a thing merely because they can give it two names, each of which they use to define the other. Thus 'Truthfulness is veracity'; but 'What is veracity?'—'Truthfulness.' So thought often remains nebulous when it is believed to be definite and distinct.

Living languages both grow and decay.

Every living language is subject to processes of growth and of decay. New words are invented to express new ideas, and, whilst new, they are generally the most definite in meaning of all the words in the vocabulary. At the same time old words drop out of use or undergo a gradual modification of meaning. In this last process we have a fertile source of ambiguity and confusion. Such a gradual change of meaning may be brought about by the word being either gene-

ralized or specialized, or by an alteration in the way in which the idea it represents is generally regarded. The word *virtue* is an instance of the last. It bore a very different meaning in the mouth of a pagan philosopher from that which it bears when uttered by a Christian moralist. Not only did it formerly include elements, as pride, which would now be regarded as utterly repugnant to it, but it excluded others, as humility, which form an important part of the modern notion.

The meaning of a word may change because the complex idea has changed.

Generalization occurs when the same word is extended to cover different ideas not before included under it. This frequently happens when something new has to be named; the tendency is to give it the name of that familiar object which it most nearly resembles. So it may happen that, after several such extensions, the same verbal symbol represents ideas or things which have little or nothing in common. We may take the word *court* as an instance. It originally denoted what we now call a court-yard. From that, the name was transferred to the palace to which the yard was attached, then to the inhabitants of that palace who surrounded the sovereign. From this use of the noun was formed the verb *to court*, meaning to practise the arts in vogue at court; soon this was generalized to cover all cases of seeking favour. Finally, it has been specialized, in one of its meanings, into denoting seeking in marriage.

Generalization extends the ... tion of words and so lessens their fixed meaning, and thus allows the same word to have different senses.

The transfer of names by analogy to objects which bear a real or fancied resemblance to those to which they first belonged is another example of the generalizing process. Thus, sounds are called *sweet*, and griefs *bitter*, on the analogy of tastes. In fact, all our higher mental pleasures and pains are described by words taken by analogy from the physical world; we speak of a *sharp* pain, a *light* heart, a *heavy* trouble.

Names are transferred by analogy.

The word *oil* is a good instance of generalization. It originally meant, as the Latin name, *oleum*, shows, olive-oil only; but its application has been gradually extended, till it is now used to denote many substances, animal, vegetable, and mineral, which resemble the original 'oil' in some qualities. In such a case as this, too, the name decreases in

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fixed meaning, and depends more and more on the context, though its various senses do not differ so much as in such a word as 'court.'

Some cases of generalization lead to confusion.

None of the above examples can be regarded as likely causes of confusion in thought, or of misunderstanding; but all cases of generalization are not so harmless. The word *law* may be cited as an instance where confusion has arisen and has led to much error and controversy. Its original meaning was the command of a superior, and this is still its signification in Theology and Politics. As such a command led to uniformity of conduct in some particular on the part of the subjects, the word 'law' was generalized so as to cover all cases of uniformity in the occurrence of phenomena. Thus arose the term 'Law of Nature.' But from this use of the word the idea grew up that a Law of Nature meant something more than a mere uniformity; and, thus, confusion of thought was caused by this ambiguity of language and led to much fruitless controversy.

Words with no definitely fixed meaning are unsuited for use as scientific terms.

In many cases words which have been generalized have a meaning so indeterminate and fluctuating that they may call up very different ideas in different minds, or in the same mind at different times. Such terms are particularly unsuited to scientific discussion, and, when they are used in it, they generally lead to misunderstanding and dispute. Political Economy is the most striking example of this; the use of such ambiguous words as 'capital,' 'wealth,' 'rent,' 'labour,' etc., has led, not only to endless arguments, but to contradictions and errors through writers using the words in varying senses and assuming what is true in one meaning to be true in all.

Specialization restricts the application of a word, but increases its meaning.

The opposite process of *Specialization* is due to an occasional meaning being gradually imposed on the general meaning, and, perhaps, gradually substituted for it. If a word is often applied in a special manner, that which was merely an occasional part of its meaning may become an essential part. Thus *fowl* meant originally any bird, but is now restricted to one particular domesticated species. *Vitriol* originally denoted any crystalline body with a certain

degree of transparency, but is now restricted to one or two such substances, and its fixed meaning embraces many qualities besides the two originally implied by the name. Similarly, most of the ecclesiastical terms used in the Christian Church have attained their present signification by a process of specialization; for instance, a *bishop* was originally any overseer; a *priest* was an elder; a *deacon* an administrator.

The same word may be, in turn, generalized and specialized.

These processes are continually going on side by side, and very often the same word is subjected to both in turn. *Pagan* is a good instance of this. Originally it denoted a villager. Then, when Christianity spread through the Roman Empire, and the old heathen faith lingered in the country districts long after it had practically disappeared from the towns and cities, the name became associated with heathenism, and so was specialized. Gradually this became the most important part of the meaning of the word, and then generalization became easy, and the word was used to denote *any* heathen, the original signification being entirely forgotten.

Lang should be continually checked by reference to the ideas it expresses.

Thus, we see that though language is indispensable to thought, yet it may sometimes lead to confusion and mistake. The remedy is to continually check the symbolic use of language by a reference to, and examination of, the ideas underlying it.

CHAPTER II.

DEFINITION AND SCOPE OF LOGIC.

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*4. Origin of Logic.

A Science of Logic is possible because men can examine their thoughts.

Although men possess the power of thought, they do not always employ that power so as to fulfil the great object of thought—the ascertainment of truth. The action of men's minds is not infallible; and, thus, false judgments are formed, or false inferences drawn even from true judgments. In other words, men reason sometimes well, sometimes ill. But, they are not only able to think and reason about external objects, they can reflect on those thoughts and reasonings. Now, as false reasoning generally leads to conclusions which are seen to be erroneous because they are rejected by others, by comparing the mental processes which led to the untrue results with those which, at other times, led to true results, the *reasons* why the former processes were invalid, and the latter valid, become manifest; and thus general principles are discovered to which thought conforms whenever it is valid. The collection of these principles into a systematic whole forms the Science of Logic.

5. Definition of Logic.

Logic is the science of the principles which regulate valid thought.

Logic is the science of the principles which regulate valid thought.

A *Science* is, in all cases, a systematic body of knowledge relating to some particular subject-matter. Knowledge of isolated facts is not science—it can only become so when such isolated facts are brought under general laws forming part of a consistent whole. The subject-matter of each science

is some definite part of the material of human knowledge; of Algebra it is the relations and properties of numbers, of Botany, vegetable life, and of Logic, thought.

A *Principle* (or Law) is the statement of a general truth; that is, a truth which holds good universally in that science, as contrasted with a particular truth, which holds good in some cases only. Thus, it is a principle in Physics that all material bodies attract each other in direct proportion to their mass and in inverse proportion to the square of their distance, and this principle we call the Law of Gravitation; but, that metals sink in water is not such a general truth, for it does not hold true in all cases.

Thought is used to denote both the process and the product of thinking. Logic is concerned primarily with the validity of the process, that is, of reasoning. But it also takes account of that of the products; of the concept, whether it is true or in agreement with reality; and of the conclusion of an argument, which is expressed in a judgment, whether it is consistent with itself, and whether it expresses the relations existing between the things concerning which it is made.

Thought is *valid*, in the narrower sense, when it does not involve self-contradiction in any one of its processes. In the wider and truer sense, it is valid when it agrees with the objective world—when things are thought of as holding that relation to each other which they really do hold. And such thought is knowledge, and, therefore, our definition is equivalent to saying that 'Logic is the science of the method of knowledge.' But validity of thought in this wider sense also resolves itself into consistency; for, as we saw in § 1, reality is for us just that mode of thinking our experience which is forced upon us by the attempt to make that experience consistent. This wider validity is, therefore, a wider consistency; a consistency not limited to any one process of thought, but embracing the whole of experience and the whole of mental life. Valid thought in the wider sense is that which constitutes the world for us as a consistent and systematic universe.

According to the view of validity taken by different writers on Logic the scope of the science has been enlarged or contracted [*see* §§ 8, 9, and 10]. As thought may be invalid, it follows that the principles of Logic do not *regulate* all thought, as the law of gravitation applies to all material bodies. Invalid thought only becomes indirectly part of the subject-matter of Logic, when the cause of the invalidity—or Fallacy—which is always, of necessity, a violation of one of the principles of valid thought, is investigated.

6. Is Logic a Science or an Art?

Much dispute has arisen on the question whether Logic is a Science, an Art, or both. The writers of the *Port Royal Logic* called it 'The Art of Thinking,' and were followed in this by Aldrich and others. Mansel and Thomson, on the other hand, denied that it is an art at all, whilst Whately combined the two views, and defined it as 'The Art and Science of Reasoning.' Mill agreed that it is both an Art and a Science, and it was called both by ancient Greek writers.

Logic is a Science and not an Art, though it has a practical as well as a theoretical side.

There is no doubt that it is a *Science*, as it is an organized system of knowledge (*see* § 5). Whether we call it an *Art* depends on our use of that term. That Logic has a practical as well as a theoretical side can hardly be denied; for, by the very fact of laying down the principles of valid thought, it furnishes rules for avoiding and detecting false reasoning; and it provides, moreover, principles for investigating the relations between things. Thus, the scholastic writers distinguished between the *Logica docens*, or purely theoretical part, and the *Logica utens*, or practical application of the former. But here we have a *Practical Science*, not an Art in the strict sense of the term. Rightly understood, an *Art* is a body of precepts for performing some work, and is, thus, not limited to one object-matter; for instance, the art of music involves a knowledge of musical instruments, of the human voice and its management, etc., in addition to the knowledge of musical theory, which last alone can be called Science. Logic is not an art in this sense; if it were it would, of necessity, be the widest of the arts—*Ars Artium*, as it has, indeed, been called—and would embrace special rules for reasoning in every branch of knowledge. This it does not do; its principles, and the rules drawn from them, are quite general. It does not even profess to teach men to reason accurately; it only gives

the principles and rules to which accurate reasoning conforms. A man may be able to arrive at true conclusions and yet have never learned Logic; and, on the other hand, one may know all logical principles and yet reason falsely.

It is true that, if thought be guided by logical rules and principles, the result will be valid reasoning, but this is not the object for which the principles and rules are stated; they are rather general results arrived at from an examination of valid processes of thought.

7. Uses of Logic.

Logic is a guard against error in reasoning,

One important use of Logic follows from what has been said above. Though it does not make false reasoning impossible, it does furnish rules and principles by which error can be detected and, therefore, avoided. By bringing an argument to the test of Logic we can ascertain whether or not reason has, in that instance, been employed rightly; and, if not, what was the cause and origin of the error. This is by no means unimportant; a man may feel sure that a conclusion is false, and yet be unable to say *why* it is so. This is frequently the case with men of 'sound common-sense,' who often see at once that an argument is invalid, but are unable to point out where the fallacy lies; in which case there is, of course, no guarantee against committing the same error in reasoning again, when, perhaps, the conclusion may not be so obviously at fault. But, if the source of the fallacy can be traced, there is hope that similar mistakes will be avoided in the future.

and the best mental discipline.

But the chief use of Logic is found in the fact that it is pre-eminently a mental discipline; and to train the mind should be the one great object of all study. One's object should never be so much to acquire knowledge of various facts and sciences, as to develop and perfect the reasoning powers. As Hamilton said, 'In the world there is nothing great but man, and in man there is nothing great but mind;' and, truly, a well-balanced and evenly-developed mind is the noblest possession man can enjoy. And a man who, by a study of Logic, has trained his mind to reason justly, will reap the advantage in every department of study or practice to which he may devote himself.

8. The Divisions of Logic.

Judgment is the essential form of thought.

The aim of thought is to arrive at knowledge, and knowledge, as we have seen (*see* § 1) is the mental assertion of what we are constrained to regard as true. In other words,

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the essential form of thought is judgment, and every judgment is a partial interpretation of reality. The simplest form of judgment is an interpretation of an isolated fact of perception, as when we say 'It rains,' or, 'This is a rose.' But as thought advances, judgments become more and more complex, and involve a wider and wider range of reality. Indeed, the ideally perfect judgment would embrace and interpret all reality. That judgment, of course, we cannot make; it would involve complete and perfect knowledge. But every actual judgment makes an assertion of some kind about reality.

We may analyse judgments into *Concepts*,

It is plain that in judging we both analyse and construct. We analyse, in that we decompose complex reality into elements standing to each other in certain relations, such as substance, identity, space, time, causality, etc.; and we synthesize in that we construct each of these elements out of a complex of such relations. These elements of reality are thought as concepts, and our concepts are true in so far as they express the nature of reality. As every actual judgment asserts some relation between elements of reality thus mentally held apart, it is obvious that we may regard judgment as the assertion of a relation between two concepts. But it must be kept in mind that this is only half the truth, the other half being that the whole judgment itself is one act of thought, refers to one single aspect—though it may be a complex one—of reality, and may itself be included in one concept richer than either of those, regarded separately, which it connects.

and combine them into *Inferences*.

But judgment is in no case a simple affirmation of perception; it always involves more or less interpretation. Now, interpretation is inference. This is implicit in most cases of direct perception; for instance, I inhale a certain odour, and say, 'There are some violets near.' But when the grounds of the interpretation are clearly set forth, we have that explicit inference with which alone logic is concerned. Now such explicit inference obviously involves a synthesis of judgments, for each of the grounds of the inference is expressed in a judgment. These propositions are called the premises, and the further judgment of which they are the ground, and which

follows necessarily from them, is called the conclusion. When we have only one premise, the only inference possible is interpretative, and renders explicit what was implicit in the original proposition. Such inferences are called *Immediate.* But when we have two premises the inference is called *Mediate,* because the conclusion is only possible through the union of two judgments dealing with one common element of reality. All cases of Mediate Inference which involve more than two premises can always be analysed into series of steps of inference, in each of which only two premises are concerned. Mediate Inference is commonly said to be of two kinds, according as we approach the aspect of reality to be interpreted from the side of some established general principle, or from that of individual facts. In the former case the inference is called *Deductive,* and in the latter case *Inductive.* The relation of these to each other will be more fully considered later on [*see* § 146 (iii.)].

Inferences may be (1) Immediate. (2) Mediate. (a) Deductive. (b) Inductive.

Lastly, inferences are not isolated, but proceed in trains towards some definite end. The orderly arrangement in discourse of such trains of inferences, is named *Method.* Here also the one fundamental aim is the unification of experience, and the more perfect comprehension of the universe.

***Method* is the orderly arrangement of inferences.**

It thus appears that the four commonly accepted divisions of logical doctrine—Conception, Judgment, Inference, Method—are rather different aspects under which we may regard the one fundamental act, always in its essence the same, of interpretation of experience by thought. When we regard them from the view of language the distinction between them is more marked, and appears as one of varying complexity. The verbal expression of a judgment is a *proposition,* and in such expression the two concepts into which the judgment can be analysed are necessarily made distinct and represented by separate *terms.* Indeed so distinctly are the terms held separate that there is always a danger of being misled into forgetting the fundamental unity of the judgment itself. Again, when an inference is expressed in language the three judgments into which it can be analysed stand out distinct and separate, and there is a similar danger of forgetting that

The four divisions of logical doctrine, (1) Conception, (2) Judgment, (3) Inference, (4) Method, are only aspects of one fundamental process.

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an inference also is a single act of thought referring to a single aspect of reality. Similar remarks apply to trains of inferences.

(i.) Views as to Conception.

Different views as to the nature of a concept.

Very different views have been held by philosophers as to the nature of General Notions or Concepts. These opinions may be divided, broadly, into three classes: Realism, Nominalism and Conceptualism.

(a) *Realism*—that the universal had a real objective existence.

(a) *The Realists* held that some real substance existed in nature corresponding to every general notion, which combined its constituent properties; that, for instance, corresponding to the concept 'horse,' there existed something which was no horse in particular, but in which every individual horse participated—a universal horse which consisted as it were of horse-essence, and which was the only *real* horse existence. This school of thought, to which many of the Schoolmen belonged, is now quite obsolete.

(b) *Nominalism*—that the name is the only general, and is represented by images.

(b) *The Nominalists* go to the other extreme and hold that General Notions are mere matters of words; that a class is constituted by its name alone, and that the name is the only general element. They hold that, every time a General Name [*see* § 27 (ii.)] is used, an image is present to the mind. They say that the class is thought of either under the image of one individual member of it, with a kind of mental reservation that the particular attributes of this representative are to be disregarded, or else by a rapid succession of images of various members of the class. On this view, it is difficult to see how we could have any concept of abstract qualities, such as truth and justice, for of such we can certainly form no images. Hobbes, Berkeley, and Prof. Bain may be cited as advocates of this view.

J. S. Mill held a modified view which made some approach to Conceptualism. He says, "We have a concrete representation, "certain of the component elements of which are distinguished by a "mark [*i.e.*, the class name], designating them for special attention; "and this attention, in cases of exceptional intensity, excludes all "consciousness of the others" (*Exam. of Hamilton*, p. 323. But he adds, "there is always present a concrete idea or image, of which "the attributes comprehended in the concept are only, and cannot "be conceived as anything but, a part" (*ibid.*, p. 337). He, therefore, held that a concept is a mere generic image.

(*c*) *The Conceptualists*, to which school most modern writers, including Kant, Mansel, Dr. Ward, and Mr. Stout, belong, hold that a concept involves no image. It is an intelligible, not a sensible, synthesis of attributes [*cf.* § 2 (ii.)], and thought is constantly carried on by means of concepts without the accompaniment of any images, whether particular or generic, except those auditory or motor images of the names which symbolize the concepts of which the train of thought is composed.

(*c*) *Conceptualism*—that a concept is an intelligible synthesis of attributes.

(ii.) Views as to Judgment.

Equally widely divergent views are held as to the nature of a judgment, and these views lead to differences of opinion as to the validity of thought with which Logic has to deal (*see* § 5), and hence as to the scope of the science. There are three chief schools :—

Different views as to the nature of a judgment.

(*a*) *The Nominalists* hold that propositions are merely statements about names, and that the whole scope of Logic is bounded by names and their relations. Whately is a representative of this school. He limits the science to a regard for mere verbal consistency; but, on the other hand, Mill and Prof. Bain, though Nominalists in their views of the concept, yet take the Objective view of the scope of Logic.

(*a*) *The Nominalist*—that judgments are about names only.

(*b*) *The Conceptualists* regard Logic as concerned not with language but with the thought it represents. The chief representatives of this school are Kant, Mansel and Thomson. They define Logic as 'the science of the pure (or formal) laws of thought,' or as 'the science of thought as thought,' meaning by this, of thought entirely separated from, and independent of, the things thought about. The most extreme and consistent writers of this school (as Mansel) hold that all which can be expressed by a judgment is that one concept is contained in, or forms part of, another, so that no judgment can ever do more than unfold and make explicit the content of a concept; it can never be a statement involving additional information. Logic, from this point of view, is a mere 'Logic of Consistency' (as Hamilton called it); it can have no concern with the real relations between things. In other words, it takes the narrower view of validity of thought (*see* § 5).

(*b*) *The Conceptualist*—judgments are about concepts only.

It will be seen that the words 'Nominalist' and 'Conceptualist are ambiguous. They may refer to the views held of a concept, or to the views held as to the nature of a judgment. It is quite possible for a Nominalist or a Conceptualist in the former sense to hold

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the Objective view of Logic, as, in fact, most modern writers do, in a more or less modified form.

(c) The *Objective*—that judgments are about things only.

(c) Those holding the *Objective* or *Material* view of Logic give it a much greater scope, for they take the wider view as to the validity of thought (*see* § 5). Some, indeed, would make it coincident with the whole realm of reality. They hold that propositions do not express relations between mental concepts, but between the things those concepts represent. If, for instance, we say 'Grass is green,' we do not mean to say that our concept of grass contains, or agrees with, our concept of green, but that the *thing* grass possesses the attribute of greenness. Mill takes this view. He defines Logic, in his *Examination of Hamilton* (p. 388), as "the Art of Thinking, "which means of correct thinking, and the Science of the Conditions "of correct thinking" (p. 391); and by 'correct thinking' he explains himself to mean thought which agrees with the reality of things (pp. 397-8). In his *Logic* he adopts the definition, "Logic "is the science of the operations of the understanding which are "subservient to the estimation of evidence" (*Logic*, Introd., § 7), which clearly makes the science conversant with reality; and a material treatment is adopted throughout the entire work.

Whewell takes a similar view. He says, "The Logic of Induction "is the Criterion of Truth inferred from Facts, as the Logic of "Deduction is the Criterion of Truth deduced from necessary prin-"ciples" (*Novum Organon Renovatum*, p. 98).

Mr. H. Spencer takes an extreme view, and regards all reference to thought as of quite minor importance. He defines Logic as the Science which "formulates the most general laws of correlations "amongst existences considered as objective" (*Principles of Psychology*, vol. ii., § 302). He further says, "The propositions of Logic "primarily express necessary dependencies of things and not neces-"sary dependencies of thought; and, in so far as they express "necessary dependencies of thought, they do this secondarily— "they do it in so far as the dependencies of thought have been "moulded into correspondence with the dependencies of things" (*ibid.*).

G. H. Lewes draws a distinction between Subjective and Objective Logic. The former "is occupied solely with the codification "of the processes of Proof," whilst the latter is synonymous with Metaphysics, and is concerned with "the codification of the most "abstract laws of Cause" (*Problems of Life and Mind*, vol. i., p. 75).

(iii.) Remarks on Inference.

Of course, writers who hold that logic is only concerned with the formal self-consistency of isolated processes of thought, deny that it can really treat of inductive inferences at all, for all such inferences are essentially material. Such logicians, therefore, confine the science to deductive inference and to these so-called 'Perfect Inductions' which consist of a mere summing-up of individual judgments of perception.

Deduction will be discussed before Induction.

As Induction furnishes many of the general propositions which are the bases of deductive reasonings, it would seem natural to treat it before Deduction. But this branch of the subject has only been treated fully in comparatively recent times. For a long time Deduction was looked upon as synonymous with Logic. Thus, it is customary to treat Deduction first, and the usual plan will be followed here; because the limits of Deduction are more clearly defined, because it is simpler, and because a knowledge of its principles is necessary to the understanding of Induction.

The terms 'Deductive' and 'Inductive' apply to Inference only.

It may be pointed out that the whole doctrine of Concepts (Terms), Judgments (Propositions), Immediate Inferences and Deductive Mediate Inferences, is frequently spoken of, somewhat loosely, as 'Deductive Logic,' whilst Inductive Inference is called 'Inductive Logic.' It seems better and more accurate to restrict the terms 'Deductive' and 'Inductive' to Inference, to which alone they rightly belong.

9. View of Logic here adopted.

Logic is concerned with reality as presented in thought.

It follows from what has been said in previous sections (*cf.* §§ 5 and 8) that we regard Logic as dealing not with processes of merely abstract and symbolic thought, nor with mere processes of an external reality out of all necessary relation to thought, but with reality as *known*, *i.e.*, as interpreted by thought. The growth of inference depends on the difference between knowledge and ignorance; were all reality known, there would be no room left for fresh inference; were all reality unknown, inference could not begin, for it would have no starting point. The possibility of inference is found in the fact that the world is a rational and systematic unity and can, therefore, be understood—partially at any rate—by a mind which is itself a rational unity. With

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the raw material of knowledge — sensations and sense-impressions—Logic does not deal; with the unconscious inference involved in perception it only deals indirectly and in so far as the process is, at bottom, one with the explicit inference with which it does deal. It is with the conscious judgments and inferences by which rational mind interprets sensuous experience, that Logic is concerned. We cannot say, then, that Logic is either purely subjective or purely objective; indeed subjective and objective are only aspects distinguishable in thought of that reality which exists for us only as embracing them both. We do not, then, restrict Logic to that merely formal and barren validity of thought which consists in the absence of self-contradiction in each of its processes, regarded in isolation from the rest; we hold that it must take that wider test of validity which is found in a complete and consistent system, and which tries every process of thought by reference to that system.

We shall speak of Terms and Propositions or of Concepts and Judgments according to the shade of meaning to be expressed.

As to the nomenclature which will be adopted it may be observed that there seems no good reason for adhering strictly to the language either of the Nominalists or of the Conceptualists. We shall very frequently speak of Terms and Propositions, as the reference will, in many cases, be more especially to the verbal expression of thought. But we shall feel at perfect liberty to use the terms Concept and Judgment, when they seem to be the more appropriate; that is, when the reference is chiefly to the mental idea or process.

10. Pure, or Formal, and Material, or Applied, Logic.

Formal Logic is concerned with the self-consistency of thought; *Material Logic* with its objective truth.

Though Logic takes note of the wider sense in which thought can be said to be valid, it does not lose sight of the narrower sense (*see* § 5). It must furnish principles to test the consistency of thought, or it will be useless as an instrument to determine the worth of thought when exercised on reality. These principles compose that *Pure* or *Formal Logic*, or *Logic of Consistency*, which the Conceptualists regard as forming the whole of the science [*see* § 8 (ii.) (*b*)]. When we apply Logic to the investigation of Objective reality, we are in the domain of *Material* or *Applied Logic.*

All Induction is, of necessity, material, for its end is to determine the actual truth or falsity of propositions about things. A great deal of the subject-matter of Book I is also material; for instance, the doctrines of Definition, Connotation of Terms, Predicables, Categories, and, to a great extent, Division and Classification. The validity of a Concept—of which the Term is the verbal symbol—is, of course, a material question. As the doctrine of Propositions (Bk. II) deals to a great extent with the form in which the thought is expressed, it is largely formal; but in so far as reference is involved to the reality of which the judgment is made, as in considering the Import of Propositions (Bk. II, Ch. II), it is material. Most of Deductive Reasoning (Bks. III and IV) can be treated formally; for its purpose is to determine the relative truth of propositions; that is, what propositions can be inferred from others; and this relative truth depends solely on the form of the argument, and is entirely independent of the matter.

Deductive Reasoning and much of the doctrine of Propositions is Formal; the rest of the Science is Material.

By the *matter* of thought is meant the thing or things thought about; by its *form*, the way in which the mind thinks about them. The matter may vary whilst the form remains the same. In a similar way, many pieces of music may be written in the same measure and have the same rhythm, though the series of notes or chords may vary infinitely; the rhythm or measure is the form, the notes or chords are the matter. On the other hand, the same thought may be expressed in different ways; that is, the form may vary whilst the matter remains unchanged. So, to revert to our illustration, the same series of chords or notes may be adapted to several different rhythms.

***Matter of thought* is the thing thought about; *Form of thought* is the way the matter is considered.**

As the validity of formal reasoning depends on the form alone, we may express our terms in symbols; and there is a great advantage in doing this, as the attention is thus fixed solely on the form, and we are not led to think a piece of reasoning is necessarily correct because the conclusion which has been drawn is, as a matter of fact, true. If we say, for instance,

Formal Reasoning may be expressed symbolically.

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All metals are fusible
Some substances are not metals
∴ *Some substances are not fusible*

we assert, as a conclusion, a proposition which is undoubtedly true; but the argument is invalid, for that proposition does not follow from the premises. Our purpose here is to decide, not as to the objective truth of the conclusion but, as to the validity of the reasoning. Had we written the premises symbolically, thus

All M's are P
Some S's are not M

we should have been more likely to examine the argument, as we should have no prejudice in favour of the conclusion *Some S's are not P*, as we have in favour of *Some substances are not fusible*. And such an examination would show that conclusion to be unjustifiable by the premises. For they do not assert that nothing is *P* except the *M's*, and, therefore, we do not know that the *some S's* which are not *M's* are, in consequence, not *P's*. In such a symbolic manner all propositions may be written in formal reasoning, where *S*, *M*, and *P*, stand for any matter whatever; and the validity of propositions and arguments so expressed can, evidently, depend on their consistency alone. Were we to assert that *No S is S*, the proposition would be formally invalid, for it is self-contradictory; or, if we have accepted as true the proposition *No S is P*, we should fall into formal contradiction were we to assert that *Some S is P*, whilst *S* and *P* remain unchanged in their reference. When we pass from propositions to arguments the same thing holds. If we assert

Every M is P
Every S is M

we cannot avoid the conclusion that *Every S is P* without self-contradiction. Of course, we may give such meanings to *S*, *M* and *P* that the conclusion is materially false, but this can only be the case when material error is to be found in one or both the premises. If we say

All volcanoes are mountains
All geysers are volcanoes

and draw the conclusion *All geysers are mountains*, our argument is formally valid; for, if expressed in symbols, it is identical with the one just considered; but the conclusion is materially false because the premises are false. But, to establish the truth of the premises is the province of Induction; Deduction simply furnishes principles and rules for drawing consistent conclusions from premises which are given us.

Logic must be both Formal and Material; and must embrace both Deduction and Induction.

The consideration of such examples as these shows the necessity for embracing both Formal and Material Logic in our science, if we would use it as an instrument for attaining a knowledge of truth. It may be thought, and has, indeed, been said, that Deduction is useless, and that Logic should be Inductive only. This is not the case, for when once a general proposition has been arrived at by Induction, Deduction enables us to apply it to numerous cases which were not before known to be instances of it; and, thus, the use of Deduction dispenses with the need of innumerable new Inductions. For example, when the general proposition which sums up the Law of Gravitation—that all material bodies attract each other in direct proportion to their mass and in inverse proportion to the square of their distance—was arrived at by Induction, it was immediately applied to explain not only the fall of unsupported bodies to the earth, but the motions of the planets and their satellites, the occurrence of tides, and many other phenomena which were not previously suspected to be instances of the same law of nature. Induction and Deduction are not so much two mutually helpful processes as two aspects of one process.

CHAPTER III.

RELATION OF LOGIC TO OTHER SCIENCES.

11. General Relation of Logic to other Sciences.

The principles of Logic must regulate thought in all branches of knowledge.

Logic has often been called the 'Science of Sciences,' because it treats of those regulative principles of thought to which, however various may be their methods, all branches of knowledge must conform. Logic does not profess to furnish rules or means of investigating any particular branch of science; its province is purely general, and is confined to that common basis of all science—the laws which must be universally observed by all valid thought. Logic has, thus, not simply an absolute value, as scientifically an end in itself; but, through the influence which as the science of thinking it exerts upon the process of thinking, a relative value also. For the very enunciation and examination of the regulative principles of thought further their practical application, since they are certain to be more fully and exactly employed by those who are scientifically conscious of them than by those who reason by the simple light of nature. As De Morgan says: "I maintain that logic tends to make "the power of reason over the unusual and unfamiliar more "nearly equal to the power over the usual and familiar than "it would otherwise be" (*Budget of Paradoxes*, p. 330). Logic also aids scientific investigation by pointing out the most appropriate procedure for arriving at conclusions from the premises with which observation has furnished us. This practical value of Logic has earned for it the name *Ars Artium*—the Art of Arts—as well as that of 'Science of Sciences' (*cf.* § 6).

Though thus related to all sciences, yet Logic has closest relations with those sciences which treat of Being, of Mind, and of Language, for it investigates thoughts about things expressed in speech. We will, then, consider more at length its connexion with Metaphysics, with Psychology, with Rhetoric, and with Grammar.

Logic has the closest relations with Metaphysics, Psychology, Rhetoric, and Grammar.

12. Logic and Metaphysics.

Metaphysics investigates the nature of Reality.

Metaphysics enquires into the nature of Reality as such. It thus goes beyond the various sciences, each of which deals with some branch of the phenomena or appearances in which Reality appeals to our senses. Each science makes certain assumptions; *e.g.* that matter has some kind of existence, that 'things' are constant in their nature, and exist in space and time, that the changes observed in the world are not random accidents but are regular and connected together in causal relations. It then goes on to ask *what* causal uniformities are to be found in its special province, and what uniformities of nature can be discovered in the 'things' with which it deals, its aim being to establish a connected body of doctrine concerning one portion of the contents of human experience. Every science is thus only a partial description of the phenomenal world, and rests on assumptions which it does not verify. The investigation of the validity of these assumptions is the province of Metaphysics. Metaphysics, therefore, deals with the presuppositions which underlie all experience, and these presuppositions it tries to arrange into a system by showing that they are necessary deductions from one ultimate first principle. Hence, Metaphysics does not aim at knowing all things, but at explaining all knowledge by making explicit the very forms of all existence.

Logic leaves its ultimate assumptions to Metaphysics.

Logic holds a sort of intermediate relation between Metaphysics and the special sciences. For Logic aims at knowing the process by whch knowledge is attained. It assumes that there is an absolute standard of truth, and that our thought can grasp, at least in part, the true nature of Reality, or, in other words, that our mental construction of the world is

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Logic accepts certain metaphysical postulates.

neither a mere mechanical copy of the phenomena around us, nor an arbitrary synthesis of ideas evolved from our inner consciousness. In brief, it assumes that experience can be analysed and known. It is thus wider than any of the special sciences, as it systematizes the formal conditions of all knowing. But it must hand over all these ultimate assumptions to Metaphysics. Indeed that very relation of thought to phenomena which forms experience, and which Logic accepts as given, is itself the very central problem of Metaphysics. Further, Logic accepts without question the assumptions of the special sciences as to the existence and relations of 'things.' Its province is to state these principles exactly and definitely; it must leave to Metaphysics the question as to their validity.

13. Logic and Psychology.

Logic is regulative and ideal; Psychology is empirical and actual.

Psychology is the science which investigates the actual phenomena of the mind and their development. It is wider than Logic in that it takes account not only of thought, but of all mental processes; though, in that all its investigations must be conducted in accordance with logical principles it is narrower than Logic, which, as has been said (*see* § 11), analyses the methods of all sciences. Psychology is essentially *empirical:* it enquires into the genesis and character of all mental activities—whether of thought, of feeling, or of willing—and their relation to each other. It investigates the mental processes subsidiary to thought and the nature of thinking, turning its attention in all to what actually occurs in mind. It seeks to arrange its results as uniformities, and to deduce therefrom knowledge of the way in which, in reality, concepts are formed, judgments made, and inferences carried out, on what conditions mental states depend and what is the nature of those states.

Logic, on the other hand, is *normative;* it furnishes criteria by which false reasoning may be discriminated from true. It does not enquire how men *do* think, but lays down laws in accordance with which they *should* think; it is *ideal* whilst Psychology is *actual.* Logic does not take account of

all the ways in which men reach conclusions; it does not enquire how ideas are recalled by the laws of association, or how belief arises from such association; of the actual process of reasoning it takes no account. It is concerned with reasonings only in respect to their validity; with the dependence of one judgment on another only so far as it is a dependence of proof. Given certain laws, it determines the form correct thinking ought to exhibit, but does not enquire whether men's actual thoughts do conform to that standard.

Though the provinces of Psychology and Logic are thus distinct, yet the latter can only be satisfactorily studied in connexion with the former. To thoroughly understand Logic it is necessary to know what is the nature of the thinking mind, what are its limitations, what is the character of the process of thought, and how it unites with the other mental elements to form those concepts and judgments which are the materials with which Logic deals.

14. Logic and Rhetoric.

Logic deals with proof; Rhetoric with persuasion.

Logic is connected with Rhetoric in that both have a common object—to lead to the formation of certain conclusions. But they proceed about this in very different ways: Logic appeals to the reasoning faculty alone, whilst Rhetoric rather aims at stirring up the emotions; the former attempts to convince, the latter, by an appeal to the passions, to persuade. Whilst it is true that such an appeal will be more powerful if at the same time it is based on valid arguments, yet by an adroit flattery of men's prejudices it is often found possible to instil an opinion which not only is supported by no sound reasoning, but is actually repugnant to it. Rhetoric is connected with Psychology in so far as the latter deals with the emotional side of mind, whilst Logic touches it where it treats of the intellectual or thinking side. The province of the science of Rhetoric is to investigate the principles on which discourse should be founded in order that it may be *persuasive*—its main ends are, not the ascertainment of truth but, the enforcement of conclusions without regard to their validity, and the incitement to action; Logic, on the other hand, as dealing with the relation of truths, investigates the principles on which discourse should be founded in order that it may be *convincing*, and its main end is the ascertainment of truth.

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*15. Logic and Grammar.

Logic is related to universal—not to particular—Grammar.

Logic and Grammar are connected through the medium of language, which is the general instrument of thought. In considering their relation we must, of course, have regard only to Universal Grammar; for the history and idiomatic peculiarities of particular languages are obviously not directly connected with a general science like Logic.

Universal Grammar deals with the general laws of all language.

Universal or *General Grammar* is the science of the universal laws which all languages must observe. It is distinct from special grammar, which is the application of those laws to a particular language, under the influence of the habits and idiosyncrasies of a particular people. Or it may be defined as "the science of the relations which the constituent parts of speech bear to each other in significant combination" (Stoddart). Whilst particular grammar partakes largely of the nature of an art, universal grammar is a science, and is evidently connected primarily and necessarily with language, which is, indeed, its subject matter; its province is to trace the connexion between certain given signs and the thoughts they are supposed to represent.

Logic is only indirectly concerned with language.

With Logic this is reversed; it is concerned primarily with thoughts and concepts, and their relations to each other and to reality; it is concerned only secondarily with language, as the means by which these thoughts, concepts, and relations are invariably expressed. Logic thus considers language simply as the instrument of thought, and only analyses it to that point which is necessary to express the simplest element of thought—the concept. All non-significant words, *i.e.*, words which cannot by themselves express a concept, are, thus, beyond the range of Logic, whilst all words which can express a concept are regarded by it as of the same class—they can form Terms, and it is immaterial whether they are, in grammatical language, substantives, pronouns, adjectives or verbs. Logically, the form of attribution belongs to all characteristics of a subject-matter which are not self-dependent, whether they are expressed by nouns, as Cæsar was a *Roman*, by adjectives, as Cæsar was *ambitious*, or by a verb,

Logical analysis of language ends with the Term, and takes no note of Parts of Speech.

as Cæsar *conquered*. It may be pointed out that all verbs are logically reducible to the verb 'to be' and a participle, and when this analysis is made the attributive force of the verb appears plainly, as Cæsar was *conquering*. Personal pronouns are for Logic the same as substantives, demonstrative pronouns the same as adjectives, whilst, as adverbs bear exactly the same relation to verbs which adjectives do to nouns, they are not a distinct form of the content of thought. Again, Logic requires that relations should be expressed between things and between concepts, but it is immaterial to it whether those relations are expressed by inflexions of words or by a preposition. Thus, Logic takes no note of that division of words into parts of speech which is so marked a feature in the grammatical analysis of language.

Logic and Grammar treat and analyse sentences differently.

Again, though the proposition is the unit of thought-expression both in Logic and in Grammar, yet its treatment is different in the two sciences. Whilst Grammar acknowledges no sentence in which subject and predicate are not distinctly expressed, but, given that, deals with sentences of the most varied construction, Logic accepts judgments made in any form, even so rudimentary a one as the Exclamatory, but demands the power of re-stating the meaning of all in one fixed and simple logical form (*see* § 68). The logical analysis of the sentence, too, often differs from the grammatical. The grammatical subject is always the noun or pronoun in the nominative case, and is thus definitely fixed in each sentence. But there is no such fixity in the case of the logical subject, which is that known part of the experience spoken of from which the judgment starts, whilst all the rest of the sentence is the predicate. Frequently, then, only the context can determine how much of any given sentence is interpretative, *i.e.* belongs to the predicate, though the more accurately our sentences express our thoughts, the more closely do the logical and grammatical analyses agree

CHAPTER IV.

THE LAWS OF THOUGHT.

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16. General Character of the Laws.

The *Laws of Thought* are the fundamental, necessary, formal, and *à priori* forms which regulate all valid thinking.

The Laws of Thought, *Regulative Principles of Thought*, or *Postulates of Knowledge* are those fundamental, necessary, formal, and *à priori* mental laws in agreement with which all valid thought must be carried on. They are *à priori*, that is, they result directly from the processes of reason exercised upon the facts of the real world. They are *formal;* for, as the necessary laws of all thinking, they cannot, at the same time, ascertain the definite properties of any particular class of things, for it is optional whether we think of that class of things or not. They are *necessary*, for no one ever does, or can, conceive them reversed or really violate them, because no one ever accepts a contradiction which presents itself to his mind as such. It is true that fallacious reasoning is common enough, but this springs from a misapprehension of the meanings of terms, or from a confused use of terms, for which the ambiguities of language give abundant scope (*see* § 3). Especially in long and involved reasonings, the force of terms is often unconsciously modified, and even entirely changed, with the result of invalidating the chain of argument; but, at no stage of the process does the reasoner *consciously* accept a contradiction. As always really obeyed by all minds, they are *laws* in the scientific sense of uniformities; when applied practically to govern and test arguments, they are laws in that other sense of the word in which we speak of laws of the land (*see* § 3). They are Postulates of Knowledge because they are involved in all attempts at interpreting experience, *i.e.* they are assumptions without which thought cannot even begin the work of

reducing to order the chaos of sense impressions. Into the justification of these postulates Logic does not enter; that is the task of Metaphysics (*see* Bk. I, Ch. III, § 12). Logic assumes them because it finds them assumed in every piece of correct thought, and it aims at expressing them as perfectly as possible.

They are treated here, as they are necessary forms of *all* thought.

Much dispute has arisen amongst logicians as to the number and expression of the necessary laws of thought, and as to the place they should occupy in an exposition of Logic. Mill says they should, at the earliest, be placed at the beginning of the treatment of Judgment (*Exam. of Hamilton*, p. 416), and Lotze gives them the same position. Ueberweg puts them still later, at the beginning of Inference, and calls them "Principles of Inference." As, however, they are the necessary forms of *all* thought, and are, consequently, required for the full comprehension of Concepts as well as of Judgments and Reasonings, and as they also form the basis for all logical division of Terms, we prefer to treat them as introductory to our consideration of the science. Moreover, the Concept is nothing apart from the Judgment, and therefore the treatment of the Proposition really begins with the consideration of its elements —Terms.

With regard to their number, formal logicians generally recognize only three such laws of thought—the Principles of Identity, of Contradiction, and of Excluded Middle. But on the view of Logic we are advancing the Principle of Sufficient Reason must be added to these.

We will now consider, in some detail, each Principle, and the various forms in which it has been expressed.

17. The Principle of Identity.

Principle of Identity—*A is A*.

The simplest statement of this law is the formula *A is A*, or, as Leibniz put it, '*Everything is what it is.*' It has also been expressed, 'Whatever is, is,' (Jevons); 'Every object of thought is conceived as itself' (Mansel). It demands that, during any argument, we use each term in one unvaried meaning. On this principle rests the justification of the Judgment.

No difficulty can be experienced in understanding, and assenting to, such propositions as *A is A*, *B is B*. But, in

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All Identity exists amidst diversity.

such statements there is conveyed no real information. To say a thing is itself tells no more about it than does the bare mention of its name. Identity must be interpreted in such a way as to cover such propositions as *A is B*, which we are continually making, and which experience tells us are justified by facts. We say 'Gold is yellow,' 'Lions are fierce,' and such statements are capable of conveying real information. No doubt, if fully analysed, such propositions may be brought to the form *A is A*. 'Gold is yellow,' does not mean that all yellow things are gold—that is, that gold and yellow are convertible terms; nor yet that gold is any yellow, but only gold-yellow. But this analysis is not actually made in thought, nor is it necessary. Identity is really expressed in the proposition *A is B*, viz., the identity of the things to which *both* names, *A* and *B*, can be applied. But this identity is expressed amidst a diversity of meaning; the two names have not the same signification, and, hence, the proposition, in which they are conjoined, is capable of giving real information. In truth, it is only amidst some diversity that we know identity at all. I am the identical person I was ten years ago, and yet I have changed ; individual men all differ from each other in many points, yet all share in the common nature of humanity. When, then, we say *A is A* we mean that a thing remains itself even amidst change, and that a common nature is manifested in different individual instances.

This view of the Principle, from the subjective side, is brought out in the statement adopted by Archbishop Thomson (*Laws of Thought*, p. 212): "Conceptions which agree can be united in "thought, or affirmed of the same subject at the same time." Mr. Bradley regards the principle to be affirmed as that "Truth is at all "times true," or "Once true always true, once false always false," and he adopts the statement, "What is true in one context is true "in another." Or, "If any truth is stated so that a change in "events will make it false, then it is not a genuine truth at all" (*Princ. of Logic*, p. 133). Ueberweg gives an *Axiom of Consistency*, which he regards as akin to that of Identity. He expresses it, "*A which is B is B; i.e.*, every attribute which belongs

"to the subject-notion may serve as a predicate to the same," for "the attribute conceived in the content of the notion inheres in "the object conceived through the notion, and this relation of in- "herence is represented by the predicate" (*Logic*, Eng. trans., pp. 231-2). Mill's expression of the Principle (*Exam. of Hamilton*, p. 409): "Whatever is true in one form of words, is true in every "other form of words which conveys the same meaning," though an indispensable postulate is really a law of *expression* rather than of thought.

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18. The Principle of Contradiction.

This Principle, which would be better named **The Principle of non-Contradiction**, is most simply expressed by the formula *A cannot both be B and not be B.*

Principle of Contradiction—*A cannot both be B and not be B.*

The law has been enunciated in various other ways, a consideration of some of which may help in making its scope and meaning clear. Thus: *A* cannot be both *B* and *non-B; A is not non-A;* Nothing can both be and not be (Jevons); The same attribute cannot be at the same time affirmed and denied of the same subject (Aristotle); The same subject cannot have two contradictory attributes; No object can be thought under contradictory attributes (Mansel); Judgments opposed contradictorily to each other (as *A is B*, *A is not B*) cannot both be true (Ueberweg); The attribute cannot be contradictory to the subject, or, A predicate does not belong to a thing which contradicts it (Kant); What is contradictory is unthinkable (Hamilton); Denial and affirmation of the self-same judgment is wholly inadmissible (Bradley). Mill's statement, "The "affirmation of any assertion and the denial of its contradictory are "logical equivalents which it is allowable and indispensable to make "use of as mutually convertible" (*Exam. of Hamilton*, p. 414) is, again, rather a postulate referring to *expression* than a principle of thought.

On this axiom, together with that of Identity, is based all Immediate Inference from Affirmative Propositions. It denies that the same thing can, at the same time, both possess a certain attribute and not possess it; and, as thought must agree with reality, that we can conceive a thing as at once both possessing and not possessing the same attribute. The same statement cannot be, nor can we conceive it as

Contradictory propositions refer to the same subject at the same time.

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being, at the same time both true and untrue; nor can the same thing at once be strong and yet not be strong. Different parts of the same object may, of course, possess incompatible attributes; one end of a bar of iron may be hot, and the other, in common parlance, cold, but the *same* end cannot at once both be hot and not be hot to the same person; and our propositions must refer to the same end, as otherwise, not being made of identically the same subject, they would not be contradictory of each other. Similarly, the same end of the bar may at one time be hot, and, at another time not be hot; but there would be no contradiction in asserting this, for judgments referring to the same subject at different times are not the same judgment. A judgment does not change with time, but once true is always true. Contradictory judgments, therefore, must refer to identically the same subject at identically the same time; they must assert incompatible attributes as standing in the same relation (including that of time) to the same subject. Of course, there must be perfect sameness of sense both in the single terms of the contradictory propositions and in their affirmation and negation; the propositions must be contradictories not merely apparently and in words, but in reality and meaning.

Principles of Identity and Contradiction are proved indirectly.

It has been disputed whether this axiom and that of Identity are really underivable. Ueberweg thinks they can be deduced from "the idea of truth, *i.e.*, the consistency of the content of perception "and thinking with existence" (*Logic*, p. 238). Anyhow, as Aristotle said, the validity of the axioms can only be proved indirectly, viz., by showing that no one can help recognizing them in actual thinking and acting, and that, were they destroyed, all distinctions of thought and existence would perish with them.

19. The Principle of Excluded Middle.

Principle of Excluded Middle—*A either is, or is not, B.*

The Principle of Excluded Middle between two contradictory judgments is most clearly expressed by saying *A either is, or is not, B.*

Other expressions of it are: *A* is either *B* or *non-B;* Two contradictories cannot both be false at the same time; Everything must

either be or not be (Jevons); Either a given judgment must be true or its contradictory, there is no middle course (Thomson); Of contradictories one must be true and the other false; Of two contradictories one must exist in every object; Judgments opposed as contradictories (such as *A is B*, *A is not B*) can neither both be false nor can admit the truth of a third or middle judgment, but the one or other must be true, and the truth of the one follows from the falsehood of the other (Ueberweg); The double answer, *Yes and No*, cannot be given to one and the same question understood in the same sense (Ueberweg); Of contradictory attributions we can only affirm the one of a thing, and if one be explicitly affirmed the other is denied (Hamilton). Mill again asserts a corresponding postulate of *expression:* "It is allowable to substitute for the denial of either "of two contradictory propositions the assertion of the other" (*Exam. of Hamilton*, p. 416).

Contradictories admit no third alternative, but contraries do.

This principle of Thought has been questioned, and even denied, by writers who have confounded contradiction with other forms of incompatibility, especially contrariety (*see* § 29). But, whilst contrary terms mark the utmost possible divergence, contradiction is simple negation. There are, of course, many intermediate stages of grey between the contrary attributes, black and white; and many varying degrees of warmth between the contraries, hot and cold. There are, then, many alternatives besides the propositions, This paper is white—this paper is black, This water is hot—this water is cold. But there is no third alternative whatever between the contradictory assertions, This paper is white—this paper is not white, This water is hot—this water is not hot. It has been urged, as proof that contradiction is not thus exhaustive, that there is a mean between *plus* and *minus*, viz., *zero;* but here again, we have contraries, not contradictories. A mathematical quantity must either be positive, or not be positive; and, if the latter, it may be either zero or negative. Similarly, one thing need not be either greater or less than another given thing, because 'greater' and 'less' are not contradictories, and there is a mean, 'equal to,' between them; but a thing must either be greater or not be greater than another given thing, and, if it be not greater,

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it may be either equal to it or less than it. Mill thought he had discovered a mean between the true and the false, which are both contradictory and contrary terms, viz., the unmeaning: "Between the true and the false there is a third pos-"sibility, the Unmeaning" (*Logic*, Bk. II., ch. vii., § 5). But to this it has been answered that an unmeaning possibility is no possibility at all; "a proposition which has no meaning "is no proposition; and . . . if it does mean anything it is "either true or false" (Bradley, *Prin. of Logic*, p. 145). In short, great care is necessary to avoid confusing judgments whose predicates are contrary terms with those whose predicates are contradictories; it is so easy to make the negation, which should only deny a strict agreement in all points, imply a thorough-going and complete divergence. If a man is declared not guilty of a certain crime people are inclined, thereupon, to attribute to him perfect innocence; whereas there may have been any degree of approximation to full guilt which yet fell short of it. The denial of guilt *as the accusation puts it* leaves open the possibility of some less degree of guilt; in many cases, further enquiry is invited rather than barred.

Non-B is not a true concept.

Lotze objects to expressing the principle of Excluded Middle by the formula *A* is either *B* or *non-B* instead of by the formula *A* either is, or is not, *B*, because, he says, *non-B* is really unmeaning as it embraces everything in the universe except *B*; for instance, 'not-green' would not only embrace all other colours, but all other qualities and things whatsoever—as hot, cold, long, etc.—which are not included under the term 'green.' With this understanding we may correctly say 'Honesty is not-green'; but the proposition is practically meaningless.

In practice negative terms are usually limited in application.

This is certainly true, and when we do use such a negative term as *non-B*, not-green, which is but seldom, we really, in intention, confine it to the genus of which *B* is a species; in the case of not-green to the genus of colour. Of course, with this limitation it is not possible to affirm either green or not-green about every subject, but only about those which possess the attribute of colour.

The Axiom of Excluded Middle is necessary, in addition to those of Identity and Contradiction, to form a basis for

some forms of Immediate and Mediate Inference. It limits the thinkable in relation to affirmation, and declares the necessity of affirming one or other of two opposed contradictory judgments, but it does not decide which of them is true. Of course, the same limitation to a definite point of time holds here as in the Principle of Contradiction (*see* § 18). By the Principle of Contradiction we are forbidden to think that two contradictory attributes can be together present in the same subject; by that of Excluded Middle we are forbidden to think they can both be, at once, absent; but no help is given us to decide which must be present and which absent.

The Principle of Excluded Middle does not decide which of two contradictory propositions is true.

From the point of view of language the three principles above discussed may be summed up by saying that whenever we use a name we must be understood to use it in its full meaning both (1) positively and (2) negatively, and (3) it must either be given or denied to everything whatever. That is, the use of a name asserts all the attributes it implies and denies all others which are incompatible with those; and everything must either possess all those attributes or be without some, or all, of them.

The three Principles regulate the use of names.

20. The Principle of Sufficient Reason.

Principle of Sufficient Reason—*Every judgment must have a reason.*

The Principle of Sufficient Reason was first distinctly formulated by Leibniz in the words, "In virtue of this "principle we know that no fact can be found real, no proposition true, without a sufficient reason, why it is in this "way rather than in another;" and again, "Whatever exists, "or is true, must have a sufficient reason why the thing or "proposition should be as it is and not otherwise" (*cf. Monadologie*, §§ 31-39). Other statements of the principle are: Every judgment must have a sufficient ground for its assertion (Mansel); Every proposition must have a reason (Kant); A judgment can be derived from another judgment (materially different from it), and finds in it its sufficient reason, only when the (logical) connexion of thoughts corresponds to a (real) causal connexion (Ueberweg).

As we necessarily regard reality as a systematic unity we

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attribute the external invariable connexion between different phenomena to an inner conformability to law. This may be symbolically expressed $A+B=C$, where we mean that any subject A, together with the condition by which it is influenced B, is identical in content with the consequent C, which is the subject itself as thus altered. For example, if A=gunpowder and B=the high temperature of a spark, then $A+B=C$ which is the explosion of that powder. This relation between the reason $(A+B)$ and the consequent (C) we necessarily conceive as universal; we could not conceive $A+B$ as a *reason* for C at all, if it did not always produce C. The principle, in brief, expresses the necessary postulate of knowledge, that explanation is attainable, and that the explanation of any element of reality must be sought in its relation to other elements, and ultimately to the whole system of reality. The law of causation is the aspect of the principle of Sufficient Reason which is most frequently appealed to (*see* Bk. v., ch. i.).

The acceptance of the principle necessitates that if we grant the reason we must accept the consequence which follows from it, and it is, thus, one of the foundations of syllogistic, and, indeed, of all other, reasoning. It follows from this, moreover, that logical necessity is not absolute but hypothetical; a consequence appears *if*—and only *if*—the appropriate conditions are secured.

21. Hamilton's Postulate.

Hamilton's Postulate—*We may state explicitly in language all that is implicitly contained in the thought.*

Hamilton, in his *Lectures* (vol. iii., p. 114) thus states what he regards as a necessary postulate of Logic. "Before dealing with a "judgment or reasoning expressed in language, the import of its "terms should be fully understood; in other words, Logic postulates "to be allowed to state explicitly in language all that is implicitly "contained in the thought." Some of the consequences which Hamilton deduced from this postulate will be noticed in the chapter on the Import of Propositions, and reasons given for dissenting from them (*see* § 86). But the Postulate itself may be accepted, and taken to assert that it is permissible to vary the mode of stating a judgment so long as the meaning is left unchanged, for the mean-

ing, and not the form of words in which it is expressed, is the important point. Read in this sense it becomes practically the same as Mill's statement of Identity (*see* § 17). Such variation in the wording is frequently necessary in order to reduce the sentences of ordinary discourse to the strictly logical form, as propositions consisting of Subject, Copula, and Predicate [*see* §§ 8 (ii.), 68].

22. Mathematical Axioms.

All valid arguments are not treated in formal Logic.

Valid arguments need not be based entirely on the Principles of Identity, Contradiction and Excluded Middle, though they must always be in conformity with them. Equally cogent are those founded on mathematical axioms, such as the *argumentum à fortiori*; If A is greater than B, and B is greater than C, then A is greater than C; or the axiom, Things which are equal to the same thing are equal to one another. But arguments based on these, though perfectly valid, are not expressed in that form of reasoning which is treated of in formal Logic.

Other logical principles, such as the *Dictum de omni et nullo*, the Canons of the Syllogism, the Postulates of Induction, will be discussed in connexion with those parts of Inference to which they respectively apply.

BOOK I.

TERMS.

CHAPTER I.

GENERAL REMARKS ON TERMS.

23. Definitions of Term and Name.

A logical proposition consists of Subject, Predicate and Copula.

The simplest element of thought is the judgment, and the verbal expression of a judgment is a proposition (*cf.* § 8). When a proposition is expressed in its perfect logical form —*S is P* or *S is not P* [*see* §§ 8 (ii.) and 68]—it is seen to consist of three parts :—

(*a*) Something of which the assertion is made, called the *Subject*, and denoted in the symbolic form of the proposition by *S*.

(*b*) Something affirmed or denied of the subject, called the *Predicate*, and symbolized in the formal statement by *P*.

(*c*) The verb *is*, either alone or accompanied by *not*, by means of which the assertion is made, called the *Copula*.

A *Term* is the Subject or Predicate of a logical proposition.

The Subject and Predicate are called the Terms (from Lat. *terminus*, or boundary) **of the Proposition,** and they are the verbal representatives of the things, and of our concepts of them, between which the judgment affirms a relation (*see* § 9). Both, therefore, must be names of substances or

of attributes. This leads to a wider use of the word *Term* as synonymous with *Name*, whether forming part of a proposition or not; but Logic considers names only when they are regarded as actual or possible terms in the stricter sense.

A *Name* is the verbal mark of the idea of a thing.

* The usually accepted definition of a name is that of Hobbes, and is as follows: "A *Name* is a word taken at "pleasure to serve for a mark which may raise in our minds "a thought like to some thought we had before, and which, "being disposed in speech and pronounced to others, may be "to them a sign of what thought the speaker had or had not "before in his mind" (*Computation or Logic*, ch. ii.). "Had not" is included in order to embrace purely negative terms which simply imply the absence of an idea (*see* §§ 19 and 29). To this definition it has been justly objected that, on no known theory of the origin and growth of language, can it be said that names are words "chosen at pleasure" to denote things; there has been no voluntary and arbitrary affixing of certain words as signs to certain things, but a natural and gradual growth of language; those words had better, therefore, be omitted from the definition. Some phrase should, also, be added to it to bring within its scope such 'many-worded names' as 'First Lord of the Treasury,' which, though consisting of five words, is yet only one name as it denotes only one object of thought. We may then, finally, say that,

A Name is a word, or combination of words, serving as a mark to recall to our own minds, and to raise up in the minds of others, the idea of some object of our thought.

24. Single-worded and Many-worded Terms.

A Name may consist of any number of words.

The simplest names consist of a single word, as 'horse,' 'London.' Such names are given to all the more important objects with which we are acquainted and which we require to name most frequently. But the multitude of things in the world is so enormous that, not only can we not give each a separate name of its own, but we cannot even form them all into definite classes, each with its own name. Many of them we must name by a kind of description; thus, many, perhaps the majority, of names or terms consist of a

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combination of several words, and are, consequently, called *Many-worded Names*. Such names always contain one or more words which, if used in a different context, would be themselves names, but with these are usually other words which cannot be used as names. For instance, in the proposition 'The First Lord of the Treasury is the present leader of the House of Commons' both the terms are many-worded names, and both contain words — lord, treasury, leader, house, commons—which are capable, by themselves, of forming either the Subject or the Predicate of a proposition; and others—first, present—which can be used as predicates though not as subjects. At the same time, there are other words—the, of—which cannot, by themselves, form terms at all.

25. Categorematic and Syncategorematic Words.

We have, thus, in Logic, two, and only two, classes of words:—

A Categorematic Word can form a Term. *A Syncategorematic Word* cannot form a Term.

(*a*) **A Categorematic word is one which can, by itself, be used as a term.**

(*b*) **A Syncategorematic word is one which cannot, by itself, form a term; but can only enter, with one or more categorematic words, into the composition of a many-worded term.**

(The terms 'Categorematic' and 'Syncategorematic' are derived from the Greek κατηγορέω, *I predicate*, and σύν, *with.*)

This is the only division of words recognized by Logic.

* This division is exhaustive; every word must fall into one or other of these two classes; and no word, used in the same sense, can fall into both. It is the only division of words, as words, recognized by Logic, for that Science pays no regard to the grammatical division into parts of speech (*see* § 15). All words which can form a Term belong to one logical class though they may be distinguished by grammar as Substantives, Pronouns, Adjectives or Participles; all those which cannot form Terms belong to the other class though Grammar calls them Adverbs, Prepositions, Con-

junctions and Interjections. This division apparently contains no place for verbs; the reason of this is that formal Logic recognises only the verb *is* (or *are*) which forms the Copula of all propositions expressed in true logical form [*see* §§ 23 (*c*) and 68]; all other verbs are, therefore, represented in formal logic by *is* or *are* and a participle. It is plain that the Nominative and Possessive Cases of nouns and pronouns are Categorematic, and the Objective Case is Syncategorematic; thus the logical division of words cuts across the grammatical. Adjectives and Participles, like the possessive cases of substantives and pronouns, can always be used as predicates, but not as Subjects except by an ellipsis, as when we say 'The virtuous are happy' where the full subject is 'Virtuous People.' Adjectives and their equivalents form true terms (predicates) and are, therefore, Categorematic words. From the very nature of the case, Adverbs, Prepositions, Conjunctions and Interjections, as such, cannot be used as Terms.

Some logicians have called many-worded names *Mixed Terms*, because they contain syncategorematic as well as categorematic words; but no object is gained by this, for Logic regards a many-worded name as a whole. Symbolically it is expressed by a single letter, as *S* or *P*, exactly as a single-worded term would be.

Of course, as the division of words into categorematic and syncategorematic is really into terms and non-terms, it is as absurd to speak of syncategorematic terms as it is tautologous to speak of categorematic terms; for, by definition, every term *must* be categorematic. Yet, at least one writer on Logic has been found who divides Terms into Categorematic and Syncategorematic (*see* Jevons, *Elem. Less. in Log*, p. 26; *Studies in Deductive Logic*, p. 9), forgetting that to speak of a 'Syncategorematic Term' is to violate, in language, the Law of Contradiction (*see* § 18).

CHAPTER II.

DIVISIONS OF TERMS.

26. Table of Divisions of Terms.

Terms may be divided on five bases.

Terms may be divided in various ways according to the point of view from which we regard them. The following Table sets forth these different divisions, and the principle upon which each is founded. Of course, each group is exhaustive and independent; every Term must fall under one or other of the members of each. It may be remarked that (i.), (ii.) and (iii.) are the only divisions which are logically important, for they alone are founded on logical considerations.

Table of divisions of Terms.

(i.) *Individual* and *General*—as names of individuals or of members of classes.

(ii.) *Connotative* and *Non-connotative*—as names capable or incapable of definition.

(iii.) *Positive* and *Negative*—as names implying the presence or the absence of some quality.

(iv.) *Concrete* and *Abstract*—as names of objects or of attributes and relations.

(v.) *Absolute* and *Relative*—as names implying or not implying a mutual determination of meaning.

A further division into *Univocal* and *Equivocal* terms is sometimes made. But this is entirely a matter of language. Whenever the same word serves as name for two or more distinct classes of things—as, *e.g.*, sleeper, which may mean either an individual asleep or the support of rails on a railroad—we have logically a plurality of terms, for the word in

each of its meanings is a distinct and separate term, and represents a distinct and separate concept.

27. Individual and General Terms.

An *Individual* or *Singular Term* can be applied in the same sense to only one object.

(i.) **An Individual Term is one which can be affirmed in the same sense of only one single thing.** Thus, 'London' can be used in the same sense of only one place, though more than one place may have this same name; 'honesty' denotes only one quality, though it may be possessed by many individuals; 'this book' is limited to one single volume, and can only be understood by a person who knows what particular book the speaker is indicating. So, 'The present Queen of England,' 'The richest man in the world,' 'The longest river in Europe,' are all Singular or Individual names. But an examination of these and similar examples will show that, though they are all names of individuals, yet they differ from each other in that, whilst some of them tell us of some quality possessed by the thing they denote, others do not. Of the latter kind are 'Honesty' and 'London.' Such terms as honesty will be discussed in § 30, under the head of Abstract Terms.

(*a*) 'London' belongs to that subdivision of Singular Terms called *Proper Names*, which may be thus defined :—

A Proper Name is an arbitrary verbal sign whose sole province is to indicate an individual object.

A *Proper Name* is an arbitrary verbal sign which merely indicates the object of which it is the name.

It may be thought that such names tell us a great deal about individuals; that 'London,' for instance, tells us that the object spoken of is a large city, situated on the Thames, the Capital of the British Empire, and many other particulars with which we may happen to be acquainted about it; but this is to confuse our knowledge of the thing obtained from all kinds of sources with the meaning implied by the name. The *word* 'London' informs us of none of these things; it may *suggest* them by the law of Association of Ideas, in the same way as hearing a song which we have

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heard before may suggest the room in which we first heard it or the person who then sang it; but 'London' no more *means* these suggested particulars than the melody of the song means a place or person. Care must then be taken in considering Proper Names to distinguish between implication and suggestion. Suggestion is purely a psychological fact. Logically, the point is that a proper name is not given on account of a certain meaning, *i.e.*, on account of the possession of certain attributes, but as a mark of recognition. No doubt, Proper Names originally had implication, and are continually tending to assume such meaning: but to the extent to which they succeed they cease to be purely Proper.

More than one object may bear the same Proper Name, but not in the same sense.

The fact that more than one object may receive the same Proper Name is no objection to the assertion that all such names are Singular or Individual. Thousands of men may be named Brown, and the same name may be borne by many dogs, horses, and other things; for instance, a town could be named Brown as appropriately as Washington, Gladstone, or Peel, all of which names have been thus employed. But the name is given to no two of these objects in the same sense. As it is simply a mark of identification, it does not matter logically to how many people or things it is applied.

(*b*) *Significant Individual Terms.*

A Significant Individual Name is a General name limited, by some word, in its application to one individual.

Proper Names are the simplest kind of Singular or Individual Terms. But the individual things we may wish to refer to are too numerous for us to give each of them a Proper Name of its own, and, sometimes, when a Proper Name has been given it is unknown to us. We are, therefore, often driven to use a General Name [*see* § 27 (ii.)] with a limiting word to make definite its applicability to only one object. The simplest means of doing this is to use a demonstrative word—as *This* pen is bad; Let us go for a walk by *the* river. Here we are, in both cases, referring, in a perfectly determinate sense, to only one object, and the name is, therefore, Singular. No doubt, in the latter case, the river has a Proper Name of its own; but, in speaking of

very familiar objects, we often use such a limited General Name in preference to the Proper Name. Again, we may use a many-worded name because it is our only means of definitely indicating the object we wish to refer to, as its Proper Name may be unknown to us; thus, if we speak of 'The inventor of the Mariner's Compass,' 'The writer of the Letters of Junius,' or 'The man in the Iron Mask,' we are using the only means in our power of exactly designating the person we mean; for, in none of these cases, is the Proper Name of the individual referred to known. In other cases, such a many-worded name may be used because there is no Proper Name applicable; as when we say 'The leader of the House of Commons,' or 'The present leader of the House of Commons.' Regarded from a point of view limited in respect of time this name can only refer to one definite person, and is, therefore, Individual. Had we said simply 'Leader of the House of Commons' the name would not have been Singular, but General; for it could, then, be applied in the same sense to many individuals; the prefixing 'The' or 'The present' limits its application so long as we restrict ourselves to one point of time. In other cases this limitation in time is unnecessary, as when we speak of 'The first King of Prussia' or 'The highest mountain in Asia,' where the reference is plainly to one individual object of such a kind that no lapse of time can make it applicable to any other. But were we to say 'The highest *known* mountain in Asia' we should again bring in the limitation of time to the present; as it may well be that the highest known mountain is not really the highest in the continent; but it is to this last alone that the former name is correctly applicable.

Such names are the richest of all names in meaning.

It is evident that Singular Names of this second kind have meaning; they are *Significant*—for they not only point out one member of a class, but, at the same time, inform us that it does belong to that class, and has at least one attribute which marks it out from every other member of the class. In fact, these names are richer in implication than any other class of names [*see* § 28 (i.)].

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A General, Common, or Class Term can be applied in the same sense to an indefinite number of things.

Such a name denotes things indirectly through their possession of certain attributes.

The application of a General Name may be merely potential.

Every Term must be Individual or General.

(ii.) A General, Common, or Class Term is one which can be applied in the same sense to each of an indefinite number of things; as book, man, dog. Subjectively considered, a General Term is the name of a General Notion or Concept. Whilst a Proper Name indicates an individual directly, a General Name does so indirectly, for such a name is given because the individuals to which it is applied, and from an examination of which the concept is formed, possess some attribute or attributes in common [*see* § 2 (ii.)]. The name, then, implies the possession of certain common qualities by every individual object which bears it, and, thus, has a meaning in itself; it not only indicates certain objects but it informs us that those objects possess certain qualities. This likeness constitutes the similar objects a class, and, hence, a General Term is often called a *Class Term*.

It is not necessary for a true General Term that it should be *really* applicable to a plurality of objects, or indeed to any real physical object at all; it is sufficient for it to be *potentially* thus applicable; that is, for it to represent a possibly real, or even an absolutely imaginary, class of things, because of their possession of some common quality or qualities. For instance, 'Conqueror of England,' 'Emperor of Switzerland,' and 'Centaur' are true General Terms; though the first is *really* applied to only one historical individual—William I., and the second is not applicable to any individual at all in the present or in the past, though both may, conceivably, have an actual application in the future; whilst the third is the name of a purely imaginary being. It is this potentiality of application to a class which distinguishes General Terms from the second class of Singular Terms; for the latter, though they are significant—that is, have implication—are not applicable, even potentially, to more than one individual. There is thus an antithesis between Individual and General Terms, and every Term must be one or the other.

In Class Terms the unity which in Individual Names is one of application is restricted to content or meaning; in

application it is overshadowed by the idea of plurality. In *Collective Terms* both these ideas are equally prominent. These Collective Names are sometimes treated as a separate division of Terms, co-ordinate with Singular and General, but this is not desirable; for, as has been said, every Term must, of necessity, be either Singular or General. Collective Terms are found in each class, and there is, therefore, no opposition between them on the one hand and either Individual or General Terms on the other. A short examination of such terms will make this clear.

Collective Terms do not form a class co-ordinate with Singular and General.

A Collective Term is one given to a group of similar units. It thus implies a plurality in unity; as an army, a flock, a library. It is not every group of individual objects which can receive a Collective Name; the constituents of the group must bear a general resemblance to each other; thus, an alphabet is composed of letters, a navy of ships, a library of books, a museum of objects of interest. We could find no use for a name denoting a group composed partly of ships, partly of books and partly of men, or any other fortuitous concourse of heterogeneous objects; and, though we could, no doubt, manufacture such a name, yet it would not be a true Collective Term; for it would not imply that the constituents of the group were all of the same kind.

A Collective Term is one given to a group of *similar* units.

As a rule, Collective Terms are not Proper Names, but a few instances may be found, chiefly amongst geographical names, in which they are. Thus we speak of the Alps, the Pyrenees, the Himalayas, the Hebrides, the Marquesas, the Antilles, the Orkneys, all of which are true Proper Names, for they give us no information whatever about the groups of natural objects to which they are applied, and are yet Collective, for they denote a group of similar units.

But few Collective Terms are Proper Names, and those are chiefly geographical names.

When the application of an ordinary Collective Term is limited—in the way illustrated in (i.) (*b*) of this Section—to one particular instance of the groups it denotes, it becomes a Significant Individual Term. Thus we can speak of 'The German Navy,' 'The Greek Alphabet,' 'The Bodleian Library,' 'The British Museum,' 'The French army which fought at Waterloo.'

A Collective Term may be a Significant Individual Term.

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Collective Terms are usually General.

Without such limiting words a Collective Term is General with regard to the class of which it denotes a member, as well as Collective in respect of the units of which the group is composed. Thus 'navy' is Collective as regards the ships which form it, but General, as denoting a member of the class 'navies'; 'alphabet' Collective as indicating a group of letters, General as the name of a member of the class 'alphabets.' We have as true concepts, in fact, of navy and alphabet as we have of ship and letter, and the former terms imply attributes equally with the latter. The group denoted by a Collective Term may even itself be a unit in a larger group which bears a collective name of wider generality; so we may have a series of terms, each, except the first, collective as regards the preceding one, and each, except the last in generality, forming a constituent of the group denoted by the following one: e.g. soldier, company, regiment, brigade, army. Thus the term Collective is relative in its meaning. At the same time, a General Term, which taken by itself is not Collective, may, if in the plural number, be used in a Collective sense by the prefixing of such a word as 'All' in the sense of 'All together,' as 'All these books weigh several tons.' The true antithesis is, therefore, not between Collective and General Terms, but between the

The group denoted by a Collective Term may be a unit in a group whose name is also a Collective Term.

A General Term, not by itself Collective, may be used in a collective sense.

Collective and Distributive Use of Terms. When we use a term Collectively our assertion will only apply to the group as a whole; when we use it distributively we assert something about each member of the group individually. Thus, if we say 'Half the fleet was lost in a storm,' 'The regiment was decimated by fever,' 'All the novels of Thackeray would fill a small bookcase,' 'The books filled six large boxes,' we are evidently using the terms which form the subjects of our propositions, whether they are 'Collective' or 'General' in a collective sense; and, equally clearly, if we say 'The fleet separated,' 'The army was scattered,' 'All the men were fatigued,' 'All the novels of Thackeray can be read in a day,' we are using the terms distributively. The full sense of the separate words is again seen to depend on the context (*cf.* § 3).

There is an antithesis between the collective and the distributive use of Terms. In the *Collective* use the assertion applies to the group as a whole; In the *Distributive* use to the individuals which compose the group.

Substantial Terms—or names of substances—are a peculiar kind of Collective Terms.

* **Substantial Terms.** The question has been raised as to whether names of substances, or 'Substantial Terms' as they are sometimes called, such as gold, oil, water, are Singular or General. On the one hand, it is urged that such words denote the entire collection of one species of material; on the other, that when we use them, we do not refer to the whole but to some definite, or indefinite, portion of the whole. But the question is not very pertinent, as in such terms the element of content or meaning is predominant, whilst that of application to this or that object is subordinate. In so far as we do consider this latter aspect, we may say, with Dr. Venn (*Empirical Logic*, pp. 170-1), that such terms are a peculiar kind of Collective Terms, with the special characteristics of theoretically infinite divisibility, and, at the same time, perfect homogeneity. It is this which makes them different from ordinary Collective Terms. We can divide and subdivide a number of pieces of gold into any number of parts, and again reunite them ; and any one part is a fair specimen of the others. In this they differ as much from ordinary General Terms as they do from true Singular Terms ; we can divide an animal, but we cannot reunite it.

28. Connotative and Non-connotative Terms.

Some Singular Terms imply attributes, and all General Names do so.

(i.) **What names are Connotative.** In the last section Terms were considered according to their applicability to one or more objects, but that division could not be intelligibly discussed without reference to a fundamental distinction which is, to some extent, bound up with it. It was shown that, whilst an Individual Name may be a mere indicative sign, implying no attribute, all names which are applicable to a plurality of objects are essentially significant, and imply some attribute or attributes possessed in common by those objects ; for, only on this principle, could the same word be, in any intelligible sense, the name of each member of a class of things. This distinction between significant and merely indicative names is expressed by the terms Connotative and Non-connotative, which we may, therefore, thus define, nearly in the words of Mill (*Logic*, Bk. I., ch. ii., § 5) :—

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A Connotative Term denotes a subject and implies an attribute or attributes.

A Non-Connotative Term merely denotes a subject.

A Connotative Term is one which denotes a subject and implies an attribute or attributes.

A Non-connotative Term is one which merely denotes a subject.

When we speak of a *subject* in this connexion we mean anything which can possess an attribute; whilst under *attribute* we include all that belongs to the subject, not only the outward marks by which it is known—as its shape, size, colour, weight, etc.—but all its properties and relations whatsoever.

All General Names are connotative.

From what has been already said [§ 27 (ii.)], it is evident that all General Terms are connotative, for they all denote—or are applicable to—certain objects, and imply that those objects agree in possessing some attribute or attributes in common; in fact, it is the possession of these attributes which entitles any particular object to bear the name. Thus, if we use the name 'horse,' we not only refer to an indefinite number of animals which are so styled; but we imply that they all agree in possessing certain well defined characteristics; any new animal brought under our notice which possessed those attributes we should, without hesitation, call a horse. Under the head of General—and, therefore, Connotative—Terms must be included all Adjectives, for they express qualities regarded solely as exhibited by things, and, if we wish to use them as subjects of propositions we must name the things they qualify (*cf.* § 25). So with all those Collective Names which are not Proper Names; they are all Connotative, for they are General when viewed as members of a class [*see* § 27 (ii.)]; for instance, 'army' implies the attributes of being composed of soldiers, armed, trained, and maintained for warlike purposes, as well as denotes each collection of men which possesses these attributes. When any General Name, whether Collective or not, is restricted in its application by some limiting word or phrase, of course its implication is not lost. Indeed, that implication is increased, and thus we have the class of Significant Individual Names [*see* § 27 (i.) (*b*)], which, though they denote only one object, yet imply the possession of many attributes

All Adjectives are General and, therefore, Connotative Terms.

Collective Terms are connotative, except they are Proper Names.

Significant Singular Terms are connotative.

by that one object. Thus, if we speak of 'a mountain' we imply the attributes 'height' and 'composition of rock'; if we add 'in Asia' we increase the number of characteristics, though we limit the number of things to which the name applies; by adding 'high' we carry both these processes a step further; and if, finally, we make the term Singular, and speak of 'The highest mountain in Asia' we, manifestly, retain all the attributes previously implied, and add to them uniqueness. All these attributes are implied by the name, and anybody using the name must be supposed to intend to convey them to his hearers.

Proper Names are non-connotative, for they can only *suggest* not *imply* attributes.

But, were we to use, instead of this significant name, the Proper Name 'Everest,' which, in our present state of knowledge, we believe to be the name of the same object, no such information would be given. To anybody who knew the geographical fact that Everest is the highest mountain in Asia, the name 'Everest' would, doubtless, *suggest* all that the words 'The highest mountain in Asia' *imply*. But *a word is not Connotative because it may suggest facts or attributes otherwise known, but because it implies them*, so that the name by itself is, when understood, sufficient to impart the knowledge that they are possessed by every object it denotes. This distinction between suggestion and implication is the distinction between connotative and non-connotative. No terms are without some meaning to those who use them, but only in connotative terms is the meaning a group of implied qualities.

No doubt Proper Names were originally significant, and implied attributes. Thus, Avon in old English meant water; Jacob meant a supplanter; Smith or Butcher, one who followed a certain trade. But even as so given their main function was distinction, and the name was retained even though the attribute it at first implied was removed; to deduce connotation from this original descriptive character is to confuse connotation with etymology. With surnames there is a very strong suggestion, amounting almost to implication, of family relationship. This was even stronger with old Roman names. For instance in the name Caius Julius Cæsar, whilst the prænomen, Caius, was non-signi-

ficant, the nomen, Julius, indicated the gens, and the cognomen, Cæsar, the family in that gens, of which the individual was a member. But as a surname can be changed at will, it seems clear that now, at any rate, its true function is merely to distinguish the individual, and that it has no necessary implication of meaning.

Proper Names used typically become General.

° The absence of real implication in Proper Names, especially to denote persons, is, probably, to be partially explained by the fact that an individual possesses such an innumerable number of different attributes that no one (or more) is specially identified with him; directly some attribute does show itself as a more marked characteristic, the use of a descriptive 'nick-name' is likely to become common, as every schoolboy can testify. In a similar way we may account for the use of the names of some prominent historical personages to imply the possession of the quality which the type possessed in an exceptional degree; thus we speak of 'a Cicero,' 'a Napoleon,' 'a Caligula,' etc., but the names have then ceased to be true Proper Names and have become General, and applicable to all objects showing the indicated qualities in a marked degree.

If, then, we distinguish between implication and suggestion, we must come to the conclusion that the definition given in the last section of a Proper Name as "an arbitrary verbal sign" is strictly accurate, or, in other words, that Proper Names are non-connotative.

The only class of names which remain to be examined in this connexion are Abstract Names, and it will be more convenient to postpone our consideration of this point till we have discussed the nature of those terms [*see* § 30 (iii.)].

The *Connotation* of a name embraces those attributes, and those only, on account of which the name is given, and wanting any of which it would be denied.

(ii.) **Limits of Connotation**. All the attributes *directly implied* by a name form its *Connotation*, and it is clear from what has been said above that this does not include all those which are common to all the members of a class denoted by a General Name, nor, consequently, all those which are possessed by the individual object to which a Significant Singular Term is applied, but *only those on account of the pos-*

session of which the name is given, and wanting any of which it would be denied. The disputes which have arisen about the connotation of terms generally, and especially of Proper Names, have owed their origin to an ambiguous use of this word.

If Connotation embraced *all* common attributes it would become entirely a matter of objective existence, not of knowledge.

Some writers hold that the Connotation of a name includes all the attributes common to the members of the Class of which it is the name. Thus Mr. E. C. Benecke says: "Just "as the word 'man' denotes every creature, or class of "creatures, having the attributes of humanity, whether we "know him or not, so does the word properly connote the "*whole* of the properties common to the class, whether we "know them or not" (*Mind*, vol. vi., p. 532). But this usage would have many logical inconveniences; it would divorce connotation and definition, and make connotation a matter, not of knowledge, but entirely of objective existence, and it is with such existence only as known that logic is concerned. If, to avoid this objection, it is said that the connotation should embrace all the *known* attributes common to a class, then it must be pointed out that some of these cannot be regarded as essential; for instance, though every kangaroo is an Australian animal, yet were such an animal found elsewhere it would not be excluded from the class of kangaroos. Similarly an animal which chewed the cud would be regarded as ruminant even though it did not agree with all known ruminants in possessing cloven feet. The name cannot, therefore, be said to strictly imply the possession of those attributes. Again, some attributes are derivative from others. Thus, that an equilateral triangle is equiangular, that a right-angled triangle is inscribable in a semi-circle, etc., are attributes derivable from those primary ones which the name directly implies (*cf.* § 37). Such attributes are, then, indirectly implied, and are a necessary consequence of those directly implied. But it is most convenient not to regard them as forming part of the Connotation; as that is to confuse primary with secondary implication. It would be convenient to use the term *Content* to express all the attributes which are either directly or indirectly implied by a name.

It must be granted that the limits of connotation are con-

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ventional; but this is made necessary by the imperfection of knowledge. It is essential that our terms should have a fixed and definite value at any given time, and that this value should express the knowledge which has been attained.

Connotation is a matter of knowledge.

It is thus seen that the question of connotation is, in essence, a question of knowledge. It is neither entirely objective, nor entirely subjective, but has both an objective and a subjective reference. To make it purely subjective would be to say that every individual should consider the connotation of a name to consist of these qualities which he himself may know to be common to the class. But, as Dr. Bosanquet well remarks, "Surely the question for logic "is never what a name means for you or me, but always, "what it ought to mean" (*Knowledge and Reality*, p. 60). And what it ought to mean is determined by the fullest knowledge attained, and is expressed in definitions accepted by all competent authorities. Without such common agreement as to the correct implication of words, language would soon cease to be available as a medium of the communication of anything like exact thought.

Connotation is conventional but practically definite enough in most cases.

(iii.) **Difficulties of assigning Connotation.** When we say the connotation embraces all those attributes, and those only, which are directly implied by the name, and that this is determined by the fullest knowledge attained, and is expressed in definitions accepted by competent authorities, we undoubtedly show that connotation is not only conventional, but may be in some cases, especially in comparatively new branches of knowledge, somewhat vague. No doubt, in the case of most terms, it is found sufficiently definite; it is, for instance, clear that the connotation of the term 'square' is that it is a plane, rectilineal, right-angled, quadrilateral figure with equal sides; none of the other numerous qualities of squares are included in the connotation; we need not think of them when we use the term, they do not form part of our concept of a square. Still, in some cases, it is, undoubtedly, far from easy to decide how much a particular term does or does not connote. When this is the case with words in common use it may lead to much confusion, but

with many names it is an advantage for the connotation not to be too rigidly limited, so that they may be applied to newly-discovered objects which most closely resemble those which already bear the name. And, at all times, the connotation of terms must be subject to revision, should occasion arise through the discovery of such objects.

Connotation is made indefinite by transference of the name to objects not at first denoted by it.

In § 3 instances were given of the way in which connotation becomes vague and difficult to assign through the transference of a name by analogy, metaphor, or partial resemblance, to things other than those to which it was originally applied. Through these processes the same name may come to denote objects entirely different, but must then be regarded as separate and distinct terms accidentally written and spoken alike, *e.g.*, 'post,' meaning a piece of wood inserted in the ground, and 'post' meaning the conveyance of letters.

This process can be represented symbolically.

This process may be represented symbolically thus: A class of objects X possesses the common attributes abc; the class Y possesses the common attributes ade; the class Z possesses dfg. Now, the name of X is transferred to Y because of the common attribute a, it is then passed on from Y to Z because of the common attribute d, and so the same word denotes X and Z which have not a point in common. As their connotation is then entirely different, they must be regarded as entirely different terms—the name of different classes. But shall Y be regarded as forming part of the class X or as part of Z? It could only do so by restricting the connotation in the former case to a and in the latter case to d; it would, therefore, probably be regarded as constituting yet a third distinct class, and its name would be a third and independent term for all logical purposes—the history of its origin and development, though interesting from the point of view of Philology, is of no value from that of Logic. All cases of ambiguity in language are instances of indeterminate connotation of names, and a vast number of fallacious reasonings are due to this indefiniteness.

***Denotation* of a Term—the objects to which it is applicable in t... sen...**

(iv.) **Denotation of Terms.** A comparison of the last section with the present will show that significant names may be viewed in two lights—their implied meaning or connotation and their range of application to a number of objects—this latter aspect is called their *Denotation*, which may, therefore, be defined as *the number of individual things*

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Logically, Denotation is fixed by Connotation, but practically, they determine each other.

to which the term is applicable in the same sense. From what has been already said it is clear that the denotation is logically fixed by the connotation; objects receive a certain name, and so form part of the denotation of that name, because they agree in its connotation. Nevertheless, practically each helps to determine the other. The connotation expresses the concept which is formed after an examination of part, at least, of the denotation; and, at all times, not only is the connotation likely to be modified by an increase in the denotation, but also conversely, making the connotation more definite or more elastic may decrease or enlarge the denotation [*see* (v.)]. In truth, neither is absolutely fixed, though, for the purposes of Formal Logic, it is necessary to regard the connotation as strictly invariable, at any rate throughout the same argument, or the Law of Identity would be violated (*see* § 17).

All terms have Denotation.

According to the definition of Denotation given above, it follows that all terms have denotation whether they have connotation or not, though in the case of Proper Names and of some Abstract Names the denotation is reduced to the least possible limit—the unit.

Dr. Venn holds that purely mental notions have no denotation.

Dr. Venn, however, dissents from this view. He says: "The "conception of Denotation becomes appropriate only when we are "concerned with objects whose existence is limited in some material "way" (*Emp. Log.*, p. 178). If we speak of the denotation of a purely mental concept, such as a perfect mathematical figure, *e.g.* a circle, the only meaning we can give to the term is to say that it embraces every circle which ever has been, or could be, conceived; for every one of them would possess the full connotation; but none with material existence, for no perfect circle has ever been drawn. Thus the denotation of such a term would be absolutely infinite, entirely notional, and, in large part, merely potential. To include in the denotation with these perfect imaginary circles, the actual 'circles' traced on paper would be to include objects which only approximate more or less roughly to the connotation. These latter, being material objects, give, of course, a denotation to the word 'circle' when it is understood to refer to them, and not to be strictly limited to the mathematical concept. This denotation embraces all such figures which are now in existence; those which have been drawn formerly

but have since been destroyed, or those which may be drawn in the future, cannot be said to form part of the denotation of the word *now*, though they did, or will do so, at a different point of time. Thus the denotation of physically real things is limited in time, and so varies with time; for instance, the denotation of 'man' includes all human beings now living, and none else. If we speak of an extinct animal "like the Dodo or Moa, then I do not think we can "avoid a reference to the element of time, and must say that it has "*now* no denotation" (Venn, *Empirical Logic*, p. 179). Though there is force in this, it seems better, on the whole, to hold to the ordinary view—that is, to make 'Denotation' wide enough to cover all things to which the name can be correctly applied. If we do not, it becomes necessary to employ some other word—such as Application or Denomination—in this wider sense.

The Denotation of fabulous objects is to be sought in mythology, etc.

But we may speak of creatures purely fabulous, as dryads, centaurs, or griffins; can the terms we then use be said to have denotation? We agree with Dr. Venn (*op. cit.*, p. 180) that they can, and that their denotation must be sought in the appropriate sphere of existence—that of mythology, fable, or heraldry, as the case may be. This is, of course, using the word 'existence' in a somewhat wider sense than is common in ordinary speech, but it does not seem to do violence to it, and the extension is necessary to enable it to include entities having an existence only in thought or fancy, such as the characters of romance.

It may be said, then, that the Denotation of a Term is the aggregate of all which, when presented to us, we should mark by the name; and that this aggregate must be sought in the appropriate realm, whether of fact or of fiction (*cf.* Venn, *Emp. Log.*, p. 176).

Universe of Discourse—the limited sphere within which a term is intended to apply at any particular time.

* In connexion with the subject of Denotation it will be well to mention what is called the *Universe of Discourse*, that is "not the whole range of objects to which a general term "can be correctly applied—which is the denotation—but "merely the restricted range to which the speaker at the "time being intends his remarks to apply" (Venn, *Emp. Log.*, p. 180). Of course, were terms always precisely used, their denotation would coincide with this universe, but in speech they are always modified and limited by the context

expressed or understood (*cf.* § 3), and thus, both speaker and hearer constantly restrict the application of a term to some portion only of its denotation. If, for instance, we say 'Everybody says so' we certainly do not intend the term 'everybody' to be taken in its full extent so as to embrace all the inhabitants of the world; we probably refer, and are understood to refer, to a very few persons. Similarly the term 'Europeans' is restricted, in most cases of its use, to human beings. The limits of this universe, which are purely arbitrary, are left to be tacitly understood, and cannot, of course, be expressed symbolically; the restriction is material, not formal. Nevertheless, it is important to bear in mind that terms are continually joined together into propositions in a sense narrower than the words themselves warrant, and that such propositions are only intended to apply within this limited sphere.

The phrase is not, however, always used in a limited sense. It always denotes the whole idea under consideration, and this may coincide with the denotation of the term. Thus Boole says: "The universe of discourse is sometimes "limited to a small portion of the actual universe of things, "and is sometimes co-extensive with that universe" (*Laws of Thought*, p. 166). And Mr. Keynes adds: "It must be "clearly understood that the universe of discourse is by no "means necessarily identical with the region of what we "ordinarily call 'fact'; it may be the universe of dreams, "or of imagination, or of some particular realm of imagina-"tion, *e.g.* modern fiction, or fairy-land, or the world of the "Homeric poems" (*Formal Logic*, 3rd Edition, p. 183).

As a general rule an increase in Connotation reduces the number of sub-classes of things denoted, and *vice versâ*; an increase in number of sub-classes denoted decreases the Connotation, and *vice versâ*.

(v.) Relation between Connotation and Denotation.

As Connotation implies attributes, and Denotation refers to the individual objects which possess those attributes, and which usually form various sub-classes, it is evident that, as a general rule, an increase in either one will cause a decrease in the other. As we augment the number of attributes implied by a name we diminish the number of things to which that name is applicable, for we exclude some of the sub-classes; there are, for instance, fewer white

horses than horses. Conversely, if we wish to include under a name a group of things not before included under it, and so to enlarge the borders of the class which the term denotes, we can, usually, only do so by removing from the implication of the name those attributes which before marked the difference between the two classes, or, in other words, by decreasing its connotation. For instance, if we unite the classes, white men' and 'not-white men' we must omit from the connotation of the common term all specification of colour; similarly, if we wish to include both sailing ships and steam ships under one common name, we must omit the points of difference, 'sailing' and 'steam,' and retain only the term 'ship,' which will be applicable to all the members of both classes but which implies less than the separate name of either. In short, generally ~~speaking~~, the less a name implies, the more groups of things it is applicable to, and the more it implies the narrower is its range of application. If, as a very simple instance, a word is taken which connotes only one attribute, such as 'white,' it is at once evident that no increase of meaning is possible which will not decrease its denotation, for now it embraces all white things whatever, but the addition of any attribute must limit it to some only of those objects, as, for instance, if we speak of white animals, white cloth, white paper, etc. It was shown in contrasting Significant Individual Names with Proper Names how the continued addition of attributes increases the connotation and decreases the denotation of a term, till at length the latter is reduced to unity, and the former has become the fullest which that term is capable of bearing, so that connotative singular terms are the most significant of all names [*see* § 27 (i.) (*b*)].

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This general relation between the connotation and denotation of terms can be represented symbolically. If the connotation of the term *A* be *wxyz*, of the term *B* be *pwxy*, of the term *C* be *pwxz*, and of the term *D* be *pwyz*, then the union of any two of the classes denoted by those terms decreases the connotation by one element, the addition of a third class reduces it by one more, whilst the union of all four classes into one causes the connotation to become *w* alone, for

This can be represented symbolically.

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that is the only element common to the connotation of all. Thus the connotation of the class $A+B$ is wxy, of $B+D$ is pwy, of $A+B+C$ is wx, and $B+C+D$ is pw, and so on with each different combination of the classes denoted by A, B, C, D. Conversely, if we have the connotation w it will embrace all the groups of things denoted by $A+B+C+D$, if we increase this connotation by adding x to it, the denotation is reduced to $A+B+C$, a further enlargement of the connotation to wxy again diminishes the denotation to $A+B$, and the final addition of z to this connotation makes the name applicable to the class A alone. But we do not know how many individuals are included under A, B, C, or D, nor can we tell how many attributes may be included under the symbols p, w, x, y, z, each of which may represent a whole group.

Connotation and Denotation do not vary in inverse ratio.

It cannot, therefore, be said that Connotation and Denotation vary in inverse ratio to each other; such a mathematical conception is quite inappropriate. We can speak intelligibly of halving or of doubling the denotation of a term, but it is meaningless to talk about doubling or halving its connotation; and even could we do so there would be no ratio maintained in the variation of the two aspects of the term. The application of a term is limited by the addition of some attributes much more than by that of others; thus, to add 'white' to man would not limit the denotation nearly so much as to add 'red-haired' for there are many more white men than there are red-haired men. Similarly, in the example we before considered [§ 27 (i.) (*b*)] —mountain—mountain in Asia—high mountain in Asia—the highest mountain in Asia—it is evident that some of the additions to the connotation of 'mountain' decreased its denotation much more than others. Moreover, it is not true that an addition to the connotation of a term will always cause a decrease in its denotation; for as a name does not usually connote every attribute common to a class, the addition to the connotation of any number of these common attributes not included in it will not affect the denotation; there are, for instance, as many mortal men as there are men; so, though 'mortal' is not part of the connotation of man, yet to speak of 'mortal men' does not

Every addition to the Connotation of a Term does not decrease the Denotation.

narrow the limits of the class 'men.' Attributes of things are in Nature very often found in groups, so that where one is found others are found too; and it is evident that, when this is the case, the addition of any of these attributes to the connotation of a term will not limit its denotation so long as the one member of the group with which they are all connected already forms part of that connotation. Thus, when a triangle is equilateral it is also equiangular, and to speak of equiangular equilateral triangles does not, therefore, limit the denotation given by equilateral triangle. So to add to 'right-angled triangle' the attribute 'having the square on the hypothenuse equal to the sum of the squares on the sides' brings in no fresh limitation, for that attribute is one of a group necessarily found wherever the property 'right-angled' is joined to triangle. There may, thus, be many additions to the connotation of a word which will have no effect on its denotation.

The opposite variation of Connotation and Denotation only exists in a series of classes.

It is, perhaps, scarcely necessary to point out that the idea of an opposite variation of Connotation and Denotation is only applicable to classes which can be arranged in a series of varying generality, so that each smaller class forms a part of the next larger; such as, figure, plane-figure, plane rectilineal figure, plane triangle, plane isosceles triangle, plane right-angled isosceles triangle; vehicle, carriage, railway carriage, saloon railway carriage, first class saloon railway carriage, first class dining saloon railway carriage. It would be absurd to say that an increase or decrease in the number of members of any one class affects the connotation of the class name; that, for instance, the birth of every baby must decrease the number of attributes implied by the term 'human being,' and that the death of each man, woman, and child, must increase that number. It is only when we add an attribute not common to the whole class that we exclude some members of the class from participation in the class name and so decrease the denotation; or when we introduce into a class some things not possessing all the attributes connoted by the class name, that we have to omit part of its meaning, that it may cover the whole of

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this more extended class; and thus we decrease the connotation. The increasing the connotation and thereby limiting the application of the term is a process of *Specialization*, the opposite process of decreasing the connotation so as to embrace a larger number of objects is one of *Generalization* (*cf.* § 3).

Of the Synonyms of Connotation and Denotation, *Intension* and *Extension* are the only two which have been generally used.

* (vi.) **Synonyms of Connotation and Denotation.** Both the Connotation and the Denotation of Terms have been spoken of in Logic under a great number of names. Thus, instead of Connotation, we find the terms Intension, Intent, Comprehension, Depth, Implication, and Force; whilst the Denotation has been correspondingly styled Extension, Extent, Sphere, Breadth, Application and Scope. None of them have come into general use except *Intension* and *Extension*, which are often used to express from the side of the concept what Connotation and Denotation express from the side of the term. Denotation (Lat. *de*, down; *notare*, to mark) and Connotation (Lat. *con*, with; *notare*, to mark) have the advantage of expressing by their etymological meaning exactly what we want to express when we use them; we 'mark down' the objects which we name and we 'mark with' them their attributes. Thus these terms are most expressive and appropriate when we deal with the forms of language in which thought is expressed. But it would be undoubtedly convenient could such a term as *Content* or *Intension* be used to express both the direct and indirect implication of a term (*cf.* ii.).

Connotation and Denotation are the most expressive.

29. Positive and Negative Terms.[1]

Incompatible Terms are those which imply attributes which cannot co-exist in the same subject.

The formal distinction of Terms into Positive and Negative is a particular case of the *Incompatibility of Terms*. *All Terms whatever which imply attributes which cannot co-exist in the same subject are incompatible.* This incom-

[1] In the whole of this section the treatment follows, generally, that of the same subject by Dr. Venn (*Emp. Log.*, pp. 191-5).

patibility may be expressed either by Contradictory, by Contrary, or by Repugnant Terms. The division into Positive and Negative is the formally logical means of marking the first of these; that distinction will, however, be more clearly understood if all three kinds of incompatibility are considered.

Contradictory Terms are mutually exclusive and collectively exhaustive in denotation.

(i.) **Contradiction—For two terms to be contradictories it is necessary that they be mutually exclusive and at the same time collectively exhaustive in denotation;** that is, they must be incapable of being predicated at the same time about the same subject, and between them they must embrace everything in the Universe of Discourse. Now, it is evident that this contradiction may be marked in two ways—knowledge of the matter may tell us that two terms are contradictories; or the very form in which the terms are expressed may imply this; the former is *Material Contradiction*, the latter is *Formal or Logical.*

Material Contradiction can only be known by examination of the facts.

* (*a*) *Material Contradiction.* In some important instances where two groups of things which fulfil both the conditions of contradiction are equally important, and equally possess a great richness of meaning, they have each a distinct name. These names do not, in any way, imply the contradiction, which can only be known by examination of the facts, for the names are not constructed for the purpose of indicating the contradictory relation which exists between them. Each is connotative, that is, each is the name of a true class of things which, like all other true classes, is marked by the presence of attributes common to every member of it. To understand the contradiction, therefore, we must know the connotation of each term. It is not necessary that these connotations should be entirely different from each other—they may and generally do possess common elements—but the attributes, or groups of attributes, which are not common must be such that every individual in the universe of discourse possesses one or the other, but no single individual possesses both. Such instances are, of course, rare, and nature is so diversified and so limitless that when

Material Contradictories, as a rule, partially coincide in connotation.

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They are generally limited in application

they do occur they never embrace the whole of existence—their application is always tacitly limited to the Universe of Discourse which was described in the last section. This Universe may be very wide, as when we speak of male and female, but usually it is somewhat narrow, and the more it is limited the more numerous are the pairs of material contradictories which can be found within it. Thus, when we speak of British and Foreign we are using terms which are contradictories in the realm of material things, but which would be utterly inapplicable to abstract ideas—no meaning can be attached to 'British Justice' or 'Foreign Honesty' except when by the words 'Justice' and 'Honesty' we mean, not the abstract quality which is the same everywhere and always but, certain acts of Justice or of Honesty. Hence, we see that the attribute 'material' is part of the connotation of both British and Foreign; as 'living organism' is part of that of both male and female; and this illustrates what was said above, that there may be common elements in the connotation of material contradictories. If we restrict our universe still more we have the contradictory terms British and Alien, which are applicable only to human beings; and a further limitation of the universe to English human beings furnishes the pair Peer and Commoner. With each limitation we see that the common part of the connotations of the contradictory terms increases, as might be anticipated from the general relation between denotation and connotation discussed in the last section; for it is the denotation of the universe we are decreasing directly, and only indirectly through that the denotation of the contradictories. However, for Logic, such material contradictories are all of the same class—they are all positive, for they all have a connotation which implies the possession of certain attributes; and, as has been said, they are comparatively few in number.

Each term is positive, *i.e.*, connotes the presence of attributes.

Words with negative prefixes are not, in most cases, true contradictories of the corresponding simple terms.

As the need of many more contradictories than these words supply was felt, common language began to form them by the addition of a negative prefix or affix. We have many instances in English in words beginning with the

prefixes *in- un-*, *non-*, *mis-*, etc., as insincere, unkind, nonsense, misfortune, or ending in *-less*, as senseless.

But these words have, in most cases, ceased to be true contradictories of the corresponding simple terms, and have only remained so when, in instances such as equal—unequal, no intermediate idea is possible. When such an intermediate idea can be formed the connotation of the negative words has tended to become more remote from that of the positive words and has itself taken on positive elements. Thus, 'happy' and 'unhappy' are not contradictories, because they leave an intermediate state of indifference between them ; we are often neither happy nor unhappy, for the latter word does not simply imply the absence of happiness but, in addition, the presence of positive misery. Common language, in fact, very seldom expresses sharp distinctions, and the meaning of a term adopted into common speech tends to approximate to a kind of average of the things to which it is applicable. Thus, as 'unhappy' would apply to anything from the simple negation of positive happiness to the most intense misery, the word gradually took into its connotation some elements of discomfort, and now signifies a state intermediate between indifference and deep misery. So with unkind, unholy, senseless, misfortune — they are no longer the simple negations of kind, holy, sensible, fortune ; for neither pair between them exhausts the universe. The same remarks are applicable to nearly all this class of words, which are therefore, although incompatible, not contradictory terms. It must also be noticed that negative prefixes and affixes sometimes do not imply the negation of any attribute at all —thus 'shameless' is not the negation of 'shameful' but almost a synonym with it, and 'invaluable' means valuable in the highest degree.

Their connotation has positive elements.

(*b*) *Formal or Logical Contradiction.* The necessity of excluding any intermediate ground is the justification of logical contradiction, which consists in prefixing *not-* or *non-* to the term—thus not-happy simply excludes 'happy,' 'not-white' shuts out white, 'not-man' removes 'man' and

Formal logical contradiction consists in prefixing *not-* or *non-* to the Term

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The connotation of such terms is purely negative.

applies to all else in our universe of discourse. The connotation of such terms is, therefore, simply negative; they imply nothing but the absence of all, or some, of the attributes connoted by the term to which they are prefixed. Thus, formal differs from material contradiction in this, that whereas in the latter the meaning of each term has to be apprehended separately, in the former the meaning of one is enough—it furnishes us, also, with the meaning of the other. As this negative particle can be applied to any name whatever, it is clear that this distinction is exhaustive of all terms; those without the negative particle are *Positive*, those with it are *Negative*. Thus formally, unhappy is positive as well as happy, not-unhappy is negative, as is not-happy. Of course, in the case of such terms as equal and unequal, where there is no third alternative, unequal is negative for it is simply the same as not-equal; it negates equal but it implies nothing else, as it has been shown such words as unhappy, unkind, etc., do.

A Positive Term implies the presence of attributes. *A Negative Term* simply implies their absence.

We may say, then, that *a Positive Term implies the presence of an attribute or group of attributes*, and *a Negative Term simply implies the absence of the attributes connoted by the corresponding positive term, but implies the presence of no attributes whatever*.

A logical negative term is limited, in practice, in its application.

As this negation is purely formal, and the class denoted by the negative term is wholly arbitrary with the presence of no common attributes implied by its name, it is evident we may make this class as wide as we please. Many Logicians, in fact, and Mill amongst the number, make it include all existence except those things which are denoted by the positive name. Thus Mill says, in reply to Bain, who would restrict the application of *e.g.* not-white to coloured things, "In this case, as in all others, the test of what a name de-"notes is what it can be predicated of: and we can certainly "predicate of a sound, or a smell, that it is not white" (*Logic*, Bk. I., ch. ii., § 6, note). It is true we can form the combination, 'This sound is not-white,' but is it not absolutely unmeaning? If we represent any term by *A*, then its negative is represented by *non-A*, and to ask us to form an idea of *non-A* which shall embrace everything in existence except those things which are *A*, is to

demand an impossibility. We may form combinations, but, like the one just quoted, they will be, for the most part, absolutely meaningless; and we decline to regard as a true expression of a logical judgment a meaningless jumble of terms simply because they are connected by the copula. This is, no doubt, the formal meaning of such a purely negative term, but we have already given reasons (*see* § 19) for holding that its application is not, in practice, thus infinite, but is restricted to the universe of discourse just as much as material contradictories are; *i.e.* that formal and material contradictories have practically the same denotation, and differ only in connotation. In the universe of human beings, not-British = alien; not-alien = British; in that of material things generally, not-British = foreign; not-foreign = British, with regard to their denotation. And so with every term; not-white applies to all colours except white, but it has no meaning when applied to things which possess no extension in space. There is no doubt that this limitation is always intended in common speech, and it can always be gathered from the context when it would be otherwise ambiguous —not-light may, for instance, belong to the universe of weight, or to that of brightness. Of course, if we are given simply the symbols *A* and *non-A*, we cannot tell what the limits of our universe are meant to be, but we know that if we had to translate those symbols into ordinary language the limitation would become apparent. Though *non-A* simply negates the possibility of *A*, yet to be real this negation must rest on the fact that some quality is present in *non-A* which is incompatible with that connoted by *A*. Now, it is obvious that heavy is certainly not-yellow if that term is to be extended from the universe of colour to embrace all existing things and attributes except yellow; yet the attributes 'yellow' and 'heavy' are by no means incompatible—they may, and do, exist in the same subject, *e.g.* in gold.

Its denotation is the same as that of a material contradictory of the same term.

We hold, then, that the so called *Infinite* or *Indefinite Terms* which simply mark an object by exclusion from a class, and are supposed to embrace all existing or conceivable things except those contained in that class, as not-white to include sounds, tastes, hymn-tunes, half-holidays, etc., etc, are not merely logical figments, but are absolutely useless and positively misleading—they take the garb of general terms but there is no concept corresponding to them.

***Infinite* or *Indefinite* Terms simply mark an object by exclusion from a class. These are logical figments.**

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* This idea of logical contradiction in terms has no real application to Proper Names, as they imply no attributes to be negated. Such a term, therefore, would obviously be a mere sham ; no idea can possibly correspond to it.

Contrary or *Opposite Terms* express the greatest possible divergence in the same universe.

(ii.) **Contrariety.** Whilst logical contradictions simply negate each other, common speech can do more than this; it can express degrees of divergence, as we saw in the case of the terms 'happy' and 'unhappy' as contrasted with happy and not-happy. **When two terms express the greatest degree of difference possible in the same universe they are said to be Contrary or Opposite Terms:** thus black—white; wise—foolish; strong—weak; happy—miserable are pairs of contraries. This distinction is entirely material and cannot be represented symbolically; Formal Logic can only take notice of and express formal contradiction. The idea of contrariety rests on the assumption that we do not simply divide our universe into two classes as in Formal Contradiction, but into a series of groups which have no sharply defined boundaries, as pleasant, indifferent, unpleasant, painful, where the extreme terms are contraries.

This distinction is material.

Under this head may be included *Privative Terms*, which are often defined as those which imply the absence of an attribute which the subject either has had or might be expected to have, as deaf, a word which is equivalent to not-hearing and which would not, in ordinary every-day life, be applied to an object unless, as a rule, things of that class possessed the capacity for hearing—not to rocks or trees or other things which never possess that attribute (*see* Mill, *Logic*, Bk. I., ch. ii., § 6). Other instances of such terms are blind, dumb, lame, etc. Thus understood, Privative Terms are of absolutely no importance, and it has, therefore, been proposed to slightly contract the connotation of *Privative* so as to make it *signify* "*the absence of an attribute in a subject capable of possessing it*" (Stock, *Ded. Log.*, pp. 35-7), but with no presumption as to the probability of its presence. This contraction is so slight and the corresponding extension of its denotation is so convenient that we shall adopt it. In

A Privative Term implies the absence of an attribute in a subject capable of possessing it.

this sense, then, Privative Terms include most of those words formed with negative prefixes or affixes, as unkind, unhappy, as well as terms similar to those already mentioned, for we should not apply 'unkind,' except to a morally responsible being and one who is, therefore, capable of being kind, nor 'unhappy' to a being incapable of enjoying happiness. It should be noticed that the connotation of a Privative Term is partly negative, in that the word implies the absence of a certain attribute, and partly positive, in that it always implies the presence of some attributes which are compatible with that denied as well as very often of some others which are incompatible with it. Thus 'unhappy' implies absence of happiness, capacity for feeling happiness, and presence of some degree of misery.

Repugnant Terms are incompatible with each other, though neither Contraries nor Contradictories.

* (iii.) **Repugnance. Terms are repugnant to each other when, without being directly contrary, or contradictory in the logical sense of exhausting the universe between them, they are yet incompatible in that they are mutually exclusive.** Very often, when we examine the denotation (in the restricted sense advocated above) of a negative term, as not-white, we find it embraces several well-defined groups, as green, red, blue, etc., no two of which can be predicated of the same thing at the same time; these terms are repugnant to each other. So we may speak of articles of furniture as being either of wood or not-wood, and when we examine the latter group we find it contains things made of gold, of silver, of brass, of steel, of iron, of stone, and of many other materials; all these, again, are repugnant terms, for no two of them are either contradictories or contraries and yet no two can be predicated of the same thing. This is as entirely a material distinction as is that of Contrariety, and we see, then, that of all the forms of incompatibility of terms employed in common speech, Formal Logic only recognizes that of Formal Contradiction, *i.e.*, the division of terms into Positive and Negative—into *A* and *non-A*.

The distinction is material.

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30. Concrete and Abstract Terms.

(i.) Relation between Concrete and Abstract Terms. The division of Terms into Concrete and Abstract is founded upon psychological and grammatical rather than upon logical reasons. It is, however, usual to consider it as part of the logical doctrine of Terms. The following definitions express the difference:—

A *Concrete Term* is the name of an object. An *Abstract Term* is the name of an attribute considered by itself.

A Concrete Term is the name of an object.

An Abstract Term is the name of an attribute considered by itself.

In the above definition the word 'object' is used widely to denote everything, whether material or not, which can be regarded as having a more or less separate existence as a whole whose parts or elements are in essential relation to each other and to the whole which comprises them. It thus includes such names as Logic and Ethics; and as point, line, etc., in their strict mathematical sense.

* Abstract Terms are formed by the process of abstracting the attention from all the qualities of a thing, except some particular one, or group, to which the name is then given. Thus, by attending to one quality only of a tree, we form the idea of greenness; by considering only the moral quality of a number of good actions we gain the concept of virtue. But it is evident that all General Terms represent concepts which are formed by Abstraction; we must not, therefore, regard this process as a sufficient ground for calling a term Abstract. If we did, we must include in that class all Terms whatever except Proper Names. A Term can only be called Abstract when it denotes a quality which, though it can only exist in some object, is yet thought of apart from all objects whatever. Thus, we can think of 'strength' by itself, although we know there can be no strength except as an attribute of strong things; or, of virtue, though it cannot exist apart from good actions.

The terms 'Concrete' and 'Abstract' have been used in various senses by logicians. This has caused some objection to their use in Logic altogether, and Miss Jones has suggested *Substantial* and

Attribute terms as more appropriate names; the reference to individual objects being prominent in the former, and to attributes in the latter (*Elem. of Logic*, pp. 12-15; 37-39). The distinction drawn exactly coincides with that taken here, and it may be at once granted that the new nomenclature, if generally adopted, would be a distinct improvement.

All Adjectives are Concrete Terms.

If it be borne in mind that an Abstract Name is not simply the name of a quality, but of a quality considered by itself, and apart from the objects which possess it, it will be immediately seen that Adjectives are not Abstract Terms; for they name qualities only indirectly, and considered in connexion with the things to which they belong. If we say 'Gold is yellow,' we do not mean that gold is a colour, but that it is a thing which possesses a certain colour. It is the colour of gold which we call yellowness, not gold itself. 'Yellowness' is, then, the name of the colour or quality; 'yellow' is the name of all *objects* which possess that quality. Whether a name is Abstract or Concrete will often depend on the sense in which it is used; for the same word may be Concrete in one sense and Abstract in another.

The distinction between Concrete and Abstract is not a fixed one.

The distinction is only logically important where the terms form pairs.

The importance of the distinction between Concrete and Abstract, however, from a logical point of view, lies in those pairs of terms wherein one is the Abstract and the other Concrete, as strong, strength; man, humanity; square, squareness; etc., and in these cases there can never be any doubt as to which is Concrete and which is Abstract. In all such pairs the Concrete term is General, and the older Logicians confined the distinction to such pairs of terms; but, in order to make the division exhaustive of all Terms, we may regard all Individual names, including Proper Names, as Concrete, though they have no Abstract terms corresponding to them. Every Concrete General Term has not, in fact, an Abstract corresponding to it, but one is always theoretically possible, and new ones are continually being coined as occasion arises. On the other hand an Abstract Term can only be expressed in Concrete terms, and that but imperfectly, by an awkward periphrasis. Thus, instead of strength we can speak of 'strong things considered only in

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Abstract Terms abbreviate thought.

their aspect of being strong.' Abstract Terms are thus a great help towards abbreviating and systematizing thought; the proposition 'Union is strength' is a much neater and more universal expression than 'Things which are joined together, considered only with respect to their being joined, are strong,' but only by such an awkward and involved sentence can we express approximately the meaning conveyed by the three words of the original proposition. If we simply say 'Things which are joined together are strong' we do not indicate that we are concerned in the subject only with the attribute 'being joined together,' and this is the very point which the proposition 'Union is strength' emphasizes. Similarly 'Justice is a virtue' would receive a fair expression only in some such sentence as 'Just acts, in so far as they are just, are virtuous.'

Abstract Names of single attributes are Singular.

(ii.) **Singular and General Abstract Terms.** Every Abstract Term which is the name of a single attribute is Singular, for though the attribute may be possessed by many objects yet it is conceived by us as one and indivisible. Thus though there are many square things, yet the attribute 'squareness,' which is common to them all, is evidently only one, we cannot imagine different species of 'squareness.' So with 'equality' and all other terms which are the names of simple attributes. But when we have a group of attributes, such as colour, which embraces red, green, white, etc.; or humanity, which includes animality and rationality; then its name will stand for all these and we must regard it as General with respect to these included notions.

Abstract Names of groups of attributes are General.

(iii.) **Connotative and Non-connotative Abstract Terms.** It has been much disputed amongst logicians whether Abstract names can ever be connotative. Mill held that they could. He says: "Even abstract names, though the "names only of attributes, may in some instances be justly "considered as connotative; for attributes themselves may "have attributes ascribed to them; and a word which denotes "attributes may connote an attribute of those attributes" (*Logic*, Bk. I., ch. i., § 5). Jevons, however, objects to

All singular Abstract Names are non-connotative.

this and holds that no Abstract Name can be connotative. Singular Abstract Names are generally regarded as non-connotative; they denote the attribute which the Concrete Names connote and there seems nothing left for them to connote But when an Abstract Term is the name of a group of attributes there seems no good reason for denying that it is connotative. Thus, virtue is a name common to justice, benevolence, veracity, and other qualities of conduct which men agree in regarding as praiseworthy; it, therefore, denotes those good qualities, and connotes the attribute 'goodness' which they possess in common. So, 'colour' denotes redness, blackness, blueness, etc., and connotes the power of affecting the eye in a certain way: 'figure' denotes roundness, squareness, triangularity, etc., and connotes shape and extension in space. We thus reach the conclusion that all General Names, Abstract as well as Concrete, are connotative, and this view seems the only one which is compatible with the nature of a General Name. For, the same name can only be applied to a number of objects of thought, whether things or attributes, because they agree in the possession of some common quality; the name must connote this common property and denote the objects of thought which possess it; hence, every General Name must, of necessity, have both connotation and denotation, that is, it must be connotative. Those, therefore, who deny that any Abstract Names can be connotative, must also, in consistency, deny that any can be General.

31. Absolute and Relative Terms.

This division of terms is based on the fact that the relations of things differ from their other attributes, in that they involve direct reference to more than one object. If, for instance, we speak of a man as strong we can confine our attention to that one individual, but if we speak of him as a friend we must at once extend our view to include some other person who stands to him in the relation of friendship. We may say, then, that

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An *Absolute Term* implies no reference to anything else.

An Absolute Term is a name which in its meaning implies no reference to anything else.

A *Relative Term* implies a reference to an object related to that which it denotes. These two are *Correlatives.*

A Relative Term is a name which, over and above the object which it denotes, implies in its signification another object which also receives a name from the same fact or series of facts which is the ground of the first name.

Each one of such a pair of terms is called the *correlative* of the other. In some cases each correlative has the same name, as friend, companion, partner, like, equal, near; in other cases the names are different, as parent, son; king, subjects; governor, governed; cause, effect; greater, less; north of, south of. But they are always found in pairs, and they always owe their names to the same fact or series of facts. Such pairs are equally common among Adjectives and Substantives, for attributes may be thus related as well as things. Neither member of the pair can be thought of alone; for the existence of each depends on, and implies, that of the other; there can be no meaning in parent, except in reference to son or daughter, nor in son or daughter except as implying parent; the Absolute Name for the same individuals would be human being.

To be correlatives, the Terms must *imply* the relation existing between the things denoted.

It must be remembered that it is the fact that the *terms imply* the relation in which the objects stand to each other which makes them Relative, not the mere *existence* of the relation; thus a king governs men, but king and man are not correlatives, for the terms do not imply this relation; king and subject are correlatives because they do imply it. In one sense it may be said that all terms are relative, for all things in Nature are interconnected—there is nothing which exists utterly by itself and out of all relation to everything else. This is true, but it is not the sense in which 'Relative' is used in Logic; if it were, we should have everything entering as a member into innumerable pairs of correlatives. *In Logic terms are only relative when the existence of the correlative is implied by the meaning of the term itself.*

All Relative Terms must connote the fact or series of facts which is the basis of the relation and which is, therefore, called the *fundamentum relationis*. Thus, each member of a pair of correlatives connotes the same fact viewed from a different standpoint ; paternity and sonship are not two different facts but the same fact viewed from two different sides, and connoted both by parent and by son. So rule and subjection imply the same condition of things regarded from the point of view of the ruler and of the subject respectively.

The *Fundamentum relationis* is the fact which is the basis of the relation expressed by Correlative Terms.

The abstract terms, then, which are the names of the fact or facts on which the relation depends, cannot be regarded as correlatives ; for they do not denote two facts but one, and a thing cannot be correlative with itself. Hence, parent and son are correlatives, but paternity and sonship are not. This will be seen more clearly in the case of those pairs of correlative terms in which each member has the same name ; friend is correlative with friend, but the abstract term denoting the connecting bond is friendship, no matter from which side we regard it ; so with partnership, which is the *fundamentum relationis* of the correlatives partner, partner. Similarly, equality and likeness are not Relative Terms though equal and like are ; for it requires two things to be either equal or like, but the fact of equality (or likeness) is one.

Only Concrete Terms can be Relative.

Thus, only concrete terms can be Relative, and of them only those which denote things whose existence absolutely alone is inconceivable ; it is impossible to imagine a parent as the only being who had ever existed in the world, for then only an absolute name, as human being, would be applicable. All terms which do not thus necessitate reference to some things other than that of which they are the names are Absolute. The distinction is not, however, of much importance in Logic.

An object whose name is a Relative Term cannot be conceived as existing absolutely alone.

CHAPTER III.

THE PREDICABLES.

32. Definition of Predicable.

The *Predicables* are a classification of the relations of the predicate to the subject of a logical proposition. A Term does not always belong to the same Predicable.

The Predicables are a classification of the relations of the predicate to the subject of a logical proposition. They do not express what a term is by itself, but only what relation it bears to the subject of the proposition of which it forms the predicate. We cannot absolutely refer any General Term to one definite Predicable; for the same term must be assigned now to one and now to another of the predicables according to its relation to the subject of which it happens, in any particular proposition, to be predicated. The consideration of the Predicables might, therefore, have been postponed till we had examined the doctrine of Propositions, and many logicians do so postpone it. The point is of no practical importance, for the whole doctrine of Terms is of logical interest only because terms are constituents of propositions; and, as this subject is concerned with those constituents and not with propositions as wholes, it seems better to treat it here.

33. Aristotle's Four-fold Scheme of Predicables.

Aristotle drew out a four-fold scheme, founded on Laws of Contradiction and Excluded Middle: (a) *Definition.* (b) *Proprium.* (c) *Genus.* (d) *Accidens.*

The first classification of the Predicables was a four-fold division made by Aristotle which was thus evolved. In every proposition the predicate must either agree or disagree in denotation with the subject. If it agrees it either has the same connotation or a different connotation; in the former case the Predicate is a *Definition*, in the latter case a *Proprium*. Thus in 'Man is a rational animal,' we have a definition; for 'rational animal' agrees with 'man' both in denotation and connotation; whilst in 'Man is an animal which

cooks its food,' the predicate is a Proprium, for though 'animal which cooks its food' agrees exactly in denotation with man—*i.e.* refers to the same individuals and to no others—yet 'to cook food' is no part of the connotation, or meaning, of the word 'man.' If, on the other hand, the Predicate has not the same denotation as the Subject, then its connotation is either partly the same or entirely different. It evidently cannot be entirely the same, for then there could be no difference in denotation; for, as a name denotes all things which possess all the attributes which it connotes and no others, it follows that if two names have identically the same connotation, they must be applicable to precisely the same things, *i.e.* have exactly the same denotation. If, then, whilst differing in denotation from the Subject, the Predicate partially agrees with it in connotation, it is a *Genus;* if it entirely disagrees, it is an *Accidens.* For example, in the proposition, 'Man is an animal,' the predicate is a Genus; for, whilst the denotation of 'animal' is greater than that of man, its connotation is less; but in the proposition, 'Some men are woolly-haired,' we have an Accidens; for the predicate, 'woolly-haired things,' differs from the subject 'men' both in connotation and in denotation. These four heads of Predicables may be thus defined:

A Definition is the aggregate of all the attributes which fully explain the nature of the subject.

A Proprium is a mark which invariably belongs to the subject, and to nothing else, but is not the attribute which would be mentioned to explain the nature of the subject.

A Genus is a mark or attribute which invariably belongs to the subject, but not to that alone.

An Accidens is an attribute which may or may not belong to the subject.

The whole scheme may be thus summarized:

Predicables.	I. Agreeing in Denotation with Subject.	1. Agreeing in connotation with subject.	*Definition.*
		2. Differing in connotation from subject.	*Proprium.*
	II. Differing in Denotation from Subject.	3. Partially agreeing in connotation with subject.	*Genus.*
		4. Wholly differing in connotation from subject.	*Accidens.*

This scheme has been superseded by Porphyry's.

This scheme, being founded on the Laws of Contradiction and Excluded Middle, is evidently exhaustive of all General Terms when used as predicates. It has, however, been practically superseded by a five-fold division, which was first advanced by Porphyry in his 'Introduction to the Categories of Aristotle,' written in the third century, and which has ever since occupied a prominent place in the traditional logical doctrine.

34. Porphyry's Five-fold Scheme of Predicables.

Porphyry's Five-fold scheme: (a) *Genus.* (b) *Species.* (c) *Differentia.* (d) *Proprium.* (e) *Accidens.*

The traditional classification of predicables is that of Porphyry, and is closely connected with the subjects of logical definition and division. It is as follows :—

Predicables are	1. *Genus*	...	γένος	of the Subject.
	2. *Species*	...	εἶδος	
	3. *Differentia*	...	διαφορά	
	4. *Proprium*	...	ἴδιον	
	5. *Accidens*	..	συμβεβηκός	

We will now briefly define these five Heads of Predicables or 'Five Words,' as they are frequently called, and will then consider them more in detail.

A Genus is a wider class which is made up of narrower classes.

A Species is a narrower class included in a Genus.

A Differentia is the attribute, or attributes, by which one species is distinguished from all others contained under the same Genus.

A Proprium is an attribute which does not form part of the connotation of a term, but which follows from it, either as effect from cause or as a conclusion from premises.

An Accidens is an attribute which neither forms part of the connotation of a term nor is necessarily connected with any attribute included in that connotation.

35. Genus and Species.

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These are not absolute terms, but purely correlative. A Genus has no meaning apart from the two or more species into which it is divided; nor has a Species apart from the containing genus. The same term may be, at the same time, a Species of the next more general class, and a Genus to the less general classes it contains; no term by itself can be styled a Genus or a Species. Thus, in the example quoted in § 28 (v.) to illustrate the relation between Connotation and Denotation of Terms, each intermediate term is a species to the preceding term, and a genus to the succeeding. If a term is so general that it is not a species of any more general term it is called a Highest Genus or *Summum Genus;* and if it cannot be further divided into species, but only into individuals, it is a Lowest Species or *Infima Species.*

Genus and Species are correlative Terms.

A *Genus* is a wider class, containing two or more narrower classes called *Species.*

A *Summum Genus* is not a species of any wider term.

An *Infima Species* can only be divided into individuals.

The Aristotelian logicians held that there were ten *summa genera*, which they called Categories or Predicaments (*see* Bk. I., ch. iv.), and each such Summum Genus, with a series of terms below it following in the order of less and less generality down to an Infima Species, was called a *Predicamental Line* (*Linea Predicamentalis*). As every genus must have at least two species included under it, it follows that each Predicament must have a plurality of predicamental lines; as many, in fact, as there are *infimæ species.* In every such predicamental line, the general inverse relation between the connotation and denotation of terms is exemplified: the *Summum Genus* is the widest in denotation, but the most meagre in connotation of the whole series; on the other hand, the *Infima Species* has a smaller denotation and a richer connotation than any other term in the series; whilst, of the intermediate terms, each is greater in connotation, and less in denotation, than the one preceding it. In every such line, the nearest genus to every term, of which that term is itself a species, is called the *Proximum Genus;* and every term in the line, except the *Summum Genus* and the *Infima Species*, is termed a *Subaltern Genus* or *Species* (Lat. *sub*, under, *alter*, the other of two).

A *Predicamental Line* is a series of subaltern Genera and Species reaching from a *Summum Genus* to an *Infima Species*, and passing through *Subaltern Genera* and *Species.*

The ten *summa genera* of the Aristotelian logicians are universally disputed by modern writers, and it is doubtful whether there can be any true *summum genus* except Being in general, or Reality; but, for practical convenience, the term is employed to denote the

widest class of things comprehended in any science; as, for instance, material substance in Chemistry. Thus it is possible that the same things may form the *summum genus* in one Science but not in another; for example, Man is the summum genus of Sociology, but is only a species under Animal in Zoology; and Animal, again, only a species of Living Organism in Biology.

Cognate Species are sub-classes under the same genus. *Cognate Genera* are classes of varying generality under which a species is contained.

Two or more classes which rank as species under the same Genus are called *Cognate Species*. A *Cognate Genus* is any one of the higher classes under which the same species falls. Thus in the following example (from Fowler's *Deductive Logic*, p. 61):—

- Figure.
  - Curvilinear.
  - Rectilinear.
    - Triangle.
      - Equilateral.
      - Isosceles.
      - Scalene.
    - Quadrilateral.
    - Polygon.

Equilateral, Isosceles, and Scalene, triangles are cognate species of the subaltern genus Triangle; and Triangle, Quadrilateral Figure, and Polygon, are cognate species of the subaltern genus Rectilinear Figure. So, Triangle, Rectilinear Figure, and Figure, are all cognate genera of Equilateral Triangle.

When a General Term is predicated of another General Term, it is a Genus and the subject a Species; but when a General Term is predicated of a Singular Term, it is a Species, for it is under *Infimæ Species* that individuals are directly included.

Genus and Species are the only classes recognized in Logic; but Botany and Zoology give them fixed places in a hierarchy of classes.

The logical use of the terms Genus and Species must be distinguished from the use of the same terms in Zoology and Botany, where a species, till recently, meant a class of animals, or plants, supposed to be descended from common ancestors, and to be the narrowest class possessing a fixed form, whilst a genus meant the next highest class. If, however, the Theory of Evolution is correct, many genera and species are really descended from a common stock,

and the distinction of genus and species becomes partly arbitrary, and dependent upon such points of resemblance as naturalists believe important. In these sciences, too, in order to express the relation of container and contained, other terms are employed besides the old logical genus and species. Thus, according to its position in a system of classification, a group is spoken of as Kingdom, Sub-kingdom, Division, Sub-division, Class, Sub-class, Cohort, Sub-cohort, Order, Sub-order, Tribe, Sub-tribe, Genus, Sub-genus, Section, Sub-Section, Species, Sub-species, Variety, Sub-variety, Variation, Sub-variation. In the language of formal logic all the intermediate classes are subaltern genera to the summum genus Animal or Plant.

Only classes separated from others by innumerable qualities were formerly called Genera and Species.

It must be noted that the ancient logicians considered as genera and species those classes only which were parted from each other by an unknown multitude of differences, and not merely by a few known and determinate ones. Where the differences were few they were considered as belonging to the Accidentia of the things, but where they were practically infinite in number, the distinction was held to be one of kind, and spoken of as an *Essential* difference. Mill adopts the same view, and speaks of species as 'Real Kinds.' It is on this distinction that the next three Predicables are founded.

36. Differentia.

Differentia—the attribute, or group of attributes which distinguish one species from all others contained under the same genus.

It has been pointed out above that a species is wider in connotation than the genus under which, in denotation, it is contained. *The excess of the connotation of a species over that of its proximate genus is called the Differentia or Difference of that Species.* Thus, in connotation, the sum of genus and differentia gives species. It is plain, however, that there can be no such thing as an absolute genus or differentia, for the same attribute may be differentia in one case and part of the connotation of the genus in another. Thus, if we have three classes of things with the respective connotations *ab*, *ac*, *bc*, whilst *a* is the genus of the first two, and *b* and *c* differentiæ; *b* is the genus of the first and third, and *a* and *c* differentiæ; and *c* is the genus of the second and third, and *a* and *b* differentiæ.

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As the connotation of a General Name only embraces those attributes which it implies, and not all those which are possessed in common by the things denoted by the name [*see* § 28 (ii.)], it is evident that the determination of what attributes form the differentia of any term depends upon the definition which unfolds the connotation (*see* § 49).

Differentiæ are spoken of as Specific and Generic. A *Specific Differentia* is that which distinguishes cognate species from each other, whilst a *Generic Differentia* is common to the whole class to which those cognate species belong, and is, to them, part of the connotation of the genus ; it is only a differentia with regard to the yet higher genus of which this genus is a species, and with regard to that, it is, of course, a Specific Differentia. So, every specific differentia of a higher class is a generic differentia with respect to the classes below it. Of course, summa genera have no differentiæ. Symbolically, if the summum genus x includes the cognate species xy, xz ; y and z are specific differentiæ. But, if we find xy to be a genus to the subaltern species axy, bxy, then y is a generic differentia with respect to those classes, their specific differentiæ being a and b.

37. Proprium.

Proprium—an attribute which does not form part of the connotation of a term, but which necessarily follows from it.

Those attributes which are common to every individual which bears the class name, and which are not included in its connotation, though necessarily connected with it, are called its Propria or Properties. Propria need not, however, be peculiar to the members of this class, for they may flow from a part of the connotation which is also part of the connotation of some other class name.

Whether an attribute is Differentia or Proprium depends on the definition of the term.

* The distinction between Differentia and Proprium is rather founded on the conventions of language than on the nature of things ; for there is often no valid reason why some, rather than others, of the common attributes of a class should be implied by the class name. Thus, with our definition of a triangle, the attribute 'three-sided' is the differentia which distinguishes that species of plane rectilinear figures from others, and 'three-angled' is a proprium ; but if we defined a triangle—as the etymology of the name, indeed, suggests—

as a 'three-angled figure' then the attribute 'three-angled' would become the differentia, and 'three-sided' the proprium. This is not so, however, in every case; and, always, propria are attributes which flow from the whole, or part, of the connotation either as effect from cause or as a conclusion from premises. Thus, that man is a tool-using animal flows from his rationality, as effect from cause—the attribute 'tool-using' is therefore a proprium; whilst, that 'the square on the hypothenuse of a right-angled triangle is equal in area to the sum of the squares on the sides containing the right angle' is a proprium; for it is an attribute common to all right-angled triangles, and which can be shown, by reasoning, to be a necessary consequence of the connotation.

As Propria are common to every individual bearing a class name we may have *Generic Propria*, which are common to every species in a genus, and which flow from the connotation of the name of the Genus, and *Specific Propria* which are attributes flowing from the differentia of the name of a species, and common to every individual included in that species. As in the case of Differentiæ, the same attribute is a Specific Proprium of a higher class, but a Generic Proprium of a lower.

The connexion of a proprium with the connotation is a necessary one; that is, its not following would be inconsistent with some law which we regard as part of the constitution either of the universe, of our minds, or of both.

38. Accidens.

Accidens—an attribute which neither forms part of the connotation of term, nor is necessarily connected with any attribute included in that connotation.

In this class are included all those attributes which are neither connoted by a term nor are connected with its connotation; that is, which are included under neither of the heads Genus, Differentia, or Proprium. We have no real definition of what an *Accidens*, or Accident, is; we can only say what it is not. An Accidens may be described as an attribute which can be removed from the class, or individual, without necessitating any other alteration; whilst to remove a

Accidentia are either Separable or Inseparable.

proprium or differentia would be to destroy the individual, or class, or at least to fundamentally change its character. There may be accidentia of a Class or of an Individual, and in both cases they may be Separable or Inseparable. An *Inseparable Accidens of a class is one which belongs to every member of the class.* It is, of course, difficult to distinguish such accidentia from propria, and a more extended investigation into the nature of things is always likely to remove an attribute from the former class to the latter. But, where there is no apparent reason why the attribute should always be found in the individuals of a class, it is called an Accidens. Thus, that all European ruminant animals are cloven-footed appears to be an invariable rule, but, as there is no apparent connexion between chewing the cud and having a cloven hoof, we regard having a cloven hoof as an Inseparable Accidens of the class European ruminant. White was long regarded as an Inseparable Accidens of swans, but the discovery of black swans in Australia has shown that it is only a *Separable Accidens*, that is, *one not common to every member of a class.* When we come to individuals the words Separable and Inseparable have, necessarily, a somewhat different meaning. An *Inseparable Accidens* of an individual is one which belongs to him at all times and can never be changed, as the date and place of a man's birth, whilst a *Separable Accidens* is one which is sometimes present and sometimes absent or which can be changed, as a man's trade, his acts or postures. These individual accidentia are of no logical importance.

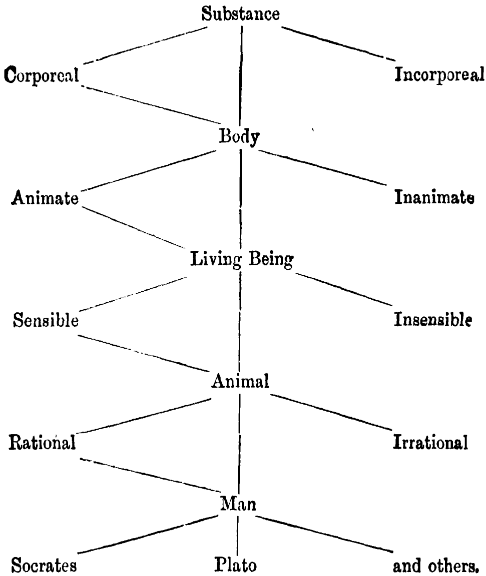
An Inseparable Accidens is common to every member of a class.

A Separable Accidens is not common to every member of a class.

* 39. The Tree of Porphyry.

The Tree of Porphyry exemplifies the chief Predicables.

An example of a portion of this scheme of Predicables is furnished by a table known as the *Tree of Porphyry* because it was first set forth by Porphyry. It is also called the *Ramean Tree* from the prominence given to it by Ramus (a sixteenth century writer on Logic). It is as follows :—



Here we have the Summum Genus, Substance—and the Infima Species, Man, which cannot be divided into any narrower species but only into individuals. The intermediate terms down the centre of the 'tree'—Body, Living Being, Animal, are Subaltern Genera and Species; each is a genus as regards those below it in the list, and a species with respect to those above it. The attributes Corporeal, Animate, Sensible (*i.e.*, able to feel), Rational, are differentiæ which divide each genus into species. Of course, the corresponding negative attributes are also differentiæ, but the species they would give rise to are omitted for the sake of simplicity; their existence must not, however, be forgotten, for every genus must be divisible into at least two species.

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* 40. General Remarks on the Predicables.

The Predicables do not consider Singular Terms as Predicates.

With regard to this five-fold scheme of Predicables it may be remarked that no provision is made in it for Singular Terms as Predicates. In fact, by the older logicians singular terms were never regarded as predicates, and such propositions as 'Lord Salisbury is the present Prime Minister of England' were looked upon as outside the scope of Logic. A Predicable was only another name for a Universal—the same term regarded in denotation was a Predicable, as being applicable to several different things; considered in connotation it was a Universal, as the attributes implied were to be found in several other and different notions.

A Proposition is *Analytic* when the predicate is a genus or differentia of the subject; *Synthetic*, if a proprium or accidens is predicated.

When a Genus or Differentia is predicated, the proposition is said to be Analytic or Verbal as the Predicate only states explicitly part of what is implicitly contained in the subject; but *when a Proprium or Accidens is predicated the proposition is synthetic or real*, as the predicate then asserts an additional fact, which no analysis of the subject would reveal. Other names for the same distinction are—Essential and Accidental, Explicative and Ampliative. Strictly speaking, a Species is only predicated of an individual; when the individual is denoted by a Proper name the proposition is, of course, synthetic, as the Proper Name implies nothing; but when the subject is a Significant Singular Name such a proposition is often analytic, as the Significant Singular Name frequently contains the species in its connotation. Thus 'Socrates is a man' is a synthetic proposition, but 'This great Greek philosopher is a man' is an analytic proposition; for, 'philosopher' implies 'man,' but 'Socrates' does not.

There is much that is valuable in this scheme, for all classification depends on the formation of genera and species, and one of the chief aims of all science is to classify accurately, and to decide what attributes are essential to the inclusion of any individual in a given class.

CHAPTER IV.

THE CATEGORIES OR PREDICAMENTS.

41. The Categories are a Classification of Relations.

Categories were regarded by Aristotle as a classification of possible predicates.

The word *Category* is derived from the Greek κατηγορεῖν, which, in Logic, meant 'to predicate,' and *Predicament* is the exact Latin equivalent for that term. The Categories were intended by Aristotle, who first drew out a list of them, as a classification of all the possible predicates of any individual subject. Thus, though he called them γένη τῶν ὄντων, or kinds of being, they were really not a classification of things, but of the relations between things. There were thus, however, the germs of two views of the nature of Categories in Aristotle—a classification of existences and a classification of relations. Of these the former was seized upon and developed by his immediate followers and by the scholastic logicians, and hence Categories were traditionally regarded as a classification of all possible things with no reference to their use as predicates of a proposition. Thus considered, they were based on the erroneous notion that the great aim of thought is to reach ultimate and independent orders of being, under which all things may be classed. On this view the establishment of a valid scheme of Categories would be the end of knowledge.

Categories are wrongly regarded as a classification of all nameable things;

But there is the other and the truer view of Categories—that which regards them as those relations conceived by the mind and applied to the interpretation of all experience, without which all knowledge would be impossible. Thus looked at they are the beginning instead of the end of knowledge.

they are forms of relation essential to knowledge.

Many attempts to draw out a satisfactory scheme of Categories from both points of view have been made by logicians, but none has yet been evolved which has met with universal acceptance. We will briefly describe the most common, though the whole subject is really of metaphysical rather than of logical import.

42. Aristotle's Scheme of Categories.

Aristotle had a ten-fold scheme: 1. Substance. 2. Quantity. 3. Quality. 4. Relation. 5. Action. 6. Passion. 7. Where? 8. When? 9. Posture. 10. Habit.

Aristotle's scheme was ten-fold, and was regarded by him as a complete enumeration of all possible summa genera, under one or other of which every term capable of being used as a predicate must be included. His Categories were all means of filling up the blank 'This is ——', and were considered by him to exhaust the possible means of doing so. They were simply enumerated by Aristotle with no attempt at arranging them as co-ordinate or subordinate, and with no statement of the principle on which the division was founded. The list is as follows :—

1. *οὐσία* - *Substantia* - **Substance** ; what a thing is ; as man, horse.
   This includes :
   (*a*) *First Essences*—names of individuals.
   (*b*) *Second Essences* — general names—Species and Genera.
2. *πόσον* - *Quantitas* - **Quantity** ; as three yards long.
   Quantity (*a*) when, as in number, its parts are not connected is *discrete*
   (*b*) when they are connected, it is either
   (i.) successive ; as time, motion, *or*
   (ii.) permanent, *i.e.*, space or extension in length, breadth, and depth.

3. ποῖον - *Qualitas* - **Quality**; as strong, black.
Of Quality there are four kinds:—
(*a*) *Habits* or acquired dispositions of mind or body; as virtue, skill.
(*b*) *Natural Powers* of mind or body; as memory, power of walking or of speech.
(*c*) *Sensible Qualities* (*i.e.*, those which the senses cognize); as hardness, weight, taste.
(*d*) *Form* or *Figure;* as round, triangular.

4. πρός τι - *Relatio* - - **Relation** (or more properly **Comparison**), which takes place when the consideration of one thing involves the consideration of another—*i.e.*, Relative Terms, as father, son, like, equal, greater.

5. ποιεῖν - *Actio* - - **Action**—whether in one's self, as speaking, thinking; or without one's self, as beating, cutting.

6. πάσχειν - *Passio* - - **Passion** (*i.e.*, endurance); as to be beaten, to be cut.

7. ποῦ - - *Ubi* - - - **Where**—answering questions respecting place; as at Rome, in the garden, here.

8. πότε - - *Quando* - - **When**—answering questions respecting time; as yesterday, last year, now.

9. κεῖσθαι - *Situs* - - **Posture**; as sitting, standing.

10. ἔχειν - - *Habitus* - **Habit**, *i.e.*, what one has as clothing, for defence, etc., as to be armed, to be clothed.

Under *Substance* is included First Essences, or Individual Names, which can be used only as Subjects; and General Names, which can be used as either Subjects or Predicates of

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Under each Category information of some kind may always be given respecting any individual person.

ordinary propositions. The other Categories can be predicates only.

* The whole scheme may be thus illustrated: "What is "this individual, Sokrates? He is an *animal*. What is his "Species? *Man*. What is the Differentia, limiting the "Genus and constituting the Species? *Rationality*, *two-"footedness*. What is his height and bulk? He is *six feet* "high, and is of *twelve stone* weight. What manner of man "is he? He is *flat-nosed*, *virtuous*, *patient*, *brave*. In what "relation does he stand to others? He is a *father*, a *pro-"prietor*, a *citizen*, a *general*. What is he doing? He is "*digging his garden*, *ploughing his field*. What is being done "to him? He is *being rubbed with oil*, he is *having his hair* "*cut*. Where is he? *In the city*, *at home*, *in bed*. When do "you speak of him? *As he is*, *at this moment*, *as he was*, "*yesterday*, *last year*. In what posture is he? He is *lying* "*down*, *sitting*, *standing up*, *kneeling*, *balancing on one leg*. "What is he wearing? He *has a tunic*, *armour*, *shoes*, *gloves*.

"Confining ourselves (as . . . Aristotle does in the Cate-"gories) to those perceptible or physical subjects which "everyone admits, and keeping clear of metaphysical enti-"ties, we shall see that respecting any one of these subjects "the nine questions here put may all be put and answered; "that the two last are most likely to be put in regard to some "living being; and that the last can seldom be put in regard to "any other subject except a person (including man, woman, "or child). Every individual person falls necessarily under "each of the ten Categories; belongs to the Genus animal, "Species man; he is of a certain height and bulk; has cer-"tain qualities; stands in certain relations to other persons "or things; is doing something and suffering something; is "in a certain place; must be described with reference to "a certain moment of time; is in a certain attitude or pos-"ture; is clothed or equipped in a certain manner. Infor-"mation of some kind may always be given respecting him "under each of these heads. . . . Until such information is "given, the concrete individual is not known under condi-"tions thoroughly determined. Moreover, each head is

"separate and independent, not resolvable into any of the "rest, with a reservation . . . of Relation in its most com- "prehensive meaning. . . . The ninth and tenth are of "narrower comprehension, and include a smaller number of "distinguishable varieties, than the preceding; but they are "not the less separate heads of information" (Grote's *Aristotle*, pp. 77-8).

* A few further observations may be offered to render this scheme perfectly clear. Under the fourth category the older logicians only included substances between which a relation exists; if this restriction is neglected this category will not differ from some of the others.

The seventh and ninth categories should be carefully distinguished.

Under the eighth category come only answers to the question 'when?' Answers to the questions 'how long?' come under the second category.

The tenth category must be distinguished from the 'Habit' which is included under the third; we have the same ambiguity in the word 'habit' in ordinary language; as 'Habits are hard to break'; 'a riding-habit.'

The whole may be thus related to the parts of speech, on which many suppose the scheme was founded: a Predicate may be

(*a*) A Substantive when it is the name of the kind of thing.
(*b*) An Adjective of quantity, quality, or comparison.
(*c*) An Adverb of time or place; no others imply existence as these do, and so no others can be used as predicates.
(*d*) A Verb either active, passive, or neuter, or expressing the result of an action.

43. Objections to Aristotle's Scheme of Categories.

Many attacks have been made on this scheme by modern writers. For example:—

The writers of the 'Port Royal Logic' object to this scheme as arbitrary and misleading.

* (i.) The authors of the '**Port Royal Logic**' speak of the divisions as of little use, and even injurious, because "they are "altogether arbitrary, and are founded only in the imagina-

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"tion of a man who had no authority to prescribe a law to "others, who have as much right as he to arrange, after "another manner, the objects of their thoughts, each accord-"ing to his own method of philosophising"; and because "the study of the categories . . . accustoms men to satisfy "themselves with words, and to imagine that they know all "things when they know only arbitrary names, which form "in the mind no clear and distinct idea of the things" (Eng. Trans., pp. 40-1).

Kant objected that the scheme was not confined to forms of the pure understanding.

* (ii.) **Kant's** criticism was founded on a misconception of Aristotle's design in drawing out the Categories. Looking upon it as the same as his own—that is, to enumerate the pure or *à priori* forms of the understanding—he objects to the scheme as founded on no principle; as containing forms of sensibility (*Quando, Ubi, Situs*) as well as of the understanding, and thus confounding empirical notions with pure notions; as classing together deduced concepts (*Actio, Passio*) with original; and as omitting altogether some original elements.

Lotze objected that the divisions were unphilosophical and empirical.

* (iii.) **Lotze** in his *Metaphysic* (Eng. Trans., vol. i., pp. 24-5) says: "In the sense which Aristotle himself attached "to his Categories, as a collection of the most universal pre-"dicates, under which every term that we can employ of "intelligible import may be subsumed, they have never ad-"mitted of serious philosophical application. At most they "have served to recall the points of view from which ques-"tions may be put in regard to the objects of enquiry that "present themselves. The answers to those questions always "lay elsewhere—not in conceptions at all, but in funda-"mental judgments directing the application of the concep-"tion in this way or that. . . . Aristotle may have had the "most admirable principles of division; but they do not "prove that he has noticed all the members which properly "fall under them."

(iv.) **J. S. Mill**—assuming the Categories to have been intended as "an enumeration of all things capable of being "named; an enumeration by the *summa genera*, *i.e.*, the

Mill objected to the scheme as unphilosophical, and both redundant and defective.

"most extensive classes into which things could be distributed; which, therefore, were so many highest Predicates, "one or other of which was supposed capable of being "affirmed with truth of every nameable thing whatsoever" (*Logic*, Bk. I., ch. iii., § 1)—objects to the list as unphilosophical and redundant and defective. He says: "It is a "mere catalogue of the distinctions rudely marked out by "the language of familiar life, with little or no attempt to "penetrate, by philosophic analysis, to the *rationale* even of "those common distinctions. Such an analysis, however "superficially conducted, would have shown the enumeration "to be both redundant and defective. Some objects are "omitted and others repeated several times under different "heads. It is like a division of animals into men, quadru-"peds, horses, asses, and ponies" (*ibid.*). He goes on to say that Action, Passion and Local Situation (*Situs*) ought to be included under Relation, together with position in time (*Quando*) and in space (*Ubi*), and he regards the distinction between Ubi and Situs as merely verbal. On the other hand all states of mind are omitted entirely as "they cannot "be reckoned either among substances or attributes" (*ibid.*). To Prof. Bain's objection to this criticism that the Categories were not intended as an enumeration of things "capable of being made predicates, or of having anything "predicated of them" but "as a generalization of *predicates*," Mill replies, "In Aristotle's conception . . . the Categories "may not have been a classification of Things; but they "were soon converted into one by his scholastic followers" (*ibid.*, note).

44. Answers to Objections.

To some of these objections more or less valid answers have been made.

Prof. Baynes denies that the Categories are arbitrary, and regards them as of Metaphysical import.

* (i.) In reply to the 'Port Royal' criticism **Prof. Baynes** says the criticism of the writers "proceeds wholly on mis-"apprehension; for the categories, instead of being arbitrary "names, or imaginary attributes, are all affections of real "being." He points out "that they are of metaphysical

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"rather than logical concernment," and acknowledges that "these heads, though full in their enumeration, are not, as "given by Aristotle, co-ordinate among themselves, and, as "a consequence, their arrangement is unsymmetrical" (*Port Royal Log.*, Eng. Trans., p. 384).

Mansel says they were not intended as a classification of *à priori* forms of thought.

* (ii.) **Mansel** points out that Kant is "mistaken in supposing that Aristotle had any intention of classifying the "pure forms of the understanding, independent of experi"ence. On the contrary, the Categories belong to the matter "of thought, are generalized from experience, and leave alto"gether untouched the psychological question of the existence "of elements *à priori*. Any objection, therefore, based on "the inclusion of empirical or the exclusion of original "elements, is untenable, and rests on a misapprehension of "the philosopher's design" (Mansel's *Ed. of Aldrich*, *Artis Logicæ Rudimenta*, 3rd ed., p. 175).

It is urged that they are useful as an aid to the examination of nature.

* (iii.) It is argued that it is no more an objection to say the number of the Categories is arbitrary than it would be to bring against a methodical arrangement of books in a library the charge that for such an arrangement another might be substituted. No scheme of Categories can, by itself, enable men to understand the nature of things, and Aristotle's, like any other orderly classification, is not only not useless, but is often of much use as an aid to a due examination of nature. Any orderly arrangement of the innumerable subjects of thought is better than none, though it must be allowed that Aristotle's scheme needs re-arrangement, and the last nine Categories should be classed under the one head 'Attributes' (*see* Walker's *Commentary on Murray's Logic*, pp. 31-2).

Mansel regards them as founded on Grammatical considerations.

(iv.) **Mansel** replies to Mill's criticism that Aristotle did not design "a classification of all things capable of being "named; at least not in that point of view in which things "are regarded according to their real characteristics as pre"sented to consciousness. The Categories are rather an "enumeration of the different modes of naming things, "classified primarily according to the grammatical distinc"tions of speech, and gained, not from the observation of "objects, but from the analysis of assertions. . . . The

"proposition, as the only assertion capable of truth and "falsehood, appears to be regarded as the unit of speech, of "which the simple term is but a fractional element. It is "therefore probable that the Aristotelian distinction of "Categories arose from the resolution of the proposition "and a classification of the grammatical distinctions indi-"cated by its parts. . . . The omission, therefore, in the "Aristotelian list, of separate heads of classification for "mental states, cannot be charged as a defect in this point "of view, so long as mind and its various states (whatever "may be their difference in other respects) are represented "by the same verbal forms as substances and attributes. "And accordingly we find various mental states . . . classi-"fied together with corresponding affections of body, under "the head of qualities. . . . We might fairly describe the "Aristotelian Categories as an enumeration of the different "grammatical forms of the possible predicates of a proposi-"tion, viewed in relation to the first substance as a subject. ". . . The Categories are enumerated, not as an exhaustive "catalogue of existing things, but as a list of the different "modes of predicating by the copula. They thus originally "belong to Grammar, rather than to Logic or Metaphysics, "though the treatment of later philosophers, perhaps in some "degree sanctioned by Aristotle himself, has brought them "into closer connection with the latter sciences, and over-"looked their proper relation to the former" (Mansel's *Aldrich Art. Log. Rud.*, 3rd ed., pp. 175-8).

It may be doubted, however, whether the origin of the Categories was an examination of the parts of speech, for that division of words was by no means sufficiently developed in Aristotle's time to favour this idea. Aristotle has distinguished not so much parts of speech as parts of the sentence (subject, predicate and different forms of the predicate).

Bain says they are an analysis of the final import of predication.

Prof. Bain says in reply to Mill: The Categories "seem "to have been rather intended as a generalization of *predi-"cates*, an analysis of the final import of predication, includ-"ing Verbal as well as Real predication. Viewed in this

"light, they are not open to the objections offered by Mr. "Mill. The proper question to ask is not—In what Category are we to place sensations, or any other feelings or "states of mind, but—Under what Categories can we predicate regarding states of mind? . . . Aristotle seems to "have framed the Categories on the plan—Here is an individual: what is the final analysis of all that we can predicate about him?" (*Ded. Log.*, p. 265). However he grants "that they are not adapted to any logical purpose; they cannot be made the basis of logical departments" (*ibid.*, p. 266).

Grote denied that *Situs* and *Ubi* are identical;

Grote, in commenting on Mill's criticism, says: "Among "the many deficiencies of the Aristotelian Categories, as a "complete catalogue, there is none more glaring than the "imperfect conception of πρός τι (the Relative), which Mr. "Mill here points out. But the Category κεῖσθαι (badly "translated by commentators *Situs*, from which Aristotle "expressly distinguishes it, . . .) appears to be hardly open "to Mr. Mill's remark, that it is only verbally distinguished "from ποῦ, *Ubi.* Κεῖσθαι is intended to mean *posture, attitude*, "etc. It is a reply to the question, In what posture is "Sokrates? Answer—He is lying down, standing upright, "kneeling, etc. This is quite different from the question, "Where is Sokrates? In the market-place, in the palæstra, "etc. Κεῖσθαι (as Aristotle himself admits . . .) is not easily "distinguished from πρός τι. . . . But κεῖσθαι is clearly "distinguishable from ποῦ, *Ubi.*

and would class mental states under *Qualitas* and *Passio.*

"Again, to Mr. Mill's question: 'In what Category are "'we to place sensations or other states of mind—hope, fear, "'sound, smell, pain, pleasure, thought, judgment,' etc.? "Aristotle would have replied (I apprehend) that they come "under the Category either of *Quale* or of *Pati*—ποιότητες "or πάθη. They are attributes or modifications of Man, "Kallias, Sokrates, etc. If the condition of which we speak "be temporary or transitory, it is a πάθος, and we speak of "Kallias as πάσχων τι; if it be a durable disposition or "capacity, likely to pass into repeated manifestations, it is "ποιότης, and we describe Kallias as ποιός τις. . . . This "equally applies to mental and bodily conditions. . . . The

"line is dubious and difficult between πάθος and ποιότης, but "one or other of the two will comprehend all the mental "states indicated by Mr. Mill. Aristotle would not have "admitted that 'feelings are to be counted among realities' "except as they are now or may be the feelings of Kallias, "Sokrates, or some other *Hic Aliquis*—one or many. He "would consider feelings as attributes belonging to these "πρῶται Οὐσίαι [First Essences or Individuals]; and so in "fact Mr. Mill himself considers them after having specified "the Mind (distinguished from Body or external object) as "the Substance to which they belong. . . . We cannot say, "I think, that Aristotle, in the Categories assigns no room "for the mental states or elements. He has a place for "them, though he treats them altogether objectively. He "takes account of *himself* only as an object—as one among "the πρῶται ουσίαι, or individuals, along with Sokrates and "Kallias" (Grote's *Aristotle*, pp. 90-1, note).

*45. Hamilton's Arrangement of Aristotle's Categories.

Hamilton arranged the Categories in grades of subordination.

Hamilton largely meets the objections against Aristotle's scheme, as wanting in arrangement, by casting it into a form of successive grades of subordination (Hamilton's *Ed. of Reid*, p. 687, note). The arrangement is :—

- Being (*ens*).
  - *Per se*, *i.e.*, (1) Substance.
  - *Per accidens* *i.e.*, mode of Substance or Attribute.
    - Absolute; either
      - Matter, *i.e.*, (2) Quantity.
      - Form, *i.e.*, (3) Quality.
    - Relative, *i.e.*, (4) Relation.
      - (5) Action. (6) Passion.
      - (7) Where?
      - (8) When?
      - (9) Posture.
      - (10) Habit.

Of course, this arrangement does away with their claim to be summa genera, for as Nos. 5-10 are species of 4, and as 2, 3, 4, are species of Attribute, whilst Substance and Attribute are themselves species of Being, we are reduced to one Summum Genus to which all the others are subaltern.

Other schemes of Categories have been given by various philosophers.

46. Other Schemes of Categories.

Other schemes of Categories have been put forward by various Philosophers :—

(i.) The **Stoics** reduced the ten Aristotelian Categories to four, which they called 'The Most Universal Kinds,' and believed to be forms of objective reality :—

1. *τὸ ὑποκείμενον* ... ... The Substrate.
2. *ποιόν* ... ... (Essential) Property.
3. *πὼς ἔχον* ... ... (Unessential) Quality.
4. *πρός τι πὼς ἔχον* ... ... Relation.

They subordinated all these Categories to the most universal of all notions—that of *ὄν* or being ; and regarded the order as necessary, each being subordinate to those before it.

(ii.) Others arranged the Categories in seven classes :—

1. Mens—mind, or the substance which thinks.
2. Materia—body, or substance extended.
3. Mensura—greatness or smallness of each part of matter.
4. Positura—their situation in relation to each other.
5. Figura—their figure.
6. Motus—their motion.
7. Quies—their rest or lesser motion.

(iii.) **Descartes** and **Spinoza** give *Substantia, Attributum, Modus ;* and **Locke** suggests *Substance, Mode,* and *Relation.* Both these schemes are related to the Stoic doctrine of Categories.

(iv.) **Archbishop Thomson** gives (*Laws of Thought*, p. 276) :—

Conceivable things are
- Substance.
- Attribute.
  - Quantity.
  - Quality.
  - Relation.
    - of Time.
    - „ Space.
    - „ Causation.
    - „ Composition.
    - „ Agreement and Repugnance.
    - „ Polar Opposition.
    - „ Finite to Infinite.

47. Mill's Scheme of Categories.

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Mill starts with the preliminary list: 1. Feeling 2. Substance. 3. Attributes. 4. Certain Relations of Feelings.

Mill, after criticizing Aristotle's list, gives the following preliminary list of Categories or enumeration of the classes of nameable things :—

I. Feelings, or states of Consciousness or mind— sensations, emotions, ideas, volitions.

II. Substances
- Bodies - occupying space—the unknown external cause to which we ascribe our sensations.
- Minds - the unknown internal subject of all feelings.

III. Attributes
- Qualities.
- Quantities.
- Relations.

IV. Certain Relations of our Feelings — co-existences, sequences, similarities and dissimilarities.

He then argues that Attributes are reducible to Feelings,

He then argues that "For logical purposes the sensation is "the only essential part of what is meant by the word "[quality]; the only part which we ever can be concerned "in proving. When that is proved, the quality is proved; "if an object excites a sensation, it has, of course, the power "of exciting it" (*Logic*, Bk. I., ch. iii., § 9). Hence "all the "attributes of bodies which are classed under Quality or "Quantity, are grounded on the sensations which we receive "from those bodies, and may be defined, the powers which "the bodies have of exciting those sensations. And the "same general explanation has been found to apply to most of "the attributes usually classed under the head of Relation" (*ibid.*, § 13). The exceptions are the relations "of succession "and simultaneity, of likeness and unlikeness. These, not "being grounded on any fact or phenomenon distinct from "the related objects themselves, do not admit of the same "kind of analysis. But these relations, though not, like "other relations, grounded on states of consciousness, are "themselves states of consciousness; resemblance is nothing

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"but our feeling of resemblance; succession is nothing but "our feeling of succession" (*ibid.*). "The attributes of "minds, as well as those of bodies, are grounded on states of "feeling or consciousness. . . . Every attribute of a mind "consists either in being itself affected in a certain way, or "affecting other minds in a certain way. Considered in "itself, we can predicate nothing of it but the series of its "own feelings. . . . In addition, however, to those attributes "of a mind which are grounded on its own states of feeling, "attributes may also be ascribed to it, in the same manner "as to a body, grounded on the feelings which it excites in "other minds" (*ibid.*, § 14). "All attributes, therefore, are "to us nothing but either our sensations and other states of "feeling, or something inextricably involved therein; and to "this even the peculiar and simple relations just adverted to "are not exceptions. Those peculiar relations, however, "are so important, and, even if they might in strictness be "classed among states of consciousness, are so fundamentally "distinct from any other of those states, that it would be a "vain subtlety to bring them under that common description, "and it is necessary that they should be classed apart" (*ibid.*, § 15). As the final result, therefore, of his analysis he gives the following four-fold scheme (*ibid.*):—

and finally gives: 1. Feelings. 2. Minds. 3. Bodies, including Attributes. 4. Successions, Co-existences, Likeness and Unlikeness.

"I. *Feelings*, or States of Consciousness.

"II. The *Minds* which experience those feelings.

"III. The *Bodies* or external objects which excite certain "of those feelings, together with the powers or "properties whereby they excite them; these "latter (at least) being included rather in com-"pliance with common opinion, and because their "existence is taken for granted in the common "language from which I cannot prudently devi-"ate, than because the recognition of such powers "or properties as real existences appears to be "warranted by a sound philosophy.

"IV. The *Successions* and *Co-existences*, the *Likenesses* and "*Unlikenesses*, between feelings or states of con-"sciousness."

The natural result of Mill's argument is to reduce everything to the one Category 'Feeling.'

This whole analysis is grounded on Mill's metaphysical position that external objects are nothing but 'Permanent Possibilities of Sensations' and Mind merely a 'Permanent Possibility of Feeling' (*see Exam. of Hamilton*, ch. xi., xii.), and cannot be accepted by any who reject that ultimate position. To one who believes that things really exist in themselves the resolution of Attributes and Relations into Feelings becomes impossible. Even if Mill's view be accepted this scheme is not satisfactory. The whole argument aims at reducing attributes to Feelings, yet they are finally included under Bodies under a plea which, certainly, has no philosophical weight. Nor is his reason for making Successions, etc., a separate class valid after he has once reduced them to Feelings. Finally, if Bodies and Minds be nothing more than Mill thinks them, it is difficult to see why they are not also included under Feelings. Thus, were he consistent, Mill would have included all existence under the one Category 'Feeling'; for that is, necessarily, his only Summum Genus.

48. Kant's Scheme of Categories.

Kant's Categories are intended to be an enumeration of the *à priori* forms of thought. He derived them from the forms of logical judgment.

Kant, in his scheme of Categories, designed, not to classify things but, to enumerate the true root-notions of pure understanding, or *à priori* forms of thought, which are immanent in the intellect and essential to the interpretation of every impression received from without. He endeavoured to attain a complete system of them by an analysis of the faculty of thought as expressed in the logical judgment, believing that the primitive notions of the understanding could be completely ascertained by a thorough examination of all the kinds of judgment. Though, then, we treat of this scheme here to render our review of the Categories complete, the discussion necessarily requires an acquaintance with the Book on Propositions, and its study should be postponed till that book has been read.

By an analysis of the forms of Judgment, Kant arrived at a division into four species, each with three sub-classes, and from each form of Judgment he deduced a Category. The whole scheme is as follows :—

Book I.
Ch. IV.

Forms of Judgment.		*Categories.*
I. Quantity.		I. Quantity.
(i.) Singular	... *This S is P.*	(i.) *Unity.*
(ii.) Particular	... *Some S is P.*	(ii.) *Plurality.*
(iii.) Universal	... *All S is P.*	(iii.) *Totality.*
II. Quality.		II. Quality.
(i.) Affirmative	... *S is P.*	(i.) *Reality.*
(ii.) Negative	... *S is not P.*	(ii.) *Negation.*
(iii.) Infinite	... *S is non-P.*	(iii.) *Limitation.*
III. Relation.		III. Relation.
(i.) Categorical	... *S is P.*	(i.) *Substantiality—Substance and Attribute.*
(ii.) Hypothetical	... *If A is B, S is P.*	(ii.) *Causality—Cause and Effect.*
(iii.) Disjunctive	... *S is either P or Q.*	(iii.) *Reciprocity.*
IV. Modality.		IV. Modality.
(i.) Problematic	... *S may be P.*	(i.) *Possibility and Impossibility.*
(ii.) Assertory	... *S is P.*	(ii.) *Existence and Non-existence.*
(iii.) Apodeictic or Necessary	... *S must be P.*	(iii.) *Necessity and Contingency.*

These Categories, or pure notions, are the *à priori* possession of the intellect, *i.e.*, they are relations under one or other of which all objects of sense are presented, and are, therefore, necessarily and universally valid. Moreover, as all our perceptions are sensuous, these Categories are only valid when applied to sensuous impressions, which are thus raised into experience. As the Categories are valid for the whole sphere of perception, they enable us to unite it into a connected whole, and thus to obtain a coherent experience of the external world; which would be impossible if sensations were not acted upon and synthesized by the understanding. As Mansel says: "Every complete act of consciousness is a compound of in-"tuition and thought; and the portion which is due to the act of "thought as such . . . will be the *form of the representative con-"sciousness.* Now, by the act of thought, the confused materials

"presented to the intuitive faculties are contemplated in three "points of view: as a single object, as distinguished from other "objects, and as forming, in conjunction with those others, a com"plete class or universe of all that is conceivable. We have thus "the three *forms* (or, as they are called by Kant, *categories*) of "*unity, plurality* and *totality;* conditions essential to the possi"bility of thought in general, and which may, therefore, be re"garded as *à priori* elements of reflective consciousness, derived "from the constitution of the understanding itself, and manifested "in relation to all its products. They are thus distinguished from "the *matter*, or empirical contents, by which one object of thought "is distinguished from another. The Matter of thought is derived "from the intuitive faculties, and consists in the several *presented* "*phenomena* which form the special characteristics of each object" (*Metaphysics*, pp. 192-3).

Some of the distinctions in the forms of judgment on which the scheme is founded are, at least, of doubtful value;

It has been objected that these Categories were derived from the forms of logical judgment, which should be their applied form, according to Kant's doctrine, and that this derivation is often forced and arbitrary. Some of the distinctions, too, in the original table of judgments are of doubtful value, if not altogether false— such are the distinctions between the Negative and the Infinite Judgments (*see* § 70), and that between the Assertory and the Apodeictic, which will be further considered in the section on the Modality of Propositions (*see* § 82). To again quote Mansel: "Besides these three [Unity, Plurality, Totality] which are classi"fied as categories of quantity, Kant enumerates nine others—viz., "three of quality,—reality, negation, and limitation; three of rela"tion, —inherence and subsistence, causality and independence, "and community or reciprocal action; and three of modality,— "possibility or impossibility, existence or non-existence, and neces"sity or contingence. But the Kantian categories are not deduced "from an analysis of the act of thought, but generalized from the "forms of the proposition, which latter are assumed without ex"amination, as they are given in the ordinary logic. A psycho"logical deduction, or a preliminary criticism of the logical forms "themselves, might have considerably reduced the number. Thus "the categories of quality are fundamentally identical with those "of quantity;—reality, or rather affirmation and negation, being "implied in identity and diversity, and limitation in their mutual "exclusion. The remaining categories are, to say the least, founded "on a very questionable theory in logic; and the two most import-

hence, the number of Categories could be reduced.

"ant—those of substance and cause—present features which dis-
"tinguish them from mere forms of thought" (*ibid.*, p. 193).

Lotze objects to the scheme that it is empirical.

Lotze also objects to Kant's scheme, on the ground that it is just as empirical and as wanting in "a principle to warrant their com-"pleteness" as Aristotle's. He says "It may be conceded to him "[Kant] that it is only in the form of the judgment that the acts "of thought are performed by means of which we affirm anything "of the real. If it is admitted further as a consequence of this "that there will be as many different primary propositions of this "kind as there are essentially different logical forms of judgment, "still the admission that these different forms of judgment have "been exhaustively discovered cannot be insisted on as a matter, "properly speaking, of methodological necessity. The admission "will be made as soon as we feel ourselves satisfied and have nothing "to add to the classification; and if this agreement were universal, "the matter would be practically settled, for every inventory must "be taken as complete, if those who are interested in its complete-"ness can find nothing new to add to it. But that kind of "theoretical security for an unconditional completeness, which "Kant was in quest of, is something intrinsically impossible." (*Metaph.*, Eng. trans., vol. i., p. 25.)

CHAPTER V.

DEFINITIONS OF TERMS.

49. Functions of Definition.

Definition is the explicit statement of the connotation of a term

Definition is the explicit statement of the Connotation of a term, *i.e.*, of all the attributes, and of those only, which are recognized by common agreement of competent thinkers as implied by the name (*see* § 28 (ii)). Every definition is, therefore, an analytic proposition (*see* § 40), or, rather, a series of analytic propositions, as a new proposition is required to state each separate attribute, and in every one of these propositions the predicate simply states in so many words what was already implicitly contained in the subject. A complete definition will exhaust the total number of analytic propositions that can be made with the defined term as subject, for it will state its whole connotation. Moreover, each one of these propositions must be universal, *i.e.*, the predication must be made of every one of the things denoted by the subject term, and the proposition must be of the form *Every S is P;* for the definition must necessarily be applicable to each object which bears the class name.

A definition sums up all the analytic predications which can be made of the term defined.

Were the necessary logical assumption—that all words have exactly the same distinct meaning for all who use them—universally true, Definition would be unnecessary. But our ideas are often *clear* without being *distinct* or *adequate;* that is, we can apply a name accurately enough to the things denoted by it without having distinctly present to our minds all the attributes on account of which it is bestowed upon them. The use of definition is to give distinctness to these clear ideas and to make them adequate—to enable us not only to use the name accurately as regards its denotation,

Definitions make our ideas distinct and adequate.

but to employ it with an intelligent apprehension of its exact implication. It is evident, then, that to form a good definition is a work of no small difficulty, and one calling for no small sagacity. It involves careful observation, comparison and analysis of the things observed, abstraction of the mind from their differences, and generalization, besides the power of distinguishing primary from derivative qualities. In short, the definition is the perfecting and completion of the process of conception [*see* §§ 2 (ii.); 8 (i.)]. Moreover, the preliminary process of *seeking* for a definition is often more important than the *finding* it. "What we gain by "discussing a definition is often but slightly represented in "the superior fitness of the formula that we ultimately "adopt; it consists chiefly in the greater clearness and ful"ness in which the characteristics of the matter to which "the formula refers have been brought before the mind in "the process of seeking for it. While we are apparently "aiming at definitions of terms, our attention should be "really fixed on distinctions and relations of fact. These "latter are what we are concerned to know, contemplate, "and as far as possible arrange and systematize. . . . And "this reflective contemplation is naturally stimulated by the "effort to define; but when the process has been fully per"formed, when the distinctions and relations of fact have "been clearly apprehended, the final question as to the mode "in which they should be represented in a definition is really "—what the whole discussion appears to superficial readers— "a question about words alone" (Prof. H. Sidgwick, *Principles of Political Economy*, pp. 49-50).

The definition completes the process of conception.

Definition is thus essentially practical, and is, therefore, a part of Applied Logic; we only need define a term when we require to use the definition as an aid to the expression of some truth.

50. Definition per Genus et Differentiam.

In unfolding the complete connotation of a name it is often practically impossible to express it in terms which denote simple attributes only; and, in nearly every case, to do

so would make the definition needlessly long and involved. It is, in all cases, allowable to employ terms expressive of groups of attributes. Hence we have the time-honoured rule that definition should be *per genus et differentiam*. In mentioning the genus we use a term which implies all the attributes common to the species whose name is the term to be defined and to all other co-ordinate species of that genus; and, by adding the differentia, we complete the statement of the connotation by giving those attributes which differentiate that species from all such co-ordinate species. In other words, when we have to define a term, we first decide what class of things it belongs to, and then we mark the attribute, or group of attributes, which distinguishes it from other members of that class. The name of the class is the Genus, the distinguishing attribute, or group of attributes, forms the Differentia. The genus selected must be a proximate genus (*see* § 35), as, otherwise, our definition will omit part of the connotation of the term we are defining. If, for instance, we defined 'man' as 'rational being' we should omit the attributes connoted by the word 'corporeal,' and our definition would allow the name to be applied to other possible beings. Or, symbolically, if we define a class term whose connotation is *abcd* by referring it to the genus *a* (instead of to the proximate genus, *abc*), and adding the differentia *d*, we plainly omit the attributes *bc* from our definition.

It is not necessary to state the Connotation in simple attributes. It is enough to give genus and differentia.

The genus must be proximate.

Which of the attributes form the genus, and which the differentia, must depend upon the classes with which we compare the term. The definition of the same name may legitimately vary in mode of expression, though, when each term employed is fully analysed, all the modes of expression will be seen to be really identical. For the object defined is the same and its essential attributes are not affected by the mode of abbreviating the definition. Thus 'Man' may be defined as 'rational animal,' where the comparison has been with other animals; or, as 'embodied spirit' where the comparison has been with other rational creatures which are not corporeal. But, if the word 'animal,' 'rational,' 'embodied,' 'spirit,' are each expressed in terms denoting simple and distinct attributes, the definitions will be seen to be identical. This may be shown in

The definition of a term may be variously expressed, but these expressions must be, at bottom, identical.

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symbols. Let the essential attributes of P be *abcd*, and X, Y, Z, be genera under either of which it may be classed. Let the connotation of X be *abc*; of Y be *bcd*; of Z be *acd*. Then P can be defined as *Xd*, or as *Ya*, or as *Zb*. But, if we analyse all these definitions into their simplest elements, we get, in all cases, that the definition of P is *abcd*. What, then, in a definition, is regarded as the genus and what as the differentia depends upon that process of comparison which, as was pointed out in the last section, is a necessary preliminary to definition.

Logic regards the demand for a definition as exceptional.

A definition *per genus et differentiam* assumes that the meaning of the name of the genus is known; but such an assumption is necessary to the science of Logic, which must regard the requirement of a definition of any particular term as an exception to the general rule that men are acquainted with the meaning of every term they use.

Definition is not *per genus et differentias* but *per genus et differentiam*.

It must be remembered that when definition *per genus et differentiam* is spoken of, it is not meant to imply that the differentia is a single attribute; it may be a group of attributes (*see* §§ 41, 43). Each species can have but one differentia—*i.e.*, one set of attributes to distinguish it from the co-ordinate species—when referred to any one particular genus. Hence it is inaccurate to speak, as Mill suggests, (*Logic*, Bk. I., ch. viii., § 3) of definition *per genus et differentias*.

51. Limits of Definition.

Only Proper Names and names of simple attributes are undefinable.

As Definition is the unfolding of the meaning implied by a name it follows that every significant name can be defined, and that the only terms incapable of definition are Proper Names which have no signification [*see* § 27 (i.) (*a*)] and singular Abstract Terms which are the names of simple attributes which as forming the ultimate limit of our analysis cannot be expressed in terms more elementary than themselves [*see* § 30 (iii.)]. No words can enable one who has never experienced pain or whiteness to conceive what either is. In such cases the utmost we can do is to clearly mark out the notion from others by a process of abstraction and isolation, and to indicate it by some accidens or accompanying mark.

The Scholastic logicians, insisting upon the necessity for all valid definition to be *per genus et differentiam*, and holding that *summa genera* and *infimæ species* were absolutely fixed, denied that the names of either individuals or *summa genera* were definable; because the former had no differentiæ but only accidentia; and because there was no higher genus under which the latter could be subsumed. Each of the ten Categories of Aristotle they, therefore, regarded as incapable of definition, as well as all sub-classes included in *infimæ species.* Thus, 'negro' would be undefinable, as it is a sub-class of the *infimæ species*, man. But the modern view does not impose this restriction; it regards 'A negro is a black man' as being, practically, a definition *per genus et differentiam*, the rigidity of the notion of *infimæ species* being relaxed. All Significant Individual Names are also held capable of definition, as their peculiar attributes can be specified in addition to those connoted by the name of the class of which they are members [*see* §§ 27 (i.) (*b*), 28 (i.)].

Terms whose Connotation is their more important element are more easily defined than those whose denotation is predominant.

Some terms are manifestly much more easily defined than others. Those in which the connotation is the more important element—such as technical terms, whose sole value lies in an exact meaning—are much more easily defined than those in which the denotative element predominates. Examples of the latter are the names of most common objects —*e.g.*, chair, horse, dog—where we learn to apply the names without any distinct idea of the attributes those names connote [*cf.* § 2 (iii.)]. That this is the case anyone may easily discover who contrasts the ease with which the connotation of such a term as 'rectangle,' for example, can be stated, with the difficulty of writing down the essential attributes of 'dog.'

Most definitions, especially those of scientific terms, are provisional, and subject to modification with advance of knowledge.

* But, though all significant names can be defined with more or less difficulty, a great number of such definitions can only be regarded as provisional. Fresh advances in knowledge may alter our estimate of the relative importance of attributes, may change propria to differentiæ, or differentiæ to propria, and so may revolutionize the connotation of the term, and thus necessitate a revised definition. In fact, with

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scientific terms the growth of knowledge must cause constant modifications of the definitions; were they fixed in any science, that science would cease to advance. Discovery and definition must go hand in hand, and finality in the latter is not to be looked for; it could only be possible with complete and perfect knowledge. So emphatically is this the case that it has been well said: "The business of Definition is part of the "business of discovery. When it has been clearly seen what "ought to be our Definition, it must be pretty well known "what truth we have to state. The Definition, as well as the "discovery, supposes a decided step in our knowledge to have "been made. . . . If the Explication of our Conceptions ever "assume the form of a Definition, this will come to pass, not "as an arbitrary process, or as a matter of course, but as the "mark of one of those happy efforts of sagacity to which "all the successive advances of our knowledge are owing" (Whewell, *Novum Organon Renovatum*, pp. 39-40).

Definitions are also modified by a change in the point of view from which a term is regarded.

Not only the growth of knowledge, but a change in the point of view from which a term is regarded may cause a change in the accepted connotation. Examples of this are most common in mathematics. Thus, an ellipse was originally defined as a conic section with the differentia that the cut goes quite across the cone, not at right angles to the axis. But in modern works it is defined as the line traced out by a point moving so that its distance from a fixed line bears always a certain ratio to its distance from a certain fixed point. Then the fact that such a curve is a conic section is deduced by a long and intricate argument; is, in fact, degraded from forming part of the connotation to the position of a proprium.*

The boundary line marked by a definition is necessarily vague, but this does not destroy its value.

Again, changes in the denotation of a term caused by its application to new classes of objects because of a real or fancied resemblance to the things of which it is originally the name (*see* § 3) results in a certain vagueness of connotation, which, of course, reacts on the denotation, and gives rise to an indefinite zone of the contents of which it is difficult to say whether they have a right to the name or not. Most common words will, if carefully examined, be seen to be thus more or less vague as to the boundary line of both their connotation and their denotation; and especially is this the case with

* *Cf.* Dr. Venn, *Empirical Logic*, pp. 284-5.

terms used in the sciences which deal with social phenomena, such as Political Economy. This vagueness of boundary does not, however, destroy the value of the definition. On this point the late Prof. Cairnes well remarked: "In controversies about definitions "nothing is more common than to meet objections founded on the "assumption that the attribute on which a definition turns ought to "be one which does not admit of degrees. This being assumed, the "objector goes on to show that the facts or objects placed within the "boundary line of some definition to which exception is taken, "cannot in their extreme instances be clearly discriminated from "those which lie without. Some equivocal example is then taken, "and the framer of the definition is challenged to say in which "category it is to be placed. Now, it seems to me that an objec"tion of this kind ignores the inevitable conditions under which a "scientific nomenclature is constructed alike in Political Economy "and in all the positive sciences. In such sciences nomenclature, "and therefore definition, is based upon classification, and to admit "of degrees is the character of all natural facts. As has been said, "there are no hard lines in nature. Between the animal and vege"table kingdoms, for example, where is the line to be drawn? . . . "I reply that I do not believe there is any absolute or certain dis"tinction whatever. External objects and events shade off into "each other by imperceptible differences; and consequently defi"nitions whose aim it is to classify such objects and events must of "necessity be founded on circumstances partaking of this character. "The objection proceeds on the assumption that groups exist in "nature as clearly discriminated from each other as are the mental "ideas formulated by our definitions; so that where a definition is "sound the boundary of the definition will have its counterpart in "external facts. But this is an illusion. No such clearly cut "divisions exist in the actual universe. . . . It is, therefore, no "valid objection to a classification, nor, consequently, to the defini"tion founded upon it, that instances may be found which fall or seem "to fall on our lines of demarcation. This is inevitable in the "nature of things. But, this notwithstanding, the classification, "and therefore the definition, is a good one, if in these instances "which do *not* fall on the line, the distinctions marked by the defi"nition are such as it is important to mark, such that the recognition "of them will help the inquirer forward towards the desiderated "goal" (*Logical Method of Political Economy*, pp. 139-141).

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* Further difficulty is sometimes due to the fact that a name is frequently used in scientific language with a meaning different from that which it bears in ordinary speech. It is, really, a different term; but the identity of the verbal symbol causes the scientific meaning and the ordinary signification to be more or less confused.

52. Rules of Definition.

A Definition must be adequate, precise, and clear, and neither tautologous nor negative.

The following rules must be observed in framing a good definition :—

I. *It should contain neither more nor less than the connotation of the term defined.*

II. *It should be clearer than the term defined, and should not, therefore, be expressed in unfamiliar, figurative, or ambiguous, language.*

III. *It should not consist of a term synonymous with that defined.*

IV. *It should never be negative when it can be affirmative.*

Or, to sum the rules into one,

A Definition should be (i.) *adequate, precise, and* (ii.) *clear, and should not be* (iii.) *tautologous or* (iv.) *negative.*

It should be noticed that in Rule IV the term 'Definition' applies to the whole proposition which states the meaning of the term, but in the first three rules it denotes the predicate only of that explanatory proposition. The term is, therefore, slightly ambiguous.

We will now discuss each rule in detail.

To add a proprium or inseparable accidens to a definition suggests the existence of objects which possess all the attributes but those.

Rule I. If the definition embraces more than the connotation of the term defined it must include either some of its propria or some of its accidentia. Of course, in the cases where either propria or inseparable accidentia are added to the connotation, the denotation of the definition remains the same as that of the name defined; but the very fact of adding these extra attributes would suggest that they were *necessary* to the true definition; and that, therefore, other

objects exist which possess all the attributes mentioned except these very ones; which is, in fact, not the case. If, for example, an equilateral triangle were defined as 'a triangle which has three equal sides and three equal angles,' this, though perfectly true of all equilateral triangles and of no other figures whatever, would yet be a faulty definition; for it *suggests* that there are triangles which may have three equal sides and yet not have their angles equal. But if a separable accidens is added to the connotation of a name as part of its definition, a graver fault is committed. In this case the definition will not refer to the whole denotation of the name defined, for some only of the things which correctly bear the name possess the attribute in question. The definition is said in this case to be *too narrow*. If, for instance, a triangle were to be defined as 'a plane rectilinear figure having three *equal* sides' the definition would be too narrow; for it would apply only to a section of the figures which are correctly called triangles, the attribute 'equal-sided being the differentia which marks off the species 'equilateral triangles' from the other co-ordinate species which are included in the genus 'triangle,' and, therefore, only a separable accidens of that genus. Or, if a labourer were to be defined as 'one who performs manual work for wages' the definition would again be too narrow, as, by the addition of the separable accidens 'for wages,' it excludes all slaves from the class labourers of which they indubitably form a part. In all these cases the definition is redundant, and, therefore, not sufficiently precise.

To add a separable accidens to a definition makes it *too narrow*, *i.e.*, limits it to a part only of the denotation of the term.

If, on the other hand, the definition contains less than the connotation of the name it is *too wide*, for evidently it will be applicable to a greater number of things than are included in the denotation of the term defined (*cf.* § 50). If, for instance, an equilateral triangle were defined as 'a plane rectilinear three-sided figure' the definition would include *all* triangles. In other words it would refer to the genus instead of to the species only, and would be inadequate. This, it may be noticed, is the most common fault of so-called 'definitions.'

If the definition contains less than the connotation it is *too wide*, *i.e.*, it applies to objects not included in the denotation of the term.

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In all cases, then, the denotation of the definition must be exactly the same as that of the term defined, and this can only be secured with certainty by its unfolding all the connotation of the term and embracing nothing else.

A Definition should be expressed in plain, unambiguous and non-figurative language.

Rule II. The violation of the rule demanding clearness in a definition is known as defining *ignotum per ignotius* or *per æque ignotum*—explaining the unknown by the more, or equally, unknown. Dr. Johnson's definition of a net as 'a reticulated fabric, decussated at regular intervals' is an amusing instance of this. To say that 'Eccentricity is peculiar idiosyncrasy' or that 'Fluency is an exuberance of verbosity' is, in each case, to give a definition which is certainly not clearer than the term defined. The so-called definitions which are expressed in figurative language are a variety of this fault. To say 'The lion is the king of beasts,' 'Bread is the staff of life' or 'Necessity is the mother of invention' gives no explanation of the meaning of the terms 'defined.' This rule, however, is not violated if a name is defined, for the purposes of a special science, in terms which to one not a student of that science would be less clear than the name itself; as, for instance, if for the purposes of Conic Sections a circle were defined as 'a section of a cone parallel to the base.'

The use of a synonym does not give the meaning of a term.

Rule III. The violation of the rule against tautology in a definition is called *circulus in definiendo*, or 'a circle in defining.' It is evidently no addition to our knowledge to 'explain' a term by itself or by a synonym (*cf.* § 3). To say that 'Truth is veracity in speech and act' is simply to affirm that 'Truth is truth,' and this, though perfectly obvious, is also perfectly useless. The great number of synonyms in English, due to the presence in the vocabulary of words derived from both Teutonic and Latin sources, offers many opportunities for committing this fault; and, it may be added, these opportunities are by no means sparingly used. But it is by no means confined to English. Ueberweg quotes the following example from the German writer, Maass: "'A feeling is pleasant when it is desired

"because of itself.' 'We desire only what we in some way "represent to be good.' 'The sensibility takes that to be "good which warrants or promises pleasure, and affects us "pleasantly;—the desires rest on pleasant feelings.' The "pleasant feeling is here explained by the desire, and the "desire again by the pleasant feeling" (*Logic*, Eng. trans., p. 175). It is, in fact, in cases of long and involved definitions, such as the above — where the three sentences are taken from different parts of the book—that a 'circle' is most frequently found. It is, however, by no means uncommon to meet with such 'definitions' as 'Life is the sum of vital functions,' 'Force is a motive power,' 'Man is a human being.' The definition once given by a Church dignitary that 'An archdeacon is one who exercises archidiaconal functions' is a very neat example of *circulus in definiendo.*

The rule against tautology is not violated when the name of the species contains that of the genus.

There is no violation of this rule when the name of the genus is repeated in defining a term which denotes a subordinate species which has no distinct name, but is specified by the addition of some limiting attribute to the name of the genus. There is no tautology, for example, in defining an equilateral triangle as 'a *triangle* which has three equal sides'; for the species 'equilateral triangle' has no separate name, and is distinguished from the species of the genus triangle which are co-ordinate with it merely by the limiting adjective 'equilateral.' Euclid, before giving this definition, has, of course, defined the name of the genus, 'triangle.' This word when it occurs in the definition of equilateral triangle is simply the name of the genus, not that of the thing defined at all; and the definition is strictly one *per genus et differentiam.*

From this rule it follows that names of simple attributes cannot be defined.

From this third rule it follows that a term which is the name of a simple and elementary attribute cannot be defined, as it can only be explained by a synonym or by itself; for instance, 'White is that attribute of sensible objects which occasions us to experience the sensation of whiteness.' We can only, in truth, *describe* such terms by analysing the conditions under which the sensations they denote are produced (*cf.* § 51).

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The name of a negative notion should be defined negatively.

Rule IV. Negative definitions are always less satisfactory than those expressed in positive terms, unless they are definitions of the names of negative notions, in which case they are to be preferred. It is, for example, simplest to define an alien as 'one who is not a citizen of the British Empire'; for the name 'alien' represents a notion whose sole differentia is just this negative attribute. The definition of Accidens is also, necessarily, of a negative character (*see* §§ 34, 38). But, to 'define' virtue as 'the opposite of vice,' or liquid as 'that which is neither solid nor gaseous,' is not to say in either case what attributes constitute the class notion, and are, therefore, connoted by the name; but to state attributes which are not so connoted. This, besides giving no positive information, is certain to lead to indefiniteness. For the number of attributes which a thing may, conceivably, possess is infinite, and to merely exclude a few of these is by no means to give a clear indication as to how many, and which, of the innumerable remaining ones must be possessed by any individual thing to qualify it to receive the name in question. Many of the objections to thoroughgoing Negative (or infinite) Terms apply, in fact, to negative definitions [*see* §§ 19, 29 (i.) (*b*)]. There is a breach of this rule in Euclid's definition of parallel straight lines as 'those which lie in the same plane, and which, being produced ever so far both ways, never meet.' Another example of the same fault is when Euclid defines a point as 'that which has no parts and which has no magnitude.' This rule is really involved in Rule I, as the connotation of a positive term cannot be expressed negatively.

But a negative 'definition' of the name of a positive notion is too indefinite to give its meaning.

53. Kinds of Definition.

From various points of view we get different divisions of Definitions into classes.

Nominal and *Real* are used by modern writers in a sense different from that of the older logicians.

(i.) **Nominal and Real.** The traditional division of definitions was into *Nominal* and *Real;* and these terms have been retained by many modern writers on Logic who have, however, used them in a sense very different from that in which they were used by the Scholastic writers on the

science. With them a *Nominal Definition* was one which unfolded the meaning of a word, and a *Real Definition* one which explained the nature of a thing.

The Scholastic writers said: A Nominal Definition explained a word. A Real Definition explained a thing.

Hamilton defined the terms thus: "By *Verbal Definition* "is meant the more accurate determination of the significa-"tion of a word; by *Real*, the more accurate determination "of the contents of a notion. The one clears up the rela-"tion of words to notions; the other of notions to things" (*Ed. of Reid*, p. 691).

Modern writers hold that a *Real* Definition implies the existence of an object bearing the name, whilst a *Nominal* Definition does not do so.

Ueberweg (*Logic*, Eng. trans., p. 164) gives the following meaning to "*Nominal* and *Real Definitions*. The former de-"fines what is to be understood by an expression. The "Real Definition has to do with the internal possibility of "the object denoted by the notion, and thus with the real "validity of the notion; for it either contains the proof of "its real validity in the statement of the way in which the "object originated, or was based upon such a proof." He thus justifies this use of the term. "The terms *Nominal* "and *Real Definition* are not thoroughly expressive; for "every definition defines not the name, nor the thing, but "the *notion*, and with it the name and the thing as far as this "is possible. But so long as the real validity of the defined "notion is not warranted, it is always possible that a notion "may have been defined which is only apparently valid, and "is in truth only a mere name or a feigned notion cor-"responding to nothing real. On the other hand, the defini-"tion of an objectively-valid notion serves at the same time "to give a knowledge of the thing denoted by the notion. "Considered in this sense these terms justify themselves" (*ibid.*, p. 167).

Mill held that "all definitions are of names, and of names "only; but, in some definitions, it is clearly apparent, that "nothing is intended except to explain the meaning of the "word; while in others, besides explaining the meaning of "the word, it is intended to be implied that there exists a "thing, corresponding to the word. Whether this be or be "not implied in any given case, cannot be collected from the "mere form of the expression" (*Logic*, Bk. I., ch. viii., § 5).

Definitions, however, are not arbitrary, but must be grounded on a knowledge of the corresponding things (*see ibid.*, § 7). This view of Mill does not seem fundamentally different from that of Ueberweg, when allowance is made for the Nominalism of the former, and it is, practically, that set forth originally by Aristotle that Nominal Definitions are those in which there is no evidence of the existence of the objects to which the name is applied. Most English logicians of the present day agree with Mill that definitions are of names only, but, of course, the name is merely the verbal symbol of the notion. On this view Hamilton's two kinds of Definitions merge into one, and we are left with the distinction drawn by Mill. But, as he himself says, both kinds are expressed by the same formula. How then shall we distinguish them? The very use of a term gives a presumption—though a presumption only—that the thing of which that term is the name exists in the external world. But the province of Logical definition is not to verify, or to disprove, this presumption; but to analyse the concept or 'notion' which exists in the mind, and which is expressed by the name. Hence Ueberweg says "every definition de-"fines . . . the notion." This analysis can be carried out safely only when it is accompanied by a continual reference to the things denoted by that name; and from an examination and comparison of which the notion was formed (*cf.* §§ 49 and 51). It would seem better, then, to finally discard this distinction from Logic; for it simply tends to confuse the whole object of logical definition by importing into it considerations with which the process of framing a definition is not rightly concerned.

But the distinction only leads to confusion.

A *Substantial* Definition enumerates attributes; a *Genetic* Definition indicates a process by which they may be secured.

(ii.) **Substantial and Genetic or Constructive.** In the former, the essential attributes of the class are enumerated as they exist in the complete concept; in the latter, a process is indicated by which they may be secured. The Genetic Definition is not a statement of the way in which the concept corresponding to the name *has been* formed in the mind, but of the way in which, by indirect means, we *may* form a concept, or a mental picture, of the notion, when it is inconvenient to say directly what it is. The method is chiefly applied in

Mathematics, but there it is frequently the simplest and clearest. For instance, the easiest way to define a ring is to say 'Let a circle revolve round a fixed axis in its own plane but outside it.' Such a definition as this necessarily postulates the possible existence of the thing whose name is defined; unless, indeed, the process be one which it is impossible to carry out. Outside the realm of Mathematics all definitions are Substantial.

An Analytically-formed Definition expresses the ordinary meaning of a term; A Synthetically-formed Definition gives a new meaning.

(iii.) **Analytically-formed and Synthetically-formed.** The former is the giving clearness and exactness to the commonly received meaning of a word, which is the ordinary work of definition; the latter is the giving a new and arbitrary meaning to an old term, or the equally arbitrary fixing of the connotation of a newly invented term, to serve the purposes of some special discussion. Such definitions can only be regarded as legitimate when a new technical term is absolutely necessary in a science; and then it is far better to invent a new term than to give a new and arbitrary meaning to an old one; for, in the latter case, both the writer and his readers are apt to revert, more or less unconsciously, to the ordinary signification of the term. As examples of Synthetic Definitions may be instanced many of those adopted by botanists and other naturalists, which are not statements of the ordinary connotation of the terms, but are based on a selection of attributes supposed to be either more fixed, or connected with a greater number of attributes, than are those included in the ordinary connotation.

These Synthetically-formed Definitions were called 'Nominal' by the writers of the *Port Royal Logic*—another meaning of that much-abused word.

This distinction, which was introduced by Kant, is psychological rather than logical; for it is based on the origin of the definition in its inventor's mind and not on the form in which it is expressed.

No Proposition which does not give the whole connotation, and nothing else, is a true logical definition.

(iv.) **Essential Definition and Distinctive Explanation.** The preceding divisions have been between true definitions, but we have here a distinction between definitions and propositions which are definitions in appearance only.

The *Essential Definition* gives the connotation, either completely, when it is a *Perfect* or *Complete Definition*, which is really the only true logical definition; or incompletely, when it is *Imperfect* or *Incomplete;* but this latter is not a true definition at all, though it is often spoken of as one. In this case

either only a portion of the differentia is given, or the full differentia is added to a genus higher than the proximate genus, as if one were to define an equilateral triangle as 'a plane rectilinear figure with equal sides.'

In the *Distinctive Explanation* likewise we have no real definition, but a proposition in which propria are given *instead of* the connotation. This must be distinguished from the case discussed under Rule I (*see* § 52), where propria were *added to* the connotation; there we had a definition, though a faulty one as it suggested false inferences; here we have no definition at all, for the attributes given are not connoted by the name. The proposition will not even enable us to identify the objects which bear the name it explains; for though they all possess those attributes yet they need not be the only objects which do so (*see* § 37).

Under this last head we may place all still less accurate means of identifying things by an enumeration of some of their attributes. Such are *Descriptions*, where inseparable accidentia are often used, with or without some of the propria, to enable us to recognize the objects denoted by the name. Such a proposition is no more a definition than would be the act of pointing out a member of the class in question and saying 'I mean something like that,' which is really a kind of the so-called *Definition by Type*. Still, though in no sense definition, description is by no means useless; it serves the very useful function of enabling us to easily identify anything which bears the name. We must not, however, speak of describing a word; we *define the name*, and *describe the thing* which bears the name. The main object of the former is to make distinct our concepts of things, and so to lead to a greater clearness and definiteness of thought and language; that of the latter is to furnish a rough and ready means of making others recognize the objects of which we are speaking. It may be pointed out that the so-called Definitions given in the ordinary Dictionary belong almost invariably to one or other of the spurious kinds discussed in this sub-section.

CHAPTER VI.

DIVISION AND CLASSIFICATION.[1]

54. Logical Division.

Logical Division is the analysis of the denotation of a term.

(i.) General Character of Logical Division — Logical Division is the analysis of the denotation of a term. By this is not meant an *enumeration* of the individuals which form the class of which the term is the name, but a statement of the sub-classes into which that class can be divided. In other words, it is the splitting up of a genus into its constituent species (*cf.* § 35). The genus which is to be divided is called the *totum divisum* (divided whole), or dividend; the species into which it is analysed are styled the *membra dividentia* (dividing members). In dividing a genus we think of an attribute which is possessed by some of its members and not by others, and this suggests the *fundamentum divisionis*, or basis of the division. The same genus may obviously be divided on several different bases into different sub-classes, according to the attributes on which the division is founded. Thus, triangles may be divided into equilateral, isosceles, and scalene, where the *fundamentum* is the relation of the sides to each other in length; or into right-angled, obtuse-angled, and acute-angled, where it is the size of the angles. So, the various divisions of terms (*see* § 26) are analyses of the same genus on different bases. When the same genus is thus divided in different ways the process is called *Co-division;* and the classes obtained by such a co-division more or less overlap each other, for every

The same genus may be variously divided; this is *Co-division.*

[1] The treatment of this subject is largely based on that adopted in Dr. Venn's *Empirical Logic.*

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member of the genus must fall into one class in each division, and the classes obtained on one basis are sure not to correspond exactly, if at all, with those resulting from another.

Sub-division is a continuing the process of division for more than one stage. Here each step is on a new basis.

When the classes resulting from an act of division are themselves again divided into their sub-classes we perform an act of *Sub-division.* These sub-classes may be again sub-divided, and so the process may go on till we reach *infimæ species*—classes, that is, which are only capable of being split up into individuals. Of course, in every step of a sub-division, we must have a new *fundamentum divisionis,* for the first step exhausts the original basis. Thus, having divided triangles into equilateral, isosceles, and scalene, it is evident that we cannot subdivide any of these classes on the basis of the relative lengths of their sides. But if we take a new *fundamentum* we may continue the analysis; for instance, we may subdivide both isosceles and scalene triangles on the basis of the size of their angles into right-angled-isosceles, obtuse-angled-isosceles, acute-angled-isosceles; right-angled-scalene, obtuse-angled-scalene and acute-angled-scalene. Or, if our original basis was the size of the angles, then we may subdivide acute-angled triangles into acute-angled-equilateral, acute-angled-isosceles, and acute-angled-scalene; whilst right-angled and obtuse-angled triangles may be divided into right-angled-isosceles, right-angled-scalene; obtuse-angled isosceles and obtuse-angled-scalene.

Division must proceed one step at a time.

Every division must be progressive; it must proceed one step at a time, and must omit no intermediate species; the division must be an enumeration of the species of the proximate genus. Hence the old logical rule *Divisio non faciat saltum* (Division must not make a leap). If this rule is broken we must not be surprised to find that some of the members of the *totum divisum* find no place at all in any of the *membra dividentia;* for in omitting an intermediate class the distinctive marks of that class will probably be at the same time overlooked, and thus, individuals having those marks, but not possessing the distinctive marks of the lower species contained in the division, may be omitted.

(ii.) **Logical Division is indirect and partially material.** But few words are needed to show the utility of Division. Every subject is much more easily treated and thoroughly comprehended when its various parts are arranged in an orderly way. Division, in fact, adds clearness to our notions, as definition makes them distinct (*cf.* § 49). As compared with Definition, however, Logical Division must be regarded as a secondary and indirect process, for it is a necessary assumption of Formal Logic that the connotation of a class term determines its denotation, and not *vice versâ* [*cf.* § 28 (iv.)]. We do not select a number of objects indiscriminately, and then seek for some attributes common to them all, which may form the connotation of the class name we affix to them; but we first get a more or less definite idea of the connotation of the class name, and then include or exclude individuals from that class in accordance with their possession of, or want of, those attributes which form that connotation. This is *practically* much less the case in some instances than in others (*cf.* § 51), but, *formally*, we must assume the class to be always determined by the connotation and not by the denotation. Besides, Division must presuppose more or less complete definitions of the names of the species into which a given genus is to be divided; for it is only by appeal to such definitions that we can determine a *fundamentum divisionis*; whilst every definition of a species-term *per genus et differentiam* suggests such a *fundamentum*.

Division depends on Definition.

Hence, we also see that no Division can be purely formal, *i.e.*, involve no appeal to knowledge outside the matter given. If we are simply given a genus we cannot even begin to divide it; for, of necessity, the attributes which separate one species from another can form no part of the connotation of the genus. Every such attribute must be a separable accidens of the genus, and can only be known by an appeal to sources of information other than the connotation of the name of that genus. Moreover, for the Division to be of any practical use, this appeal must be to the objects themselves which are included in the genus; for only thus can we be sure that we are dealing with really existing classes of

Hence no division is purely formal.

things. Thus, every Division contains, at least to some extent, a material element (*cf.* § 10).

Only General Terms can appear in a Logical Division.

(iii.) **Operations somewhat resembling Logical Division.** As a logical division is only the analysing of a genus into its species it follows that only general terms [*see* § 27 (ii.)] can appear in it. A singular term cannot be divided, for it is a name applicable in the same sense to an individual only; and the logical meaning of an 'individual' is that which is incapable of logical division. And the division must stop at *infimæ species* (*see* § 35); for to go further would be to enumerate individuals, which, it has been already pointed out, is not Logical Division. Hence, Logical Division must be carefully distinguished from

Physical Partition divides an individual into parts.

(*a*) *Physical Partition*, which is the splitting up of an individual into its constituent parts; as, for instance, a ship into hull, masts, sails, rigging, etc.

Metaphysical Analysis enumerates the attributes of a thing.

(*b*) *Metaphysical Analysis*, or the enumeration of the attributes of a class or of an individual; as when we name whiteness, ductibility, malleability, etc., as the attributes of silver.

Verbal Distinction separates the meanings of an equivocal word.

(*c*) *Distinction of the various meanings of an equivocal term* (*cf.* § 26); as when we distinguish between 'vice' meaning a moral fault, and 'vice,' a mechanical tool.

In Logical Division only can the divided whole be predicated of each dividing member.

In a Logical Division the genus can be predicated of each of the species, and of each individual member of those species. This follows necessarily from the fact that the definition of the species involves the genus (*cf.* § 50). For instance, if we divide animals into men and brutes, we can predicate of each man and of each brute that he, or it, is an animal. In none of these other processes, however, can the whole be predicated of the parts. We cannot say 'A mast is a ship,' or 'Whiteness is silver.' In the case of Distinction, of course the same *verbal symbol* can be predicated of each of the meanings: 'This tool is a vice,' or 'This fault is a vice.'

But the same definition—that is, the same connotation—cannot be so predicated, which shows that it is not the *same logical term* which is predicated in each case. This proves anew the accuracy of what was said before (*cf* § 26)—that an equivocal term is really and logically two or more terms.

55. Rules of Logical Division.

A Logical Division must avoid cross division, be exhaustive, and step by step.

The rules to which a Logical Division must conform to ensure validity may be gathered from the preceding section. They are :—

I. *Each act of division must have only one basis.*

II. *The sub-classes must be together co-extensive with the whole.*

III. *If the division be a continued one* (i.e., *embrace more than one step*), *each step must be, as far as possible, a proximate one.*

Or, more briefly : *The division must* (i.) *avoid cross division, be* (ii.) *exhaustive, and* (iii.) *step by step.* It will be noticed that for each separate act of division the first two rules are sufficient.

Other rules are frequently given, but, on examination, they will be seen to be redundant. Thus :—

(*a*) *Only class terms can enter into a division.* This is involved in the very definition of Logical Division.

(*b*) *The sub-classes must be mutually exclusive.* Rule I secures this, for if there is only one basis of division it is impossible for any individual to fall into more than one sub-class.

(*c*) *The whole must be predicable of each of the sub-classes.* This is provided for by Rule II, for, if there were a sub-class of which the name of the whole could not be predicated, then, evidently, the denotation of the sub-classes would be together greater than that of the whole instead of coextensive with it. If the whole and the sum of the sub-classes form two sides of an equation, clearly the same name is predicable of both.

Very few words will be needed to illustrate these rules.

If the Division is on more than one basis it will probably contain part of the denotation more than once,

Rule I.—Evidently a division made on more than one basis would be worthless. It would be nearly certain to include some individuals in more than one sub-class, so that the total denotation of the species would be apparently greater than the denotation of the genus. If, for example, triangles were divided into isosceles, scalene, and acute-angled, every possible triangle would fall into one or other of these classes (for every equilateral triangle is acute-angled), but some would fall into more than one; viz., those which are acute-angled isosceles, and those which are acute-angled scalene. But we have no guarantee that the opposite fault will not be committed and the division be made *too narrow* by the exclusion of some individuals from every sub-class. If we divided triangles, for instance, into equilateral, obtuse-angled, and right-angled, we should not, indeed, include any individual twice, but we should exclude all acute-angled-scalene, and acute-angled-isosceles triangles. Our division is too narrow. Very probably both faults will be committed; some individuals will be included more than once and others omitted altogether. Thus, if we divide triangles into equilateral, isosceles, and right-angled we include right-angled-isosceles triangles twice, and exclude obtuse-angled-scalene, and acute-angled-scalene triangles altogether. It may, indeed, occasionally happen that a division may be made on two principles and yet be practically accurate. But it is only in the exceptional instances when one attribute solely involves, and is solely involved by, another that this can occur. For example, a division of triangles into equiangular, isosceles, and scalene, would be both exclusive and exhaustive; but that is simply because all equilateral triangles, and they only, are equiangular, and so the division coincides with one made on the single basis of the relative lengths of the sides. Only when we have but one basis of division can we be sure that our sub-classes are necessarily exclusive of each other—that no individual can be placed in more than one.

or be too narrow,

or commit both faults.

If we omit species the Division is *too narrow.*

Rule II.—We have seen that a violation of Rule I is apt to lead to a too narrow division, that is, to the exclusion of part of the denotation of the whole from each of the sub-

classes. Of course, the same fault may be committed, even if Rule I be rigidly adhered to; for it is possible to omit one or more of the sub-classes in any division. We should get too narrow a division, for example, by dividing triangles into equilateral and scalene, and omitting isosceles. In such a simple case as this the fault is not likely to be committed, but when we are dealing with matter as complex as nature continually presents to us it requires great care to ensure that we have made a *complete* enumeration of all the species contained under a genus. Other examples of too narrow divisions are of men into good and bad, of books into instructive and amusing, of objects into useful and ornamental. The opposite fault is to make the division *too wide;* that is to include among the species some objects not denoted by the genus. This, again, is not likely to occur in simple cases; few, for example, would think of dividing coins into gold, silver, bronze and banknotes. But an indistinct apprehension of the connotation (*i.e.*, of the definition) of any of the terms we employ in our division may lead to this fault when we are dealing with complex matter.

If we include classes which do not fall under the genus we are dividing, the division is *too wide.*

It is plain that if this rule is broken we have not really divided the genus at all—but either only a part of it (when the division is too narrow) or the genus and something else as well (when the division is too wide). In a true division the sum of the denotations of the species must exactly coincide with the denotation of the whole; and only when this is the case has the genus given been really and accurately divided.

In either case the genus has not been divided.

Rules I and II may be thus expressed symbolically—

If the genus G is divided into the species g_1 g_2 g_3,

$$\left.\begin{array}{l}\text{then No } g_1 \text{ must be } g_2 \text{ or } g_3\\ \quad\text{No } g_2 \;\;,, \;\;,, \; g_1 \text{ or } g_3\\ \quad\text{No } g_3 \;\;,, \;\;,, \; g_1 \text{ or } g_2\end{array}\right\}\text{ Rule I.}$$

$$\text{and } g_1+g_2+g_3 \text{ must} = G \quad \cdot \quad \text{Rule II.}$$

Rule III.—This has been already discussed when it was remarked *divisio non faciat saltum* [*see* § 54 (i)], and it was pointed out that a violation of it usually leads to a division being too narrow.

If a continued division is not step by step it may be too narrow.

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56. Division by Dichotomy.

Dichotomy is division at each step into corresponding Positive and Negative Terms.

To ensure that none of the rules of Division are violated, many Formal Logicians have insisted that all valid Division must be by **Dichotomy** (*Grk.* δίχα, in two, τέμνω, I cut), or division at every step into a positive term and its corresponding negative. This process is founded on the Principles of Contradiction and Excluded Middle.

Dichotomy is cumbrous, and, so far as it is formal, is purely hypothetical.

A strictly dichotomous or bifid classification can always be thus formed, but it lies open to the objections

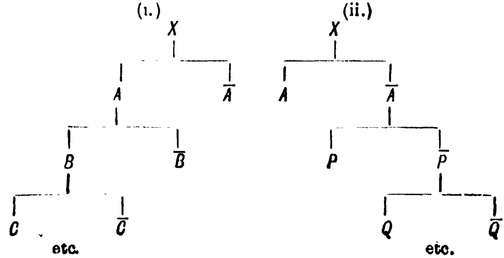
(*a*) That, at each step, one of the sub-classes—and that frequently the largest; viz., that denoted by the negative term—is entirely undefined in its extent [*cf.* §§ 19, 29 (i.) (*b*)]; and, no matter how far the process of subdivision is carried, the last term must always be formally left thus indefinite.

(*b*) That, in so far as it is formal, it is entirely hypothetical; the division does not guarantee the existence of any of the sub-classes.

(*c*) That it is excessively cumbrous. It seems absurd to divide a genus into two classes when it evidently falls naturally into some other, and equally definite, number of species, and to do so obscures the fact that these species are co-ordinate.

This process may be represented symbolically. Given a genus X, it may be divided into two species A and *non-A*, according as the objects possess or do not possess a certain attribute. [We use $\bar{A}$, $\bar{B}$, etc., to denote *non-A*, *non-B*, etc.] Either the positive or the negative member at either step may be subdivided. Thus:—

(i.) (ii.)



Or to take material examples

(iii.)

- Men
  - Europeans
    - Frenchmen
      - soldiers
      - non-soldiers (*if any*)
    - non-Frenchmen (*if any*)
  - non-Europeans (*if any*)

(iv.)

- Men
  - Europeans
  - non-Europeans (*if any*)
    - Asiatics
    - non-Asiatics (*if any*)
      - Africans
      - non-Africans (*if any*)
        - Americans
        - non-Americans (*if any*)
          - Polynesians
          - non-Polynesians (*if any*)

It is evident that this, like all other Division, in so far as it is not hypothetical, possesses a material as well as a formal element. It is by appeal to the *matter* that we know that some men are Europeans; and that some X are A. Even then, the existence of $\bar{A}$ and of non-Europeans is hypothetical unless we make a further appeal to outside matter. Similarly, 'Frenchmen' is not part of the connotation of Europeans—we must again appeal to fact. Every step is, therefore, either partly material, or wholly hypothetical, and it is evident that a purely hypothetical division is of but little practical use. In all strictly dichotomous division we must, at least, finish with a hypothetical term of whose existence, or non-existence, the division leaves us absolutely ignorant.

Every division may be reduced to dichotomy, but, as was said above (*c*), it is absurd to do this when we know definitely the number of sub-classes our *fundamentum divisionis* will give rise to. Thus we may make the division

Triangles

equilateral — non-equilateral

non-equilateral: isosceles — non-isosceles

non-isosceles: scalene — non-scalene (*if any*)

We know that the last class does not exist, for equilateral, isosceles, and scalene, form a complete enumeration of the species of the genus triangle on this basis of division, and these species are co-ordinate. This shows, however, that every division on one basis into more than two sub-classes may be expanded into several successive divisions on slightly different bases.

The Tree of Porphyry is an instance of Division by Dichotomy.

The Tree of Porphyry (*see* § 39) is, omitting the last step, a good instance of dichotomous division as treated by the older logicians. We there see that only one—the positive—term in each dichotomy is sub-divided ; the division proceeds along the predicamental line (*see* § 35) towards a certain definite end—the species 'man.' We also see its material as well as its formal character. The attribute 'animate,' for instance, is not included in the connotation of 'body,' and so with all the qualities the possession, or want, of which form the various bases of division ; they are only known by an appeal to experience.

Dichotomy is chiefly valuable as a test of the completeness of a division.

The real value of a division by dichotomy is to test the validity of our analysis—particularly to discover if it is exhaustive—and to find the position of any assigned class. Thus, in the Analytical Key prefixed to Bentham's *British Flora*, which is intended to enable anyone who has a specimen of a certain plant before him to discover its species and its technical name, the arrangement is nearly entirely

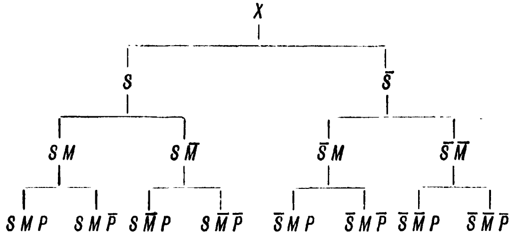
dichotomous, and, for such a purpose, this form is the most useful. But to adopt dichotomy as a *final* arrangement would be absurd. A botanist, for example, starts at once with three classes of the *summum genus* 'plant,' viz., exogens, endogens, and acrogens, and each of these is subdivided into varying numbers of orders, and these again into still further varying numbers of genera, and so on, with little or no regard to dichotomy, the object being to make the classification agree with the distinctions existing in the plants themselves. It may be added that every Definition *per genus et differentiam* suggests a division by dichotomy, and, conversely, every such Division supplies us with a Definition of that kind.

57. Purely Formal Division.

A full development of formal division leads to Symbolic Logic.

It has been shown in the preceding sections [§§ 54 (ii.), 56] that every logical division is partly formal and partly material, and these two elements continually hamper each other. The absolutely formal process of perfect and complete dichotomy is prevented by the desire not to form classes which have no real denotation, but are purely imaginary. When we discard these material considerations and develop dichotomous division to the fullest extent we enter into the domain of *Symbolic Logic*. Here we employ only formally negative terms, not, as in the Tree of Porphyry, (*see* § 39) terms such as 'rational and irrational,' which however contradictory they may be in meaning are not so in form [*see* § 29 (i.), (ii.)]. These formal contradictories are fully developed, the number of sub-divisions being determined solely by the number of terms we are dealing with. Thus, if we are concerned with three terms S, M, P, we have eight sub-divisions, viz. $S\,M\,P$, $S\,M\,\overline{P}$, $S\,\overline{M}\,P$, $S\,\overline{M}\,\overline{P}$, $\overline{S}\,M\,P$, $\overline{S}\,M\,\overline{P}$, $\overline{S}\,\overline{M}\,P$, $\overline{S}\,\overline{M}\,\overline{P}$ (where $\overline{S}$, $\overline{M}$, $\overline{P}$, denote *non-S*, *non-M*, *non-P*, respectively). But these are not regarded as *classes*, for some of the combinations may be non-existent, but as mere *class-compartments*—a framework, as it were, into which existent classes may be fitted. Every universal proposition is then regarded as asserting the *emptiness* of one or more of these compartments; that is, as denying the existence of one or more classes. Thus the proposition *No M is P* declares that M and P are never found together, *i.e.*, that the class-compartment $M\,P$ is empty, and this includes the sub-compart-

ments $S\,M\,P$ and $\bar{S}\,M\,P$. Similarly, *All S is M* denies the existence of the class $S\,\bar{M}$, that is, of the sub-classes $S\,\bar{M}\,P$ and $S\,\bar{M}\,\bar{P}$. The combination of these two propositions, therefore, will leave us with only the compartments $S\,M\,\bar{P}$, $\bar{S}\,M\,\bar{P}$, $\bar{S}\,\bar{M}\,P$, $\bar{S}\,\bar{M}\,\bar{P}$ occupied; that is, those are the classes which, under the terms of our propositions, may possibly exist. It is evident that such a process as this is purely formal, and is based entirely upon a development of bifid classification. It may be thus represented—X standing for the universe [*see* § 28 (iv.)] to be divided, which may include all existing things if necessary:—



This may, of course, be continued indefinitely, each additional division doubling the number of compartments. As all the compartments are divisions of X we might write X before each of them; for the sake of clearness this has been omitted.

Were these compartments regarded as classes, or sub-species of the genus X, the process would be invalid; for a thing is not simply the sum of its attributes, but those attributes must stand in a certain definite relation to each other. This necessity makes some classes impossible. For example, a triangle has three sides and three angles, but every combination of these—*e.g.* right-angled equilateral—is not possible. But regarded, as they are in Symbolic Logic, as mere class-compartments, this objection does not hold, and the process is formally valid. For an account of the developments of Logic to which it leads the student may be referred to Dr. Venn's excellent work on *Symbolic Logic*.

58. Material Division or Classification.

A development of the material side of division leads to *Classification*.

A similar development of Logical Division on the material side leads to the **Theory of Classification**. The object of classifying is to so arrange in order the facts with which we are dealing that we can the most easily acquire the greatest

possible command over them, and can economize statement —and so lighten the task imposed on memory—by being enabled to convey a large amount of information in a few words. To do this with success evidently requires a considerable knowledge of the phenomena we are engaged in classifying — it is nearly entirely a material process. To some extent our classification is already done for us by the mere use of language. Every giving of a General Name is a classification so far as it marks the formation of a group, constituted by the agreement of its members in the possession of certain attributes. Logic cannot tell us which attributes should form the basis of our primary, or secondary, divisions of a genus; all it can do is to warn us not to select those which are common to many different kinds of things, and which do not exercise any recognizable influence on the rest of their nature. It is only by special knowledge of, and reference to, the objects to be classified that we can select those attributes which carry with them the greatest number of other attributes as the basis of our first division, and can further decide which properties shall determine the subsequent divisions.

Every classification necessitates special knowledge of the things to be classified.

It does not follow that because a certain attribute is important in the division of one species of a genus it will be important in all the co-ordinate species. We may divide the genus X on the basis of the attribute a into the species M, N, O; and then sub-divide M on the basis of b into the sub-classes P, Q, R. It may be, however, that b is of no importance in the species N and O; for N it may happen that c is all important, and that on this basis N should be divided into S, T, U; whilst for O the important difference may be found in modifications of d which may lead to the sub-species V, W, Y. The determining of these facts, so as to make our classification practically useful for the purposes of scientific study and investigation, is a purely material process. If we thus adopt different bases of division for each new species it is evident we can have no *formal* guarantee that our classification is exhaustive. In fact, as Lotze remarks, "Classifica-"tion does not create the complete material, but assumes its

"completeness to be guaranteed elsewhere" (*Logic*, Eng. trans., vol. i., p. 167). Every such classification must, therefore, like scientific definition (*cf.* § 51) be regarded as subject to revision with the advance of knowledge.

59. 'Artificial' and 'Natural' Classification.

A '*Natural*' *Classification* was held to be one which followed strongly-marked divisions of nature, the attribute on which it is founded involving many other attributes, whilst An '*Artificial*' *Classification* was regarded as arbitrary.

If we take a genus and proceed to divide it on the basis of some single attribute, it may be that this division will not enable us to assert anything further about the species than that they possess, or do not possess, that particular attribute. But if we select some other attribute, we may find that the presence, or absence, of this property involves a great deal besides, as other attributes are invariably found where it is present, and are wanting where it is absent. It has been customary to call a classification of the former kind *Artificial* and of the latter *Natural*. The former was held to be simply an arbitrary grouping of natural objects for a certain definite purpose, but the latter was thought to involve a recognition of divisions really existing in nature. The leading modern upholder of this view was Mill, who based it on his doctrine of 'Natural Kinds.' These he regarded as classes of things existing in nature which were sharply and definitely separated from each other by an unknown and indefinite number of differences. Practically these 'natural kinds' were believed to be due, in the animal and vegetable kingdoms, to separate acts of creation — all the members of the same 'natural kind' might be regarded as descended from the same parents. The spread of the doctrine of Evolution has largely modified this view, for it is seen that many—if not all—these 'natural kinds' are descended from one stock. Moreover, natural objects do not fall into such well-defined and separate groups (*see* § 51). There is thus seen to be no essential difference between an 'Artificial' and a 'Natural' Classification : in neither case are we dealing with ready-made groups presented to us from without; but in both we are grouping objects—or rather our ideas of them, for we very seldom deal with the objects themselves—in the way most convenient for our purpose. The words 'Artificial'

But the divisions of nature are not thus definitely marked, and every classification is both Natural and Artificial.

and 'Natural' are thus misleading; for every classification is 'artificial' in the points just mentioned, and, on the other hand, every classification should aim at being 'natural,' in the sense of being based upon attributes of the objects themselves. It would be better, therefore, to discard these terms and to speak of *Classification for a special purpose*, and *Classification for general purposes*, instead of 'artificial' and 'natural' respectively. This would remove the stigma which the use of the word 'artificial' tends to throw upon classifications which are of the greatest value for certain special purposes.

It is better, therefore, to speak of *Classification for general purposes* or *for a special purpose.*

As the same objects possess many attributes, and as any of these may be made the basis of a classification, it is evident that the same things may be classified in many different ways. But it should be noticed that it is only in arranging the intermediate classes that differences of grouping can come in — the *summum genus* — *e.g.*, plants in Botany — and the *infimæ species*—*e.g.*, the tulip, hyacinth, etc.—are given us by the language of ordinary life, and must be the same in every scheme of classification. But this potential variety of grouping is of the greatest utility; the botanical arrangement of plants, for example—the 'Natural' classification of Mill—would be of little use for medicinal purposes, where another and special grouping is required on the basis of quite different attributes, and neither would serve the purpose of the farmer or gardener. For each distinct purpose we require a new classification, and the most appropriate one is the most 'natural' for that purpose. We may give, then, as a fundamental rule in classifying (in addition to those given in § 55)—

Only the intermediate groups admit of varied classification.

Rule IV. *The Classification should be appropriate to the purpose in hand.*

Every classification must be appropriate to the purpose in hand.

60. Classifications for Special Purposes.

The simplest kinds of classifications are those intended for special purposes; for in them attention need be fixed on only one attribute or group of attributes: we are not concerned with the amount of general resemblance or differ-

A classification for a special purpose is based on only one attribute, or group of attributes.

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Such a classification is useful as a key to a general classification.

An alphabetical arrangement is the simplest example.

ence. Such a special classification is frequently employed to serve as a key or index to the classification of the same objects for general purposes, which is based on our knowledge of their whole nature. A very familiar instance is the alphabetical arrangement which is found to be most serviceable in library catalogues and in indexes of books, and whose sole purpose is to indicate where a certain book, or statement, is to be found. Several attempts have been made to draw up catalogues of the books in large libraries, classified according to the subjects treated of, but this invariably leads to doubt as to where to look for the title of any particular volume, owing to the impossibility of marking off the subject matter of many books by rigid lines. If there is a class of Historical books and another of works on Philosophy, under which head should be entered those which treat of the History of Philosophy, or of the Philosophy of History? And would the 'psychological' or the religious novel find its most appropriate place in the department of fiction or in those of philosophy and theology respectively? Every such attempt has proved a more or less complete failure, and it is now pretty generally agreed that the alphabetical arrangement is practically the only satisfactory one for a catalogue. The same holds in the case of an index to a book; we can most readily find where to look for the discussion of any particular question treated of in its pages by means of an alphabetical list of the subjects discussed.

Such an arrangement is useful only as a key.

Useful as such an arrangement is, however, as a key, there its utility ends. It does not enable us to make a single general statement about any one of the groups formed, whose members may, indeed, have practically nothing in common except that their names happen to commence with the same letter of the alphabet. It is, therefore, valueless for the purposes of scientific investigation and research. Its value, too, is absolutely confined to the language in which it happens to be written; if translated, it ceases to be a classification at all, for the names of the same objects in two different languages by no means necessarily have the same initial letter.

A less simple arrangement but one whose chief use is still to serve as an index to a more general classification is such an Analytical Key as is prefixed to Bentham's *British Flora* (*cf.* § 56). Its object is to enable us to find out the name of a plant of which we have a specimen before us. As a bifurcate arrangement is found to offer the most ready means for attaining this object, that arrangement is almost exclusively adopted. Thus, flowering plants are first divided into those whose flowers are compound and those which are not compound. Then the former are sub-divided into those with one seed and those with more than one; and the latter into those in which the perianth is single and those in which it is double. Flowers with a double perianth are then sub-divided into those in which the corolla consists of one piece and those in which it does not; into those whose ovary is free and those in which it is not free; and so on. In every case the endeavour is to set forth two easily discriminated alternatives. Such an arrangement as this is less conventional than the artificial one discussed above, for it is founded on properties of the objects themselves—though not necessarily on those which directly involve others. Hence, it will bear translation into another language without losing any of its value as a classification.

Analytical Keys, as in Botany, are chiefly bifurcate. They are less artificial than an alphabetical arrangement as they are grounded on real properties.

Other classifications for special purposes are not thus intended to serve as keys to a general classification. But they all agree with the one last discussed in that no attempt is made to show by the grouping the full resemblances of the things classified, but only their agreement as to possessing more or less of the attribute, or group of attributes, important for the purpose in hand.

All special classifications are not intended as keys to a general classification.

61. Classifications for General Purposes.

In all the cases we have so far considered, the arrangement is intended to be on paper only, and the intermediate classes have been of no importance in themselves — they simply serve to point out where we may find that of which we are in search. The class of books whose authors' names begin with *M*, for instance, has no interest for us as a class—

If real objects are grouped it is with reference to general resemblances.

the books have not of necessity any other point in common —the arrangement is only useful as enabling us to find some particular work which we wish to refer to, say Macaulay's *Essays*. But the books themselves on the library shelves, or the plants in a Botanical Garden, will not be arranged in the alphabetical order, but in groups which have as much in common as possible. On the same shelves will be found, for instance, all the books relating to Logic; and in the same part of a Botanical Garden we should expect to find all plants of the lily kind. This bodily proximity of objects having much in common is an important aid to study; it saves time, and by presenting similar objects side by side renders comparison more easy and complete. Here we see every group is of importance in itself and is not merely a kind of index-finger, or meaningless label.

The same end is aimed at in a theoretical classification for general purposes.

What is attempted partially in such cases as the above in grouping the objects themselves, it is sought to attain completely in a theoretical Classification for General Purposes. Every special classification is, at least, to some extent subjective or personal; but here it is attempted to form a classification entirely objective—one which will appeal to all minds alike. The object to be attained is twofold:—

Such a classification seeks (a) to class objects on the basis of greatest general resemblances; (b) to similarly arrange classes.

(*a*) So to group individuals into classes that each class name may have the greatest possible connotation —that is, that the members of each class may resemble each other in as many points as possible.

(*b*) To arrange these classes into larger groups on the same principle.

Were such an ideal classification thoroughly attained the result would be that the whole world of thinkable things would be systematically organized. There would be one supreme genus with innumerable branching lines of species and sub-species, and each species would be so placed that it would show by its position the amount of resemblance it bears to all other species whatsoever. Nothing approaching to this has, however, ever been attempted; in all probability it never will be. The utmost that has hitherto been done is

to aim at such a classification in separate departments of knowledge only; and it is only in Botany and Zoology that this more modest task has been accomplished with any completeness. In attempting such a task we may either begin with the *summum genus*, *e.g.*, plants, and carry the division downwards by successive steps—and this is the usual way of conducting the process—or we may commence with the *infimæ species*, roses, lilies, etc., and go on to form gradually wider and wider classes by a process of aggregation. The resulting groups will be the same in both cases.

Only in Botany and Zoology has this been done with any completeness.

But when the task is attempted many difficulties present themselves, as will appear from a brief examination of the special rules (in addition to the general rules given in §§ 55 and 59) usually given to guide this kind of classification. They are:—

The wider group should be determined by the more important attributes, and the classification should be graduated.

Rule A. *The higher the group the more important should be the attributes by which it is constituted.*

Rule B. *The classification should be graduated, so that the groups with most affinity with each other may be nearest together, and so that the distance of one group from another may be an indication of the amount of their dissimilarity.*

Another Rule—that all groups should be so constituted as to differ from each other by a multitude of attributes—owed its origin to the doctrine of Natural Kinds. The more nature is understood the more it is seen that the Law of Contiguity is everywhere to be traced—one species fades into another by almost imperceptible degrees (*cf.* § 51), and it is impossible to constitute our groups in accordance with this rule, which must, therefore, be discarded.

Everything depends on determining which attributes are most important.

Rule A. Of the two rules given above it is evident that the first is the more fundamental; for everything depends on our ability to determine which attributes are the most 'important.' In a special classification, of course, that attribute must be considered important which has most connexion with the purpose in hand. But here our purpose is general, and the important attributes have been considered as "those which

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"contribute most, either by themselves or by their effects, "to render the things like one another, and unlike other "things; which give to the class composed of them the most "marked individuality; which fill, as it were, the largest "space in their existence, and would most impress the "attention of a spectator who knew all their properties but "was not specially interested in any." (Mill, *Logic*, Bk. iv., ch vii., § 2.) But how shall we determine these important attributes? The attributes of an object have no individual existence apart from our own mental analysis of them. They do not stand side by side like books on a bookshelf, but are merely our way of describing things. It, therefore, largely depends on us how far we analyse these inextricably connected phenomena and mentally hold them apart from each other as separate properties. Moreover, we can, probably, never know *all* the properties of any one thing, *i.e.*, understand its full and complete nature; consequently every General Classification must be regarded as always subject to revision with the advance of knowledge.

'Affinity' between classes in Botany and Zoology is used, under the evolution hypothesis, in its primary meaning.

Rule B. When we consider Rule B we are at once forced to consider what is meant by the word *affinity*. Mill, and the other upholders of Natural Kinds used 'affinity' in a merely metaphorical sense to imply resemblance, but not anything corresponding to that family relationship which is the primary meaning of the word. Now, however, under the influence of the doctrine of Evolution, affinity is regarded in Botany and in Zoology as meaning just this very relationship; the word is no longer used metaphorically, but in its primary meaning. The same doctrine has also led to a modification of what is meant by 'important' characteristics, by causing the evidence of descent to be regarded as an element of importance. The properties which bear witness to this are by no means necessarily, or even usually, those which are of most moment for the welfare of the individual at present. The latter bear witness rather to the more recent conditions in which the species has existed; for these conditions continually modify those properties which are

Qualities which furnish evidence of descent are important, but often neither striking nor of great present utility.

necessary to vitality and health. But qualities of little consequence to the immediate well-being of the species may remain through many generations unchanged — though, doubtless, in the course of very long periods of time they tend to disappear. No matter how trivial a property may be, yet if it appears in generation after generation, it is good evidence of descent and is, so far, an *important* attribute; but the most obvious and striking attribute, even though it involves many others, will be no safe guide in classification if it is subject to modification with changing circumstances. The habit of climbing in plants, for example, will not determine species; for striking as this characteristic is, it is due to external surroundings, and it is found that even ferns can climb as successfully as the ivy, if climbing is necessary to their existence.

A General Classification has only been attempted in sciences where it is seen to be an aid to discovery.

It is clear, then, that it is no easy matter to form a General Classification, but it does not follow that it is impossible. On the contrary, in Botany and in Zoology, and, to a less extent, in Chemistry and in Mineralogy, this has been done with very considerable completeness, and the first object mentioned above has been fairly attained. The body of such a work as Bentham's *British Flora*, or any other systematic treatise on Botany or Zoology will furnish an example. But only in these Sciences has such an elaborate attempt at classification been made. In them it is felt to be an aid to discovery and investigation, and it is this end which has been sought; but where the classification is seen to lead to nothing beyond itself, the work has not been considered worth the labour it entails.

A General Classification requires an Analytical Key.

When we are provided in any science with such a General Classification, we need some help in assigning to any individual object its place in that scheme. "This operation "of discovering to which class of a system a certain specimen "or case belongs, is generally called *Diagnosis*" (Jevons, *Princ. of Science*, p. 708). Any conspicuous and easily discriminated property which is peculiar to the class may be selected as a guide to the class to which an object belongs, and a scheme of classification based on these *characteristic*

properties forms an analytical Key such as was described in the last section. This key itself is called by Whewell the Diagnosis (*Novum Organum Renovatum*, p. 23). Such a diagnostic system should be, as far as possible, bifurcate (*cf.* § 60), and each characteristic on which it is based should be possessed by every member of the class of which it serves as the sign, and by no other object whatever.

62. Classification is not by Types.

As the type of a class is an individual it cannot furnish a basis for forming the class.

The fact, which has been already more than once referred to (*see* §§ 51, 59 and 61), that species, especially in the vegetable and animal kingdoms, are not separated from each other by rigid and definite lines, together with the further fact that even between members of the same species differences exist, led Whewell and other writers to advance the theory that classification is by Types and not by characters. Whewell defined "the Type of any natural group" as "an example which possesses in a marked degree all the leading "characters of the class" (*Nov. Org. Ren.*, p. 21). He then went on to say, "A Natural Group . . . . . is determined, not by a "boundary without, but by a central point within ;—not by what it "strictly excludes, but by what it eminently includes ;—by a Type, "not by a Definition" (*ibid.*, p. 22). But this is not classification at all. As Jevons remarks: "The type itself is an individual, not "a class, and no other object can be exactly like the type. But "as soon as we abstract the individual peculiarities of the type and "thus specify a finite number of qualities in which other objects "may resemble the type, we immediately constitute a class. If "some objects resemble the type in some points, and others in other "points, then each definite collection of points of resemblance con-"stitutes intensively a separate class. The very notion of classifica-"tion by types is in fact erroneous in a logical point of view" (*Prin. of Science*, p. 724). It is, in truth, regarding the connotation as secondary to the denotation, which is an inversion of the true logical method [*see* § 54 (ii.)]. We can no more classify by types than we can define by types [*see* § 53 (iv.)]. We can use the mental image of a type as an illustration of a class, and in this way the conception of a typical example is useful. But it is typical because the idea of the class is already formed; not because it is the one determining factor in that formation.

63. Classification by Series.

So far we have considered mainly the grouping of individuals ; we will now examine the grouping of the classes thus formed. Our object here is similar to what it was in the former process ; we desire so to group our classes that their position may show the amount of similarity, or dissimilarity, existing between them. Now, any attribute and its relations to other attributes may vary quantitatively in different individuals. Hence, it may happen that one species may pass over into another species, and this into a third, and that we thus get a serial arrangement. To take an example from mathematics. On the one side the ellipse passes into the circle, when its diameters become equal ; on the other it passes into a straight line, when the conjugate diameter becomes nil. In such a case we should regard the typical ellipse as intermediate between these extremes. If we represent the transverse diameter by x and the conjugate by y, then in the former case $x-y=o$, and in the latter $x-y=x$. If now we add these two equations together we get $2x-2y=x$. That is, we may regard that ellipse as typical and most perfect whose transverse diameter is exactly twice the length of the conjugate diameter ; for in this case the attributes and their relations are most characteristic of the figure.

We may group classes on the same principle as individual objects.

Some classes form a series,

But when we come to consider natural objects we find our arrangement is not usually serial ; it rather resembles a series of concentric circles on a globe, as many groups are at the same distance from the typical one. Nor is this type always a kind of average. It may be that the species is most perfect, not when it is halfway between the two extremes, but when it is just on the point of passing over into another species. Thus, the species M may be most perfect and typical just at the point where it tends to become N, and similarly N may reach its highest development just when ready to pass into O and so on. That is, our basis of grouping is some attribute which gives perfection to each species in exact and direct proportion to the fulness with which it exists. In Zoology, for example,

but with natural objects the arrangement of classes is more complex.

The basis of grouping is the attribute on which the perfection of the species depends,

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the attribute which forms the basis of the grouping of our species is the possession of animal life; and it is evident that each member of any species is a more or less perfect representation of that species in exact proportion to the degree in which it shares in this vitality. In such a case a species becomes more perfect as it becomes less its average self, and the whole series tends towards one highest species, in which the attribute is found in fullest perfection. In the animal kingdom, this crowning species is, of course, man. We attain, then, finally, a classification in which the different species are not simply placed side by side, but in which they follow one another in a definite order till they culminate in one point. Nor is there one series only, but rather a web of series all branching from one point; the horse is a different species from the dog, yet they probably occupy about the same relative position with regard to man.

and there are many series of species branching from one point.

Were the whole of nature classified we should have an arrangement of the whole world of thinkable things, in which species followed species in a definite and determinate direction, and in which all would be connected into one harmonious whole.

64. Scientific Nomenclature.

A Nomenclature is the system of Class Names used in any science.

A Nomenclature is a system of names for the groups of which a classification consists. No classification could long remain fixed without a corresponding nomenclature, and every good nomenclature involves a good system of classification. The two are indissolubly connected. As Whewell remarks: "System and Nomenclature are each "essential to the other. Without Nomenclature, the system "is not permanently incorporated into the general body of "knowledge, and made an instrument of future progress. "Without System, the names cannot express general truths, "and contain no reason why they should be employed in "preference to any other names" (*Novum Organon Renovatum*, p. 288). It follows that only those sciences which have a fairly complete and generally received classification

Classification and Nomenclature are essential to each other.

possess a true general nomenclature—the sciences, that is, of Botany, Zoology, and Chemistry. As the classification must precede the nomenclature it follows that the latter is a consequence rather than a cause of extended knowledge. To give a thing a name which marks its position in a system implies that its attributes are known, and that a system exists sufficiently elaborate and regular to receive it in the place which belongs to it, and in no other. Any system of names of the classes in a systematic classification is a nomenclature; there may, therefore, be nomenclatures depending on special ('artificial') or on general ('natural') classifications, but the latter are, by far, the more important.

Only Botany, Zoology and Chemistry, have true nomenclatures.

A nomenclature may depend on either a special or a general classification, but the latter is the more important.

"Every nomenclature dependent on artificial classifications "is necessarily subject to fluctuations; and hardly anything "can counterbalance the evil of disturbing well-established "names, which have once acquired a general circulation. "In nature, one and the same object makes a part of an "infinite number of different systems—an individual in an "infinite number of groups, some of greater, some of less "importance, according to the different points of view in "which they may be considered. Hence, as many different "systems of nomenclature may be imagined as there can "be discovered different heads of classification, while yet it "is highly desirable that each object should be universally "spoken of under one name, *if possible*. Consequently, in "all subjects where comprehensive heads of classification do "not prominently offer themselves, all nomenclature must "be a balance of difficulties, and a good, short, *unmeaning* "name, which has once obtained a footing in usage, is pre-"ferable to almost any other" (Herschel, *Discourse on Natural Philosophy*, § 132). When, however, the science does admit of comprehensive heads of classification, then the names should not be unmeaning, but should recall both the resemblances and the differences between classes. Such a nomenclature prevents our being overpowered and lost in a wilderness of particulars. The number of species of plants, for example, is so enormous that if each had a name which expressed no relation with any other, memory would find it

A nomenclature should, if possible, suggest relations between classes.

impossible to retain anything more than a very small fraction of the whole number. The nomenclature should, therefore, be so constructed as to suggest these relations. There are two main ways of doing this :—

(1) The names of the lower groups are formed by combining names of higher and lower generality.

(2) The names indicate relations of things by modifications of their form.

In Botany and Zoology relations are expressed by combining names of higher and lower generality.

The former method is that which, since the time of Linnæus, has been adopted in Botany and Zoology. In Botany, for instance, the higher groups have distinct names, *Dicotyledon*, *Rosa*, *Geranium*, etc. The species is marked by adding a distinctive attribute to the name of the genus, as *viola odorata*, *orchis maculata*, etc. These distinctive attributes are not the logical differentia of the species, so the specific name is not a definition. They are, on the contrary, formed from all kinds of more or less important considerations. Sometimes the name is given in honour of an individual, as *Rosa Wilsoni ;* sometimes from a country in which the plant was first observed, as *Anemone Japonica ;* sometimes from some peculiarity of the plant, as *Geranium sanguineum*. Some are purely fanciful ; for instance "Linnæus . . . gives the name *Bauhinia* to a plant which has "leaves in pairs, because the Bauhins were a pair of brothers. "*Banisteria* is the name of a climbing plant in honour of "Banister, who travelled among mountains. But such names "once established by adequate authority lose all their in-"convenience and easily become permanent" (Whewell, *Novum Organon Renovatum*, p. 308). Of course names which, in themselves, describe some peculiarity in the plant are at first of most value, but any easily remembered name serves the purpose. The names of varieties, sub-varieties, etc., are formed on the same principle as those of Species.

In Chemistry relations are expressed by modifications of form in the names.

The second method of constituting a nomenclature is employed in Chemistry. This system of names is founded on the oxygen theory. It "was constructed upon . . . the "principle of indicating a modification of relations of

"elements, by a change in the termination of the word. "Thus the new chemical school spoke of sulph*uric* and "sulph*urous* acids; of sulph*ates* and sulph*ites* of bases; and "of sulph*urets* of metals; and in like manner, of phos*phoric* "and phos*phorous* acids, of phos*phates*, phos*phites*, phos-"*phurets*. In this manner a nomenclature was produced, in "which the very name of a substance indicated at once its "constitution and place in the system.

"The introduction of this chemical language can never cease to "be considered one of the most important steps ever made in the "improvement of technical terms; and as a signal instance of the "advantages which may result from artifices apparently trivial, if "employed in a manner conformable to the laws of phenomena, and "systematically pursued. It was, however, proved that this lan-"guage, with all its merits, had some defects. The relations of "elements in composition were discovered to be more numerous "than the modes of expression which the terminations supplied. "Besides the sulphurous and sulphuric acids, it appeared there were "others; these were called the *hyposulphurous* and *hyposulphuric*: "but those names, though convenient, no longer implied, by their "form, any definite relation. The compounds of Nitrogen and "Oxygen are, in order, the *Protoxide*, the *Deutoxide* or *Binoxide;* "*Hyponitrous* Acid, *Nitrous* Acid, and *Nitric* Acid. The nomen-"clature here ceases to be systematic. We have three oxides of "Iron, of which we may call the first the *Protoxide*, but we cannot "call the others the *Deutoxide* and *Tritoxide*, for by doing so we "should convey a perfectly erroneous notion of the proportions of "the elements. They are called the *Protoxide*, the *Black* Oxide, "and the *Peroxide*. We are here thrown back upon terms quite "unconnected with the system.

"Other defects in the nomenclature arose from errors in the "theory; as for example the names of the muriatic, oxymuriatic, "and hyperoxymuriatic acids; which, after the establishment of "the new theory of chlorine, were changed to *hydrochloric* acid, "*chlorine*, and *chloric* acid.

* "Thus the chemical system of nomenclature, founded "upon the oxygen theory, while it shows how much may be "effected by a good and consistent scheme of terms, framed "according to the real relations of objects, proves also that

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"such a scheme can hardly be permanent in its original form, "but will almost inevitably become imperfect and anomalous, in consequence of the accumulation of new facts, and "the introduction of new generalizations. Still, we may "venture to say that such a scheme does not, on this account, "become worthless; for it not only answers its purpose in "the stage of scientific progress to which it belongs:—so far "as it is not erroneous, or merely conventional, but really "systematic and significant of truth, its terms can be translated at once into the language of any higher generalization "which is afterwards arrived at. If terms express relations "really ascertained to be true, they can never lose their "value by any change of the received theory" (Whewell, *Novum Organon Renovatum*, pp. 275-277).

65. Scientific Terminology.

A Terminology is a collection of terms necessary in describing individual things.

But we require not only a system of names to designate classes but **a collection of terms which will enable us to describe individual objects.** This is a **Terminology,** and it will embrace names of the properties—shape, colour, etc. —and of the parts of the objects recognised in the science. As both classification and nomenclature depend upon the knowledge of the qualities of objects, which knowledge is the result of comparison and the noting of points of agreement and difference, it follows that, unless we can express the qualities by suitable names, our nomenclature cannot be fixed and stable. Terminology is, in brief, the language in which we describe objects, and, without description, there can be no classification. All the names which form a terminology are general names; though, by their combination, we can describe individuals.

Terminology is essential to classification.

Botany is the only science which possesses a complete Terminology.

Botany is the only science which, as yet, possesses a complete terminology; this, as well as its nomenclature, it owes to Linnæus. "The formation of an exact and extensive descriptive language for botany has been executed with "a degree of skill and felicity, which, before it was attained, "could hardly have been dreamt of as attainable. Every "part of a plant has been named; and the form of every

"part, even the most minute, has had a large assemblage of "descriptive terms appropriated to it, by means of which "the botanist can convey and receive knowledge of form and "structure, as exactly as if each minute part were presented "to him vastly magnified. . . .

"It is not necessary here to give any detailed account of "the terms of botany. The fundamental ones have been "gradually introduced, as the parts of plants were more "carefully and minutely examined. Thus the flower was "successively distinguished into the *calyx*, the *corolla*, the "*stamens* and the *pistils;* the sections of the corolla were "termed *petals* by Columna; those of the calyx were called "*sepals* by Neckar. Sometimes terms of greater generality "were devised; as *perianth* to include the calyx and corolla, "whether one or both of these were present; *pericarp* for "the part enclosing the grain, of whatever kind it might be, "fruit, nut, pod, etc. And it may easily be imagined that "descriptive terms may, by definition and combination, "become very numerous and distinct. Thus leaves may be "called *pinnatifid*, *pinnatipartite*, *pinnatisect*, *pinnatilobate*, "*palmatifid*, *palmatipartite*, etc., and each of these words "designates different combinations of the modes and extent "of the divisions of the leaf with the divisions of its out-"line. In some cases arbitrary numerical relations are "introduced into the definition: thus a leaf is called *bilobate* "when it is divided into two parts by a notch; but if the "notch go to the middle of its length, it is *bifid;* if it go "near the base of the leaf, it is *bipartite;* if to the base, it is "*bisect*. Thus, too, a pod of a cruciferous plant is a *silica* if "it be four times as long as it is broad, but if it be shorter "than this it is a *silicula*. Such terms being established, "the form of the very complex leaf or frond of a fern is "exactly conveyed, for example, by the following phrase: "'fronds rigid pinnate, pinnæ recurved subunilateral pinna-"tifid, the segments linear undivided or bifid spinuloso-"serrate.'

"Other characters, as well as form, are conveyed with the "like precision: Colour by means of a classified scale of

"colours" but "the naturalist employs arbitrary names. . . "and not mere numerical exponents, to indicate a certain "number of selected colours" (Whewell, *Novum Organon Renovatum*, pp. 315-317).

Technical terms employed in a Terminology are unambiguous; terms of common speech must be made so by convention.

* In the above examples we have illustrations of both the kinds of terms of which a terminology consists—names of parts of the plants, as *pistil*, *stamen*, *calyx*, *frond*, and names of properties, as *bipartite*, *silica*, *pinnate*. Most of these technical terms being peculiar to the science have a perfectly clear and definite meaning, but when terms in use in common life—as the names of colours—are required to form part of a terminology, their meaning must be precisely, though arbitrarily, fixed by convention. For there must be no doubt as to the exact meaning of the terms used in a scientific description, as, otherwise, our scientific language will be incapable of expressing all the shades of difference which we recognize in the objects we are examining and comparing. To again quote Dr. Whewell: "The meaning of [descrip-"tive] technical terms can be fixed in the first instance only "by convention, and can be made intelligible only by present-"ing to the senses that which the terms are to signify. The "knowledge of a colour by its name can only be taught "through the eye. No description can convey to a hearer "what we mean by *apple-green* or *French-grey*. It might, "perhaps, be supposed that, in the first example, the term "*apple*, referring to so familiar an object, sufficiently suggests "the colour intended. But it may easily be seen that this is "not true; for apples are of many different hues of green, "and it is only by a conventional selection that we can "appropriate the term to one special shade. When this "appropriation is once made, the term refers to the sensation "and not to the parts of the term; for these enter into the "compound merely as a help to the memory, whether the "suggestion be a natural connexion as in 'apple-green,' or "a casual one as in 'French-grey.' In order to derive due "advantage from technical terms of this kind, they must be "associated *immediately* with the perception to which they "belong; and not connected with it through the vague

"usages of common language. The memory must retain "the sensation; and the technical word must be understood "as directly as the most familiar word, and more distinctly. "When we find such terms as *tin white* or *pinchbeck-brown*, "the metallic colour so denoted ought to start up in our "memory without delay or search.

* "This, which is most important to recollect with respect "to the simpler properties of bodies, as colour and form, "is no less true with respect to more compound notions. In "all cases the term is fixed to a peculiar meaning by conven-"tion; and the student, in order to use the word, must be "completely familiar with the convention, so that he has no "need to frame conjectures from the word itself. Such "conjecture would always be insecure, and often erroneous. "Thus the term *papilionaceous*, applied to a flower, is "employed to indicate, not only a resemblance to a butter-"fly, but a resemblance arising from five petals of a certain "peculiar shape and arrangement; and even if the resemb-"lance to a butterfly were much stronger than it is in such "cases, yet if it were produced in a different way, as, for "example, by one petal, or two only, instead of a 'standard,' "two 'wings,' and a 'keel' consisting of two parts more or "less united into one, we should no longer be justified in "speaking of it as a 'papilionaceous' flower" (*History of Scientific Ideas*, vol. ii., pp. 111-113; *Novum Organon Renovatum*, pp. 314-315).

BOOK II.

PROPOSITIONS.

CHAPTER I.

DEFINITION AND KINDS OF PROPOSITIONS.

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66. Definition of Proposition.

A *Proposition* is the verbal expression of a truth or falsity.

A Proposition may be briefly, but sufficiently, defined as **the verbal expression of a truth or falsity.** From this it follows that not every grammatical sentence is a logical proposition. The latter implies *belief* in the statement made, and claims assent; whilst the former may be the expression of a command or a wish, or some other of the many forms taken by human speech, without necessarily making a distinct statement challenging assent or dissent. Not only does every proposition express a truth or falsity, but this is the only way in which truth or falsehood *can* be expressed; a logical proposition is the one form of words of which it can be said 'This is true' or 'This is false.'

Deductive Logic does not examine into the truth of Propositions,

but Inductive Logic does.

Nevertheless, it is no part of the business of Deductive Logic to examine into the truth or falsehood of any individual proposition; it accepts those offered to it as true, and determines what inferences can be drawn from them. Inductive Logic, on the contrary, has for its sphere the investigation of this very point.

This definition of the proposition makes clear that it is the translation into language of the judgment, which, as the essential form of thought, is the ultimate subject-matter of logic (*see* § 8). All knowledge is expressed in affirmations made by thought about reality, and such affirmations expressed in words are propositions. In investigating propositions, therefore, we must constantly go behind the form of words to the judgments which are more or less perfectly expressed by them.

Judgment is the subject-matter of Logic.

67. Kinds of Propositions.

Kinds of Propositions.

Propositions are traditionally divided into different classes on the bases of Relation of Subject and Predicate, of Quality, and of Quantity.

I. Relation.

(i.) Categorical - *S is P* : *S is not P.*

(*a*) Analytic.

(*b*) Synthetic.

(ii.) Hypothetical - *If* **A**, *then* **C**.

(iii.) Disjunctive - *Either* **X** *or* **Y**.

II. Quality.

(i.) Affirmative - *S is P.*

(ii.) Negative - *S is not P.*

III. Quantity.

(i.) Universal—

(*a*) Singular - *This S is P.*

(*b*) General - *Every S is P.*

(ii.) Particular - *Some S's are P.*

The nomenclature of the classification under Relation is in a confused state. Some writers make a twofold division, subsuming Hypotheticals and Disjunctives under a wider class which they call Conditional, though by others this use of the terms Hypothetical and Conditional is reversed. The threefold division is, however, needed to mark important differences between the forms of propositions.

Quality and Quantity apply primarily to Categorical Propositions.

As the divisions under *Quality* and *Quantity* apply primarily to Categorical Propositions, it is proposed to treat them under that head, and to consider how far they are applicable to Hypothetical and Disjunctive Propositions when we treat of those forms.

CATEGORICAL PROPOSITIONS.

68. Analysis of the Categorical Proposition.

A *Categorical Proposition* simply asserts or denies a fact.

A Categorical Proposition is one which simply asserts or denies some fact; as 'Gold is yellow'; 'True bravery is not rash.'

Categorical Propositions should be considered before Hypothetical or Disjunctive Propositions.

The consideration of this class of propositions naturally precedes that of either Hypothetical or Disjunctive propositions. For we can only require to make the assertion that *S is P* dependent on a condition when we have already had experience of the presence of *P* in some instances of *S*, and desire to find the reason for that connexion. Nor can we say that *S is P or Q* unless we know that *P* and *Q* may be subsumed under a wider genus *M*; thus, the Disjunctive proposition is a more specialized form of the proposition *S is M*.

The Demonstrative and the Impersonal Judgments are simpler forms than the complete logical proposition, *S is P*; but they can be expressed in this complete form, which is the only one adopted in formal Logic.

It was seen in § 8 that the most elementary form of a complete judgment is a simple interpretation of an actual experience. From this, thought increases in complexity and generality, until the point is reached when we have a proposition of the form *S is P*, in which subject and predicate are distinct terms (*cf.* § 23). Now, all categorical propositions are capable of being expressed in this form, and, for the sake of simplicity, formal Logic so expresses them. This possibility is all that is meant when it is said that the complete logical form is 'involved' in every judgment. We do not say that an exclamation, such as 'Fire!'—which expresses a true judgment—is a *worn-down* form of some such statement as 'That property is on fire,' but that it may be *expanded* into such a form without change of meaning. The Impersonal Judgment may be similarly expanded; *e.g.*, 'It rains' may be written 'Rain is falling.' Such a reduction makes manifest the artificial character of the formal proposition with

its emphasis of the distinction between subject and predicate. For in the Impersonal Judgment—which may be regarded as the most elementary attempt to explain reality—the real subject is not made definite at all, but is simply the vague mass of present impressions. The whole force of the judgment rests in the predicate, which, indeed, as being the interpretative element may be regarded as the most essential element in every judgment. In making a reduction of all categorical judgments to one fixed form of proposition, formal logic makes a simplification which is not altogether justified by either thought or expression.

Analysis of the forms *S is P*, *S is not P*.

* We must now examine this form more closely. As was pointed out in § 23 it consists of three parts—subject, copula, and predicate—two terms and the expression of a relation of agreement, or disagreement, between them. When we say *S is P*, 'Gold is yellow,' we mean that we are referring to identical things under different names implying different attributes (*see* § 17). On the other hand when we affirm *S is not P*, 'Corn is not poisonous,' we mean that the terms used are applicable to entirely different things, and that both can never be correctly applied to any single object; for all the attributes connoted by the one term are never found conjoined with all those connoted by the other.

The *Copula* merely expresses agreement or disagreement between the terms,

* **The Copula**. The relation between the terms is expressed by the Copula; which is the verb *is* or *are*, by itself in affirmative propositions, and conjoined with the particle *not* in negative ones. Nothing but this bare relation of agreement or disagreement is expressed by the copula which, in itself, involves no assertion of existence. It is true that the verb 'to be' sometimes has this meaning of 'exists,' as when we say 'Evil is.' But in all such propositions *is* is not the copula but the copula and predicate combined, and may be expanded into *is existent*—'Evil is existent—where the *is* has its merely relational value (*cf.* § 89).

and is always in the Present Tense,

* The copula is always in the *Present Tense*. Every act of judgment is a present one and expresses a present belief. Moreover, a proposition which is once true must be always true; no change of time can affect it, for it refers to the

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which is the only one free from ambiguity,

moment in which it was first made. If, then, we express a judgment about a past or future event in a formal proposition, we, as it were, put ourselves at that point of time. Only when propositions are expressed in the Present Tense can they enter into formal arguments, for only then are they unambiguous. If we say *M was P* and *S was M* we can draw no conclusion at all, for the time when *S* was *M* may have been quite different from that in which *M* was *P*, and so we are not justified in inferring that at any one moment *S was P*. Of course, when we use such arguments, as we constantly do in every-day life, it is with the tacit understanding that all the propositions refer to exactly the same point of time. But the verb *was* does not express, or even imply, this ; and the same difficulty meets us in the use of the Future *will be*. This difficulty can only be avoided, and this tacit assumption —that all the propositions in an argument refer to exactly the same time—expressed, by writing each in the Present Tense, for that tense is the only one which expresses one simple, exact, and unmistakable point of time. This restriction to the Present Tense also marks our belief that attributes *co-exist* in the same subject. We cannot apprehend them all at once, for attention can, at most, be fixed on two or three of them at any one moment; yet we can vary the order in which we thus experience them ; and we believe that the successive knowledge of them is necessitated by the nature of our minds, not by the nature of the things themselves. We recognize that a piece of gold is *at once* yellow, heavy, and malleable, though we probably perceive those attributes *successively* in the order named. By saying 'Gold *is* yellow,' '*is* heavy,' '*is* malleable,' we emphasize this fact of the co-inherence of those attributes in the substance gold. And the same form of speech is adopted when the attributes immediately apprehended are believed to necessarily involve the presence of another in the future; as when we say of the wound of a still living man that it *is* mortal, or of a poison still in the vial that it *is* deadly.

and which marks our belief that attributes *co-exist* in the same subject.

* **Subject and Predicate.** As the copula expresses a relation between the two terms, every affirmative proposition

sets forth a *process* of synthesis. This differentiates the Proposition from the Term, in which the *product* of the synthesis is alone regarded. But the mere fact that we are in a judgment engaged in giving meaning to some aspect of reality, necessitates our regarding one Term as the more or less permanent and determined centre to which the other is to be attached, and which it will qualify. In other words it is a process of affirming *attributes* of a *thing*. This fixed and determined centre, or thing, logically comes first in thought, and its name forms the **Subject** of the proposition; whilst the name of the attribute, or interpreting notion, which we affirm or deny of it, is the **Predicate**. It by no means follows that because the Subject is logically first in thought it is always expressed first in language. This is very frequently not the case, and which term is subject and which is predicate must be decided by the meaning of the sentence rather than by the position of the words in it. If one term clearly tells us something about the other—as every Adjective does—it is the predicate. On the other hand, if one Singular Term occurs, or a General Term is explicitly used in its whole denotation in an affirmative proposition, that term is necessarily the subject (*cf.* § 72). But no general rules can be given for distinguishing in all cases between Subject and Predicate; the meaning of the sentence must decide.

The *Subject* is the fixed centre or thing.

The *Predicate* is the attribute affirmed of the subject.

But although a proposition may thus be analysed into two terms and a copula, it must be borne in mind that language here emphasizes one aspect of judgment to the exclusion of another and equally important one (*cf.* § 8). The judgment is always *one* act of mind, the interpretation of *one* aspect of reality. It is not a comparison of separate things, nor a connexion of two independent concepts, as is suggested by the verbal form in which it is expressed. The copula represents no separate element of thought, and in Aristotle, as now in common language, copula and predicate were expressed by the same word. The Subject marks the point of reality which is being interpreted, and the Predicate expresses the interpretation; but neither reality nor inter-

Every judgment is a unity.

pretation can exist as elements of knowledge apart from each other.

Propositions whose terms are synonyms express no true predication.

This Subject and Predicate form of Proposition is so general that it is used even in cases where there is no real distinction between the terms, as when they are synonyms such as 'Mercury is quicksilver,' 'Queen Victoria is Empress of India,' 'Bismark is Duke of Lauenburg.' In such cases there is no predication of attributes whatever, but simply a statement that the same object bears the two names. If we express this, and say 'The same metal is called both mercury and quicksilver,' we do get a really significant predication, but we have changed the whole form of the proposition in order to do so.

The Impersonal Proposition is most natural where there is no definite subject.

Finally, it may be noted that when we are expressing a judgment about some group of events which we cannot easily connect with a definite and fixed centre, we most frequently and naturally use the *Impersonal* form of proposition, as 'It snows.' This, however, as was shown in the earlier part of this section, can be expanded into the Subject and Predicate form, and should be thus expanded before being used in formal reasoning.

* 69. Analytic and Synthetic Propositions.

The distinction between Analytic and Synthetic Propositions is one of *origin*.

The distinction between these was pointed out in § 40. It applies, of course, only to affirmative propositions, and is really a matter of the origin of each judgment. If the judgment can be obtained by an analysis of a concept already formed, or—which comes to the same thing—of the definition of a class name (*cf.* § 49), then it is **Analytic**; if the predicate asserts an attribute which does not form part of the connotation of the subject, the judgment is **Synthetic.**

The distinction is not a subjective one.

It is often objected that this division of judgments is purely subjective; that every judgment is at first synthetic, and by familiarity becomes analytic; that, *e.g.*, 'Lions are carnivorous' is a synthetic judgment for any person whose knowledge of a lion did not hitherto embrace that attribute, but is thereafter for him, as for all who know the nature of

a lion, an analytic judgment. But this is to confound the personal history of an individual's mind with the general method of knowledge with which alone Logic is concerned. The basis of the distinction is the fundamental postulate of knowledge that reality has a constant nature, that there is a unity in the world which finds expression in a uniform constitution of things. This is at the basis of the very idea of connotation and definition, that is of the very possibility of classification. When a proposition simply states explicitly what is regarded as the constant and essential nature of the subject, it is Analytic; when it makes some additional affirmation, it is Synthetic.

A judgment which was analytic in the fullest sense would make an explicit statement of the full connotation of the subject. But this demand is not made: any proposition which states any part of the connotation is held to be analytic. But as the connotation of a term may change with increase of knowledge, it is evident that a judgment may pass from one of these classes to the other; the distinction, therefore, is not sufficiently fixed to be of great logical importance.

Every judgment is, in a sense, both analytic and synthetic.

Moreover, in another, but a very real sense of the words, every judgment is both analytic and synthetic. It is analytic, for it sets over against each other and distinguishes elements in the one and indivisible mental act of judgment; and it is synthetic in that it brings together this present element of reality and the universal idea which gives it meaning.

70. Quality of Propositions.

Affirmation and Negation are the only possible kinds of *Quality*.

In expressing the relation of predicate to subject only two courses are open to us. We must either *affirm* that the subject possesses the attributes connoted by the predicate, or we must *deny* this (*cf.* § 19). On this basis, therefore, propositions are classed as (i.) **Affirmative** and (ii.) **Negative**. In a Negative Proposition we do not deny that the Subject has *any* of the attributes connoted by the predicate, we only deny that it has them *all*. Some it must have, for the genus of which the denied predicate is a species is always under-

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In Negative Propositions the *P* which is denied always belongs to a genus which is affirmable of the *S*.

stood to be affirmable of the subject. If this were not so the proposition would be meaningless, as if we should say 'Virtue is not blue,' where there is no real predication, for the notion of colour is absolutely foreign to an unextended and abstract concept such as 'virtue.' But if we say 'Those berries are not poisonous' a real meaning is conveyed, for it is understood that they are 'edible' [*cf.* remarks on the Universe of Discourse in § 28 (iv.)].

Negation is due to *S* possessing an attribute incompatible with the proposed *P*, and this is implicit in the Negative Judgment.

Pure negation has no existence in fact, and cannot be really thought. The denial of any number of attributes of a subject *S* can only be grounded in, and justified by, the fact that *S* possesses some attribute which is incompatible with the proposed *P*, so that if *P* were added, *S* would at once lose its character and cease to be *S* at all. It is this incompatible attribute which is the real basis of the negation, though we may not even know what it is, and may only feel that if *S* were to receive *P* it would at once cease to be itself. This, though it is not made explicit, must be regarded as implicitly contained in the negative judgment.

An *Infinite Judgment* is an attempt to express negation as affirmation —*S is non-P*,

* Some logicians have endeavoured to reduce negative propositions to an affirmative form by regarding the negation as part of the predicate, and writing *S is non-P* instead of *S is not P*. Such judgments were called *Infinite* by Kant, who, however, retained the true negative judgments as well, though it is evident that to have two forms to express the same fact is not only superfluous but misleading, as it suggests a distinction which does not exist. But this simplification is only apparent. For, "in order to know that *S* "accepts *non-P*, must we not already have somehow learnt "that *S* excludes *P*? And, if so, we reduce negation to "affirmation by first of all denying, and then asserting that "we have denied,—a process which no doubt is quite legiti-"mate, but is scarcely reduction or simplification" (Bradley, *Principles of Logic*, p. 111). Besides, as has been already said (*see* § 19), if *non-P* be taken—as strictly and formally it should—to include everything which is not *P*, then *non-P* is not conceivable at all, for we cannot possibly form a concept which will embrace all the heterogeneous elements in the

but this assumes a previous denial.

Moreover, as *non-P* is not a true concept, there is no real affirmation.

universe which are excluded from *P*. In *S is non-P* we have, then, the form of an affirmation without the reality. Upon analysis it will be found that all we can possibly mean by asserting the absence of *P* is to deny its presence, and it is better to do this explicitly. In short, Affirmation and Negation are fundamentally different, and it can only lead to confusion to treat the distinction as if it were only verbal, as is done by expressing the negative proposition *S is not P* as a sham affirmative *S is non-P*.

The distinction between Affirmation and Negation is not verbal merely, but fundamental.

71. Quantity of Propositions.

The Quantity of a Proposition depends upon whether the predicate is explicitly affirmed, or denied, of the whole of the subject or not. This gives a two-fold division into :

A Term is *distributed* when explicit reference is made to its whole denotation. It is *undistributed* when the extent of denotation referred to is left indefinite. Universal Propositions distribute their subjects; Particular Propositions do not.

(i.) *Universal Propositions*, in which the subject is **distributed**—*i.e.*, explicitly stated to be used in its whole denotation.

(ii.) *Particular Propositions*, in which the subject is **undistributed**—*i.e.*, the extent of the denotation referred to is left absolutely indefinite.

The marking of this distinction evidently necessitates a fourth element in the Proposition; viz., the mark of the quantity of the subject.

(i.) Universal Propositions.

Under this head we have two sub-classes of propositions. In the one the definite whole which forms the subject is indivisible, *i.e.*, is an individual; in the other it is simply undivided, *i.e.*, it is a class, of every member of which the predication is made.

Universal Propositions are of two kinds:

(*a*) *Singular Propositions.* In a Singular Proposition the subject is a single individual *directly* indicated by a Proper Name or by a General Name with a distinctive limiting mark attached to it restricting it definitely to the one individual indicated, such as, '*This* man,' '*That* man,' '*The* man *of whom I spoke to you yesterday.*' The symbolic ex-

(*a*) *Singular*—where the *S* is a definite individual;

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pression of such a proposition in Logic is—*This S is P; This S is not P.*

(*b*) *General—where the S consists of every member of a class.*

(*b*) *General Propositions.* In a General Proposition the subject is the whole class which bears the General Name, of every individual member of which the predication is made. Each individual is here indicated *indirectly* through the General Name, not as a definitely specified individual, but merely as a member of a class to which it belongs in virtue of possessing the attributes connoted by the Class Name. The general symbolic expression of such propositions is—*Every S is P; No S is P.*

These two kinds of propositions are fundamentally one in making explicit reference to the whole extent of the *S*.

The name 'Universal' is often restricted to this latter class; but, from the point of view of denotation, there is no fundamental difference between them and Singular Propositions, for it is absolutely indifferent whether the subject be small or great in extent so long as, whatever that extent may be, the whole of it is explicitly referred to. It is this definiteness of application which distinguishes both kinds of propositions from the Particular, and, therefore, it is usual to class them under one common name.

The signs of quantity of an Affirmative General Proposition are *Each*, *Every* and *All*—the latter always used distributively.

The common signs of quantity for an Affirmative General Proposition are *Each, Every, All.* The latter word is ambiguous, as it may be understood either in a distributive or in a collective sense [*cf.* § 27 (ii.)]. In a General Proposition, however, it must always be interpreted *distributively* (*omnes*, not *cuncti*), for the predication is made of each individual member of the class, not of the class as a whole; thus 'All lions are fierce' means 'Every lion is fierce.' To avoid ambiguity we shall generally use 'Every' or 'Each' in preference to 'All'; but whenever 'All' is used it must be borne in mind that, unless the context shows the contrary, it is equivalent to 'Every.' The distinction may also be marked by writing the General Proposition *All S's are P*, and the Collective *All S is P.*

Propositions with Collective Subjects are Singular.

A Collective subject, indeed, gives us a Singular Proposition, for the predication is there made, not of individuals but, of *one group*—as when we say 'The Romans conquered Gaul,' where it is evident we are referring, not

to individual Romans but, to the Roman army as a body. So, if it is said 'All the books in this library weigh several tons,' the reference is plainly to the whole body of books, and is equivalent to 'This collection of books weighs several tons,' where we get at once the typical Singular form, *This S is P.*

A Universal Proposition is not a summing up of singulars

If we regard only the verbal form of the Universal Proposition it would appear on the face of it to be merely a summing up of a number of singular judgments—'This, that, and the other *S* is *P*'—one of which has been made of each member of the class *S*. Thus, in *form*, the proposition *Every S is P* claims to be the result of a complete enumeration of instances. But in meaning it is generally something very different. If we compare two such propositions as 'Every book on these shelves treats of Logic' and 'Every right-angled triangle is inscribable in a semi-circle', we see that they are really very different in essence and in importance. The former refers to only one collection of objects here and now; it is universal only in the sense that if the collection of books remains unchanged the same proposition will hold true of it throughout the lapse of time. But in the ordinary sense of the word we cannot say that such a judgment is necessarily true independently of limitations of time and space. It resembles, indeed, and that very closely, singular judgments of fact, such as are most exactly represented by propositions such as 'London is the largest city in Europe,' where the subject is the Proper Name. Such judgments are in a sense, universals, for the predicate is affirmed of all the subject, but they are concrete universals. If, on the other hand, we examine the latter judgment—'Every right-angled triangle is inscribable in a semi-circle'—we see at once that its ground is not an enumeration of instances, but that the proposition is true because the nature of right-angled triangles is such that the affirmed predicate *must* hold true of them, and that this can be shown by rigorous demonstration. The basis of such a proposition is found, therefore, in connexion of content, not in constant experience in perception. And this will be seen to be the case in all judgments which we feel to be really

but is the denotative expression of a *Generic Judgment*—*S is P*—which asserts connexion of content.

universal, not only in the sense of always holding true of an examined concrete totality, but in that of applicability to whatever in the universe falls at any time under the subject-concept. But as the true ground of the universal proposition is thus seen to consist in the nature of the reality dealt with, it is better to mark this by expressing the judgment in a form of proposition which does not suggest a false origin. This we find in the *Generic Judgment*—*S is P*, *e.g.*, Right-angled triangles as such are inscribable in a semi-circle; Man is mortal,—where the connotation of the terms is obviously the more prominent element. Of such judgments the ordinary categorical proposition may be taken as a statement in denotation.

The Generic Judgment has both an abstract and a concrete aspect.

There is thus in the Generic Judgment both an abstract and a concrete reference. It is abstract in that it states an essential relation of content without direct reference to the particular instances in which that connexion exists in reality; it is concrete in that such relation is always regarded as being actually so existent.

The Negative of *This S is P* is *This S is not P*, but *All S's are not P* and *Every S is not P* are not universal, but mean *Some S's are not P*.

When we wish to express a Negative Singular Proposition we need only add the sign of negation to the copula of the affirmative—*This S is not P*. But when we require to express a Negative General Proposition we cannot do it by saying *All S's are not P*, or *Every S is not P*. Each of these expressions simply means that it is not allowable to affirm *P* of *every S*, but does not mean that *P* cannot be asserted of *any S at all*. The majority of *S's* may be *P* and yet it remain true that *Every S* is not *P*, which, indeed, holds if *only one S* is not *P*. Thus, it appears that *All S's are not P*, and *Every S is not P*, do not possess the definite character which distinguishes Universal Propositions, but are quite indefinite in quantity; that is, they are really Particular, and should assume the special symbolic form of Negative Particular Propositions—*Some S's are not P*. To mark the absolute and entire separation between *P* and *S* required in a General Negative Proposition we must clearly express the fact that not a single individual which possesses the attributes connoted by *S* also possesses all those connoted by *P*, and the

only unambiguous way of doing this is by saying *No S is P*. This, then, is the most exact symbolic form of a General Negative Proposition. The force of this distinction will appear more clearly if a few examples are considered. Thus, it is true to say 'Every Englishman is not a lawyer,' for this only implies that there are some Englishmen who do not follow the profession of law; but it would be false to say 'No Englishmen are lawyers.' Similarly, 'All Englishmen are not brave' is true; but 'No Englishmen are brave' is false. Of course, if *All* is taken collectively, then *All S is not P* is a true negative of *All S is P;* but, as was remarked above, ambiguity had better be avoided by using another form of words. For instance, it would be better to say 'This collection of books is not five tons in weight' than to say 'All these books are not five tons in weight;' for the strict formal interpretation of this last expression is that some individual books in this collection do not weigh five tons; though this, of course, is not what is meant.

The true expression of a Negative Universal is *No S is P*,

Of course the ultimate justification of such a proposition as *No S is P* is found in the fact that the content of *S* includes one or more elements which are incompatible with *P* (*cf.* § 70). The basis of the negative as of the affirmative universal is, therefore, not an exhaustive examination of instances, as the verbal form suggests, but a knowledge of content which finds appropriate statement in the Negative Generic Judgment *S is not P*, of which the form *No S is P* is merely the denotative expression.

and its justification is the Negative *Generic* Judgment *S is not P*.

(ii.) Particular Propositions.

The distinguishing characteristic of a Particular Proposition is the perfect indefiniteness of the application of the subject. Its general symbolic form is—*Some S's are P; Some S's are not P.* (Some is always used in the sense of *aliqui* never in that of *quidam.*) Now, in using this word *Some* in Logic its absolute indefiniteness must always be borne in mind. Usually, no doubt, in common talk, when we say 'Some' we mean to refer to less than all. But, if the idea underlying the word be analysed, it will be found that this is not really involved in it. It would be perfectly

In a *Particular* Proposition the extent of the S is perfectly indefinite.

In Logic *some* is always the mark of absolute indefiniteness.

accurate to say 'I saw some of your friends at the theatre yesterday' when the speaker has no knowledge as to whether the individuals he saw included all the friends of the person he is addressing or not. In fact, this idea will probably not be present to his mind at all; he simply states imperfect knowledge in an appropriately indefinite form. So a scientist, when he has observed the concurrence of certain phenomena in several instances, but knows of no necessary law connecting them, would only be justified in positively affirming *Some S's are P*, even though he had never met with an *S* which was not *P*, and might even think it highly probable that *Every S is P*. Of course, Science cannot rest satisfied with anything short of a definite Universal, but the indefinite Particular is quite allowable as a stepping-stone to the more perfect stage. This may be reached either by finding a necessary connexion which enables us to affirm that *S is P*, or by discovering that when *S* is limited in a certain way it is *P*. Thus, it may be that *MS* is *P*. A new class name, *R*, would, probably, then be found for *MS*, so that, ultimately, we should get the generic judgment *R is P*. But, in common life, our knowledge is often avowedly indefinite, and should, therefore, be expressed in a truly indefinite form. As this indefiniteness must be absolute we cannot agree with those few logicians who would depart from the traditional use of 'Some' so as to exclude 'All' from its possible range, thus reducing slightly (but, of course, not abolishing) its indefinite character. Nor would we define it as 'not none' for this leads us to a circle, as our only definition of 'none' must be 'not some.' It is better to say at once that 'Some' is a confession of limited knowledge, and means 'I know I can make this predication of at least one *S*, but of what part of the denotation of *S* it holds good I do *not* know.' Every indefinite subject gives us, therefore, a Particular Proposition. We may even know that the predication can only be made of one individual, still if that individual is merely referred to as a member of a class and not *definitely* marked out from the other members of that class—as *An S is P*—the proposition is indefinite, and, therefore, Particular. In such a case we do not know

Science aims at Universal Propositions,

but common life often requires indefinite ones.

Some does not exclude *All* from its scope,

and is a confession of limited knowledge.

Every indefinite subject, even if singular in reference, gives a Particular Proposition.

which S is *P* and the proposition is equivalent to *Some* (*one*) *S is P*. Particular Propositions, indeed, would be more appropriately named '*Indefinite*' Propositions, but it is not advisable to alter the old and long established nomenclature of the Science.

Particular Propositions are the denotative expression of the Modal Particulars *S may be P*, *S need not be P*.

Particular Propositions in their form express an indefiniteness in denotation only; in other words they suggest that their origin is an enumeration of instances, either avowedly incomplete, or, at least, not known to be complete. But, as this suggestion of enumeration was found to be misleading in the case of the universal proposition, so it is here in most cases. Sometimes, no doubt, the judgment is the outcome of a more or less extended experience, but, even then, the fundamental point of uncertainty is not whether the enumeration is complete—that is an aspect of the question quite out of the range of scientific, *i.e.* exact, thought—but whether the determination of the content of *S* is complete. And the form of proposition which most clearly expresses this doubt is what is most appropriately called the *Modal Particular*, whose affirmative form is *S may be P*, and its negative form, *S need not be P*. The meaning is that the content of *S* has not been sufficiently determined to make it clear whether or not it contains the full and sufficient ground for *P*. Of these forms the traditional particular propositions are the denotative expressions.

Indesignate Propositions. What has already been said will enable us to deal with those propositions to whose subjects no sign of quantity is attached, as 'Birds are feathered.' These are called by Hamilton **Indesignate or Preindesignate Propositions.** Some writers have held that they are quite inadmissible in formal Logic, and it is true that the traditional Logic, with its undue deference to distinctions of mere language, does not acknowledge them. But we have seen that they are the fundamental form of the judgment, without reference to which the traditional forms cannot be justified. We have also seen that they can all be translated into the traditional forms of denotative expression. When the subject term of an indesignate proposition expresses a general concept, and the predicate makes an

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assertion which is grounded in that content we have, of course, the true Generic Judgment, as when we say 'Sin is worthy of punishment,' where the predicate is a necessary consequence of a true conception of the subject. Whenever we know the predicate in an affirmative Indesignate Proposition to be a part of the connotation, or a proprium, of the subject, we know the proposition is Universal (*cf.* §§ 35-37). If the predicate is a separable accidens, we know the Proposition is Particular (*cf.* § 38). If the predicate is an inseparable accidens we are, strictly speaking, only justified in affirming it as a particular, as we know no reason for the connexion of *P* and *S* and cannot, therefore, be sure that it is really invariable. In other words, the only judgment as to connexion of content we should be justified in making would, in this case, be the Modal Particular *S may be P*. Of course, if uncontradicted experience is the ground on which we make the judgment, the wider and more varied that experience is, the greater is the probability that no instances to the contrary exist at the present time, whatever may have been the case in the past or may be the case in the future. But certainty can never be attained, and though for common practical purposes the proposition expressing such experience would be usually regarded as general, yet in logic we have no right to raise it to the dignity of a true universal, whose very essence is that it must hold true always and everywhere. Often, however, the context so limits the subject that we know it is true universally. Thus, if we have the proposition 'Crows are black,' and interpret it as 'All crows are black' we have a proposition which is probably, but not certainly, true. But if the context shows that only *English* crows are meant—or even *known* crows—we know the proposition is a really general. On examination, indeed, it will be found that Indesignate Propositions, when they are not true universals because their predicates are separable accidentia of their subjects, are yet only employed when the predication can be made of the majority—generally the great majority—of the members of the class denoted by the subject, as 'Frenchmen are vivacious,' 'Italians are musical.' Such propositions were called by the old logicians 'Moral

In Affirm. Indesignate Propositions if the *P* is part of the connotation or a proprium of the *S*—Universal.

If the *P* is a sep. accidens of the *S*—Particular.

If the *P* is an insep. accid. of the *S*—strictly Particular, but practically regarded as Universal.

Indesignate Propositions when Particular refer to most of the *S*.

Universals,' whilst the really general propositions were termed 'Metaphysical Universals.' But, of course, a 'Moral Universal' is only a Particular, and if stated universally is false, as it would be to say 'Every Italian is musical.'

Negative Indesignate Propositions are Particular if *P* is a sep. accid. of *S*;

If an Indesignate Proposition is Negative, the predicate must evidently be either a separable accidens of the subject or an attribute never found in that subject. In the former case, the propositions belong to the 'Moral Universals' just discussed, and are Particular; as 'Englishmen are not cowardly.' In the latter case they are, of course, universal, as 'Englishmen are not negroes.'

if not, Universal.

72. The Four-fold Scheme of Propositions.

Combination of Quality and Quantity gives: A. Universal Affirmative. I. Particular Affirmative. E. Universal Negative. O. Particular Negative.

If we combine the divisions under Quality with those under Quantity we get a four-fold Scheme of Categorical Propositions; viz., Universal Affirmative, Particular Affirmative, Universal Negative, Particular Negative. These it is customary to indicate by the letters **A**, **I**, **E**, **O**, respectively, those letters being the first two vowels of the Latin verb *affirmo* (I affirm), which represent the Universal Affirmative (**A**) and the Particular Affirmative (**I**); and the vowels of the Latin verb *nego* (I deny) which stand for the Universal Negative (**E**) and the Particular Negative (**O**). By writing these letters between *S* and *P* we obtain a brief symbolic mode of expressing propositions. Thus:—

These may be briefly written: **A** - - *S a P*. **I** - - *S i P*. **E** - - *S e P*. **O** - - *S o P*.

A -	*Every S is P* -	*S a P*.
I -	*Some S's are P* -	*S i P*.
E -	*No S is P* -	*S e P*.
O -	*Some S's are not P* -	*S o P*.

These four forms appear to be naturally dictated by the common needs of human speech, in which we require either to affirm or to deny, and to do both either definitely or indefinitely. They do not quantify the predicate, for that is regarded as an attribute and read in connotation (*cf.* § 84). As, however, every term has denotation [*see* § 28 (iv.)], it is possible, if we wish, to consider the denotation of the predicate, and to ask whether, if we do so, we are to consider it as distributed or undistributed in each of the above four forms.

As the Predicate is read in connotation it is not quantified, but as it has denotation we may consider its distribution.

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Affirmative Propositions do not distribute their predicates,

Distribution of Terms. Now, in every *affirmative* proposition, whether universal or particular, we assert that a certain subject possesses an attribute *P*, but we make no assertion as to the full extent of the denotation of *P*. We do not consider whether or not other objects exist of which *P* can also be predicated. In some cases there are such objects—as when we say 'All lions are fierce,' for there are certainly other fierce animals; in other cases there are not—as when we say 'All diamonds are pure crystallized carbon.' But in no case is any explicit reference made to the full denotation of *P*; the extent of its application in each proposition is determined indirectly by that of the subject of which it is affirmed. In every affirmative proposition, therefore, whilst the predicate is asserted in its full connotation, it is left indefinite as to its denotation, and is, therefore, *undistributed.*

but Negative Propositions do.

In a *negative* proposition, on the other hand, as was said above (*see* § 70), every part of the connotation of the predicate is not denied of the subject, but only the connotation as a whole. But when we look at the predicate in denotation we find that, in every case, it is *distributed.* For it is only when explicit reference is made to *every* object which can be included in the denotation of the predicate that a proposition has any negative force at all. If the subject is not definitely separated from the whole extent of *P*, it may at least partially agree with it, and then there is no negation. And this is independent of the extent of the subject. If we deny *P* of only one individual—as when we say 'This *S* is not *P*' or 'An *S* is not *P*'—yet we must deny *every P* of the *S* in question, or we have evidently denied nothing at all. For we must needs deny not only that 'This *S*' is *this or that particular P* but that it is *any P whatsoever.* A negative proposition must assert that no individual included under the subject can possess the attributes connoted by the predicate in their entirety. But this is equivalent to excluding from *S* every individual object called *P*, for only those objects which possess all those connoted attributes can possibly bear that name.

If we now sum up our results as to the distribution of

each of the terms in each kind of proposition when read in denotation, we have :—

Universal Propositions distribute *S*, Negative Propositions *P*.

(1) *Universals* (**A** and **E**) *distribute their Subjects; Particulars* (**I** and **O**) *do not.*

(2) *Negatives* (**E** and **O**) *distribute their predicates; Affirmatives* (**A** and **I**) *do not.*

Thus, **E** distributes both subject and predicate.
A distributes its subject only.
O distributes its predicate only.
I distributes neither term.

73. Other Signs of Quantity.

Other signs of quantity must be reduced to one of :—*This; Each, Every, All, No; Some.*

This; Each, Every, All, No; Some; are the only signs of quantity which are recognized by Logic. If any others occur they must be reduced to these before the propositions can be used in strictly logical reasoning. Other marks of quantity are, however, in common use in ordinary speech, some of which it will be well to briefly examine.

Strictly definite numerical statements are Exponible Propositions.

(i.) **Numerically definite statements of quantity.** We occasionally have such propositions as 'Three-fourths of the *S*'s are *P*,' 'Sixty per cent. of the *S*'s are *P*,' 'Sixty per cent. of the bullets hit the target.' If these are to be taken in their strict and literal meaning, they must imply that every *S* has been examined, and hence they involve a negative proposition in addition to the affirmative statement which is explicitly made. In this sense they come under the head of Exponible Propositions which are treated in section 75 (ii.). Thus, 'Three-fourths of the *S*'s are *P*' would imply that 'One-fourth of the *S*'s are not *P*.' If, however, such predications are of any general importance a new name will soon be found for the *S*'s which are *P*, as distinguished from those which are not *P*, and the propositions will, thus, become universal. But this is seldom the case. Such statements generally refer to some individual occurrence, and are of no general interest; for they in no way tend to the advancement of knowledge. Or the numerical statement is only meant as a rough approximation; and then, of course, it can be expressed with little loss of meaning by the indefinite logical 'some.'

If of any general importance they tend to become Universal, but this is seldom the case.

They are often more approximations and then mean 'some.'

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Any in a categorical proposition means 'every.'

(ii.) **Any.** *Any S is P*, 'Any house is a shelter in a storm.' In such cases 'any' is evidently exactly equivalent to 'every,' and such categorical propositions are universal. For we cannot assert that *any S* which may be taken at random will be found to be *P* unless we know that *every S is P*. Each is a denotative expression of the Generic Judgment.

A few means 'some.'

(iii.) **A few.** This must be regarded as equivalent to 'some.' For when it is asserted that *A few S's are P* it need not be meant to limit in any way the number of *S's* which may be *P*, but simply to imply that only a small number of instances of *S* have been examined, though every one of those instances may, possibly, have been *P*. Sometimes it is Collective and then means 'a small number,' as when it is said 'A few Greeks defended the Pass of Thermopylæ,' which they evidently did as a body. Such a proposition would be better expressed, formally, in the form 'A small body of Greeks defended the Pass of Thermopylæ,' as this shows its really Singular character.

When *A few* is Collective the proposition is Singular.

Plurative Propositions are of the form *Most S's are P.*

If *Most* and *Few* are interpreted strictly the propositions are Exponible,

(iv.) **Plurative Propositions.** In *Most* and *Few* we have signs of quantity which it is possible to interpret either strictly or vaguely, as we found to be the case even with numerical statements. If taken in the strictest sense of the words, they imply that every instance—or, at least, an extremely large number of instances—of *S* has been examined, and that in the one case a number less than half (*Few S's*), in the other case a number greater than half but less than all (*Most S's*) have been found to be *P;* but that the other instances of *S* have been found not to be *P*. Thus, *Few S's are P* would imply that *Most S's are not P*, though, at the same time, *A small number of S's are P;* and *Most S's are P* would mean that though *The majority of S's are P* yet still *A small number of S's are not P*. Such propositions would, therefore, belong to the class Exponibles [*see* § 75 (ii.)]. But it does not appear that so strict a meaning is usually intended; and, therefore, Logic, restricting itself to that minimum amount of assertion which a proposition *necessarily* implies, can only regard *Most* as meaning 'more than half,'

but this is not their logical sense.

but as not excluding 'all.' To express this, propositions of the form *Most S's are P* are called *Plurative*. Thus, *Most S's* is perfectly indefinite beyond 'half,' and that limit but roughly estimated. For instance, after being at a political meeting an observer might say 'Most of the people present wore a blue rosette,' and only mean to imply that the wearing of the rosette appeared to him to be general, not necessarily that he saw a few persons present who had no rosette. *Most* is, then, equivalent to 'Some more than half' and may in Logic be generally replaced by 'Some,' so that Plurative Propositions may be regarded as Particular. It has, however, been pointed out that, though from two really particular propositions—*Some M's are P*, *Some M's are S*—nothing can be inferred, yet from two pluratives—*Most M's are P*, *Most M's are S*—the conclusion *Some S's are P* can be drawn.

Most is perfectly indefinite above 'half,' and does not exclude 'all,'

and is logically expressed by 'Some.'

Few S's is indefinite up to the limit of 'half' (again interpreted loosely), as *Most S's* is above that limit. But when we say *Few S's are P* we usually mean to imply simply that *Most S's are not P*, and not that any *S's* necessarily exist which are *P*. *Few* does not, therefore, necessarily exclude 'none'; for, in that case, *Most* would exclude 'all,' which we have seen it does not logically do. In fact, to assert that 'Few novelists have ever been superior to Thackeray in humour' by no means implies that any have; such a sentence simply expresses, in a most forcible way, the opinion that 'Most novelists are inferior to Thackeray in humour.' Hence, *Few S's are P* must be regarded as really a Negative Plurative Proposition, and as meaning *Most S's are not P*; whilst *Few S's are not P* is really the Affirmative Plurative *Most S's are P*. Logically, then, *Few S's are P* must be treated as an **O** proposition, and expressed *Some S's are not P*; whilst *Few S's are not P* finds its logical expression as an **I** proposition—*Some S's are P*. From *Few M's are not P*, *Few M's are not S* we can infer *Some S's are P* in the way noted above; for the two given propositions are only negative in appearance, and their true force is expressed by *Most M's are P*, *Most M's are S*.

Few is indefinite below 'half' and does not exclude 'none.'

But *Few* is only another way of expressing *Most . . . not*, and *Few . . not* of expressing *Most*.

Thus, *Few S's are P* is really negative,—**O**; and, *Few S's are not P* really affirmative,—**I**.

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Hardly any and *Scarce* mean *Few.*

Hardly any S's are P, and *The S's which are P are scarce*, are both exactly equivalent to *Few S's are P;* that is, both are logically expressed by *Some* (=*Most*) *S's are not P.*

74. Propositions with Complex Terms.

Though all terms can be expressed by *S* and *P*,

As has been already stated (*see* § 68) all Categorical Propositions can be expressed in one of the forms *S is P* or *S is not P* with a definite, or indefinite sign, of quantity affixed to the subject (§ 71). But it is by no means generally the case that the propositions in use in common speech are as simple in their structure as are those which, for the sake of clearness, we have employed as examples. It is frequently necessary to qualify or limit either the subject or the predicate.

yet terms are frequently complex.

Hence, either term, or both terms, may be of any degree of complexity, and care must be taken to determine what the true predication really is. If there is but one verb there is no difficulty in this; a many-worded term is as easily recognized as a single-worded one. Thus 'The highest mountain in Europe is Mont Blanc' is obviously of the form *S is P*, though subject and predicate have been written in inverted order.

A term may contain a subordinate clause,

But frequently the qualifications required are expressed by subordinate clauses embedded in the proposition, which then, of course, contains more than one verb. In such cases the predications contained in the subordinate clauses must be carefully distinguished from that of the proposition as a whole. The question is, however, of grammatical, rather than of logical, bearing, as it concerns merely the various ways in which a given meaning can be expressed in words. Still, a short discussion of it will probably conduce to a clearer understanding of some of the more complex forms which a proposition may take.

Introduced by a Relative Pronoun and equivalent to an Adjective.

A subordinate qualifying, or limiting, clause is introduced by a Relative Pronoun, expressed or understood, and is equivalent in meaning to an adjective or adjective phrase, by which, indeed, it may be replaced. Thus '*In Memoriam* is a poem which contains many beautiful thoughts' may be equally well expressed '*In Memoriam* is a poem containing many beautiful thoughts.'

There are two kinds of these subordinate clauses:

Of these subordinate clauses there are two kinds :—

(i.) **Explicative.** In this case the qualification contained in the subordinate clause applies to every individual denoted by the class name to which it is attached. The sentence quoted above is an example of this, where the qualification belongs to the predicate. In 'The natives present, who all wore garlands of flowers, greeted us kindly' we have a similar qualification of the subject, the true predication being evidently contained in 'greeted.' Whenever a subordinate clause is Explicative, the name which is qualified can always be substituted for the Relative Pronoun, and the proposition thus formed remains true when removed from its context. Thus, in the example given above, 'The natives present all wore garlands of flowers,' is a statement whose truth is guaranteed by the given sentence as a whole.

Explicative, which qualify the whole denotation of the term;

(ii.) **Determinative** or **Limiting.** Here the subordinate clause restricts the name it qualifies to a certain part of its denotation. Thus, in 'All men who are over six feet in height are eligible for enlistment in the Life Guards' the qualifying clause evidently curtails considerably the denotation of the subject. In such cases the name qualified cannot be substituted for the Relative Pronoun; to say 'All men are over six feet in height' would be obviously false. These limiting clauses always really affect the subject, even when it is not immediately apparent that they do so; for the subject is the more definitely determined term in every proposition (see § 68). The occurrence of a limiting subordinate clause is, therefore, a guide in deciding what is the logical subject of an involved statement. Thus, 'I have read all the books in this library which treat of Politics' is logically expressed by 'All the books in this library which treat of Politics are books which I have read.' The subordinate clause which here appears in the predicate is, of course, explicative. The general form of propositions containing such limiting clauses is, therefore, *Every S which is M is P*, and their meaning is *If any S is M that S is P*, which shows there is no hard and fast distinction between Categorical and Hypothetical Judgments.

and *Determinative,* which limit that denotation.

These last always affect the subject.

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*75. Compound Categorical Propositions.

A *Compound Categorical Proposition* —two or more propositions joined in a single statement.

We have now to consider cases in which two or more propositions are really involved in a single statement, and not merely apparently so as in the instances considered in the last section. This compound structure may be apparent either from the form or only upon analysis of the meaning. We thus have two kinds of Compound Propositions :—

(i.) **Compound in Form.** These will require but few words. They are of three classes :—

A *Copulative Proposition* —simple union of Affirmative Propositions.

(*a*) *Copulative Propositions*, where we have a simple combination of two or more Affirmative Propositions. There are two or more subjects, or two or more predicates, or a plurality of both ; as *S and M are P; S is P and R; S and M are P and R and Q*, etc. These are evidently merely briefer ways of expressing each predication separately; *S is P, M is P; S is P, S is R*, etc. Thus 'Gold and silver are precious metals' is plainly equal to 'Gold is a precious metal, and Silver is a precious metal.' There will, of course, be as many simple propositions as the product of the number of separate subjects into the number of separate predicates; for each predicate is united to each subject to form a distinct proposition.

A *Remotive Proposition* —union of Negative Propositions.

(*b*) *Remotive Propositions*, where we have a similar union of two or more Negative Propositions, as *No S nor M is P; No S is either P or R; No S nor M is either P or R or Q;* etc., which are equivalent to *No S is P, No M is P; No S is P, No S is R*, etc. Everything said above of Copulative Propositions applies to Remotives.

A *Discretive Proposition* —union of two affirmative propositions by an adversative conjunction.

(*c*) *Discretive Propositions*, where two affirmative propositions are connected by an adversative conjunction—*but, nevertheless, although*, etc. Here some opposition is implied between the propositions joined, which are not expected to be true together. Thus 'He is poor but honest' would imply that most poor people are not honest. The proposition may evidently be contradicted by denying either the poverty or the honesty, a sure proof of its compound

character; for every simple proposition admits of but one contradictory. Book II. Ch. I.

(ii.) **Exponible Propositions,** *i.e.*, those whose composition is not obvious from their form, and which, therefore, require explanation to show what this hidden composition really is. For example, all Numerically Definite Propositions [§ 73 (i.)] and Plurative Propositions [§ 73 (iv.)] if strictly interpreted would be Exponibles, for each implies both an affirmative and a negative proposition; but, as was said above, such strict interpretation is logically incorrect in the latter case, and by no means universal in the former. Any proposition, however simple it may at first sight appear, which can be contradicted in more than one way, is really compound, and falls under this head.

An *Exponible Proposition* is a compound proposition whose composition is not obvious.

All propositions which can be contradicted in more than one way are compound.

Exponible Propositions may be classed as :—

(*a*) *Exclusive Propositions.* These contain a word such as *alone*, which limits the subject, as 'Graduates alone are eligible.' This is equivalent to the two propositions 'Graduates are eligible' and 'No non-graduate is eligible,' and it can be contradicted either by denying the eligibility of graduates or by affirming that of others. It must be noted that such an exclusive form distributes the predicate but not the subject. It can, therefore, be expressed by the **A** proposition 'All eligible persons are graduates,' but this is really an Immediate Inference from the original proposition [*see* § 102 (ii.)]. If we would refrain from inverting subject and predicate we must use both the propositions *Some S is P*, *No non-S is P* to express the Exclusive *S alone is P*.

An *Exclusive Proposition* —*S alone is P*

(*b*) *Exceptive Propositions.* These exclude a portion of the denotation of the subject-term from the predication by some such word as *except*, *unless*, as *Every S except MS is P*, or *Every S is P unless it is M*. If the exception is purely indefinite the proposition is particular. Thus 'Every man except one assented' does not permit us to assert of any individual whether he assented or not, for the one dissentient is unknown. We can, therefore, only say 'Some (=all but one) assented' and 'Some (=an unknown one) did not assent.'

An *Exceptive Proposition* —*Every S is P unless it is M.*

If, however, the exception is clearly specified we can make a definite assertion about each individual, negative or positive according as he falls within or without the excepted part. Thus 'Every Frenchman is bound to perform military service unless he is physically incapacitated' enables us to say definitely 'No physically incapacitated Frenchman is so bound' and 'All other Frenchmen are so bound.' And the contradictory of either of these propositions denies the original statement.

Exclusives and Exceptives are interchangeable forms.

It must be noticed that Exclusives and Exceptives are only two somewhat different forms for expressing the same meaning. Either can, therefore, be changed into the other, the excepted part of the one becoming the exclusive subject of the other, or *vice versâ*, and the quality being changed. Thus 'The miser does no good except by dying' may be expressed exclusively as 'Of all the acts of the miser his death alone does good'; so the Exclusive 'Virtue alone can render a man truly happy' is the same as the Exceptive 'Nothing can render a man truly happy except virtue.'

Inceptive and *Desitive Propositions* state the beginning or end of something.

(*c*) *Inceptive and Desitive Propositions.* In these something is said to begin or to end. They are resolvable into two propositions, the first declaring the state of things before the change and the second the state after the change. Thus, 'After the Black Death there was a great dearth of labourers in England' implies (1) There was no such dearth just before the time mentioned, (2) There was such a dearth after it. Similarly, 'Ploughing by oxen has been discontinued in England for many years' implies (1) Ploughs were formerly drawn by oxen in England, (2) The practice has been discontinued for many years. As these propositions make two assertions relating to two different times they may be contradicted by a denial referring to either time. Thus, the last example may be contradicted by 'Ploughs were never drawn by oxen in England' or by 'The practice has not been discontinued for many years.'

HYPOTHETICAL PROPOSITIONS.

76. Nature of Hypothetical Propositions.

A Hypothetical Proposition asserts that the Consequent is grounded in the Antecedent.

A Hypothetical Proposition is one in which the predication made in one proposition is asserted as a consequence from that expressed by another. The proposition containing the condition is called the **Antecedent** or **Protasis**, and is introduced by some such word as *If;* that containing the result is termed the **Consequent** or **Apodosis**. For example, in the sentence 'If all prophets spoke the truth, some would be believed,' the Antecedent is 'If all prophets spoke the truth,' and the Consequent is 'some (prophets) would be believed.'

The most general symbolic form is If A then C, but the most expressive is If S is M it is P.

The most general symbolic expression of the hypothetical proposition is *If* **A** *then* **C**, where **A** and **C** represent not terms but propositions. Other forms frequently given may be included under this general expression. Of these one of the most common is *If A is B, C is D*, but the one which most truly represents the nature of the judgment expressed by the proposition is *If S is M it is P*, where both the protasis and the apodosis have the same subject. This form indicates that the essence of the judgment is the explicit assertion that the ground of the attribution of *P* to *S* is found in the fact that *S is M*.

Propositions containing four terms can generally be reduced to this form,

When the proposition contains four terms, and, therefore, falls at once under the form *If A is B, C is D*, analysis of the meaning frequently shows that this is a mere accident of expression, and that the real subject of thought is the same in both antecedent and consequent, so that the judgment may be equally well expressed by a proposition of the form *If S is M it is P.* Such reduction from one form to another is always allowable when it does not affect the meaning of the proposition, *i.e.* the real judgment. For example, the judgment involved in the proposition 'If the government of a country is good, the people are happy' finds perfect expression in 'If the people of a country are well governed, they are happy.' So, 'If the barometer falls, we shall have rain' is reducible to 'If the state of the atmosphere causes a fall

of the barometer, that atmospheric state will bring rain.' 'If we ascend a mountain, the barometer falls' is equivalent to 'If a barometer is taken up a mountain, it falls.' 'If a child is spoilt, its parents suffer' may be resolved into 'If a child is spoilt, it brings suffering on its parents.' 'If you take a large dose of arsenic you will be killed' is expressed by 'If arsenic in undue quantity is taken into an animal organism, it causes death in that organism.' 'If patience is a virtue, some virtue may be painful' is the same as 'If virtue includes patience, then virtue may be painful.'

and only by such reduction can the essential unity of the judgment be made explicit.

In other cases the reduction is not so easy, and in order that it may be effected links have to be supplied which are not explicitly stated in the original proposition. In other words, a hypothetical expressed with four terms conceals the essential unity of the judgment it expresses, as there is in the symbolic statement no obvious point of union between the antecedent and the consequent. But the union is always there in thought when the proposition is expressed—as all real judgments always are—in significant words and not in mere empty symbols, and is generally found without difficulty. For example the point of unity involved in the judgment 'If some agreement is not speedily arrived at between employers and workmen, the trade of the country will be ruined' is the recognition of the injurious effect of strikes on trade, and the whole judgment may be expressed 'If trade continue to be injured by strikes, it will soon be ruined.' Sometimes the union is found in the recognition that the subject of the apodosis is a species under the wider subject of the protasis, as in 'If demagogues are mischievous, this stump orator is mischievous,' 'If violent emotion is followed by a reaction, your fit of anger will lead to a reaction'; 'If all savages are cruel, the Patagonians are cruel.' In other cases, both are recognized as species under the same genus, as in 'If virtue is voluntary, vice is voluntary.' But, in every case, where the judgment is really hypothetical—*i.e.* asserts the consequences of a supposition—such unity is present. No doubt, the hypothetical form of proposition is occasionally used when no such judgment is really involved,

as when Mr. Grimwig in *Oliver Twist* said, "If ever that boy returns to this house, sir, I'll eat my head"; which was only a forcible mode of asserting disbelief in the realization of the supposition stated in the protasis; and was, therefore, in its essence, categorical. Such propositions are obviously of but small value in a theory of knowledge.

The Hypothetical Judgment makes explicit the ground of connexion of content implied in the Generic Judgment,

In discussing the universal categorical proposition we found that its justification must be sought in a relation of content which is most appropriately expressed in the Generic Judgment, *S is P*, and that this form of judgment has both an abstract and a concrete reference [*see* § 71 (i.) (*b*)]. If such a judgment is true, it is because there is something in the nature of *S* of which *P* is the necessary consequence. If we make this explicit we have the hypothetical judgment *If S is M it is P*, where the sufficient ground for *P* is found in *M*. Such a relation is nearly as explicitly stated in a Generic Judgment of the form *S which is M is P*, a fact which shows that the categorical and hypothetical forms are not separate and distinct species of judgment, but merge into each other, and are distinguished chiefly by the highly abstract character of the latter [*cf.* § 74 (ii.)]. For in the hypothetical judgment we have got away from the concrete; our proposition is an abstract universal, and deals with only one element in a complex whole. The judgment, if true, is necessarily and universally true, and yet may be incapable of concrete realization. This, indeed, is so with geometrical judgments, such as 'If a triangle is right-angled, it is inscribable in a semi-circle,' for no concrete diagram is ever a perfect right-angled triangle or a perfect semi-circle. Still more clearly, perhaps, is this seen in such a judgment as 'If a body is given a certain movement, and if no counteracting conditions are operative, it will continue for ever to move in the same direction and with the same velocity.' This is impossible of realization in sensuous experience, and yet is a fundamental law of physics; that is, a necessary element in our mental construction of the material world.

and is an abstract universal.

A Hypothetical Judgment may be true though its realization is impossible.

It is evident from what has been said that the hypothetical judgment is essentially abstract, and, as such, states con-

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Many Hypothetical Judgments justify Enumerative Conditional Judgments,

nexion of content. But as the generic judgment finds an enumerative or denotative expression in the universal categorical proposition, so many hypothetical judgments can be represented by what we may, perhaps, call the concrete conditional proposition, whose general symbolic expression is *If any S is M that S is P*, or *Whenever an S is M that S is P.* The latter statement is to be preferred, as the use of *If* in the former suggests an abstract connexion of content, rather than that simultaneity of occurrence in experience, which is what the denotative form explicitly asserts. The denotative form, then, has a distinct reference to occurrence in time and space; it expresses connexion of phenomena, and is, therefore, only appropriate when such occurrence is possible. In other words, it contains a distinctly categorical element, and is practically equivalent to the proposition *Every S which is M is P.* In form, like the proposition *Every S is P*, it suggests that its basis is enumeration of instances, but its real justification is connexion of content expressed by the pure abstract hypothetical, and found to be realized in phenomena.

which as asserting connexion of phenomena contain a categorical element.

77. Relation of Hypothetical to Categorical Propositions.

The transition from the categorical to the hypothetical is gradual.

We have seen that as judgment becomes less occupied with concrete and complex phenomena regarded as wholes, and concerns itself more and more with abstract relations of content, it gradually passes from the categorical to the hypothetical form (*see* § 76). But the fact that the generic judgment mediates this transition, whilst the denotative conditional form mediates a transition in the opposite direction, shows that no strict line of demarcation can be drawn between them as modes of thought. With their form as propositions, the case is, of course, different; here language makes fixed and definite a distinction which is far from being so fixed in thought. Sometimes, it is an accident whether a judgment is expressed in the hypothetical or the categorical form; for instance 'Right angled triangles have the square on the hypoteneuse equal to the sum of the squares on the

sides' really gives the ground for attributing the predicate to the subject and would appropriately take the hypothetical form 'If a triangle is right-angled, the square on the hypoteneuse is equal to the sum of the squares on the sides.' But, in all cases, it should be considered whether the categorical or the hypothetical form is the more appropriate, and this depends upon the degree of abstraction involved in the judgment.

The two forms cannot be reduced to each other, as each makes prominent an element only latent in the other.

The question whether the categorical and hypothetical forms can be reduced to each other without change of meaning has been much disputed. From what has been said above it is evident that though the one essential nature of judgment pervades both, yet that each emphasizes just that aspect which is only implicit—and often but vaguely so—in the other. Thus, the categorical emphasizes reference to concrete reality existing in space and time; whilst the hypothetical brings into prominence the element of relation of content which is the implicit justification of the categorical. The two forms cannot, therefore, be regarded as interchangeable. The element of supposal which is prominent in the hypothetical disappears if the judgment is written in the categorical form; and on the other hand it is introduced *ab extra* when a categorical proposition is translated into the hypothetical form. In all cases where the categorical is abstract, such translation is, no doubt possible; but in many cases, especially when the proposition is a definition, it is inappropriate. We *can* say 'If gold is, it is yellow,' or 'If a lion is, it is carnivorous,' but the form suggests the possibility of the non-existence of the subject, and is, consequently, not an adequate expression of the judgment really made. At the same time it must be granted that it is not always easy to say which kind of proposition will most appropriately express a given judgment, for most human knowledge is neither entirely in the realm of concrete facts in all their particularity, nor in that of pure abstract relation, which exists only in idea. There are both categorical and hypothetical elements in most of the judgments men actually make, as is, indeed, shown by the frequency with which the

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generic form of judgment—in which we have, side by side, both an abstract universal and a concrete character—is adopted.

78. Quality and Quantity of Hypothetical Propositions.

Hypothetical Propositions may be negative.

(i). **Quality.** Hypothetical Propositions admit of distinctions of quality. Of course, a negative antecedent does not make a hypothetical proposition negative; for the consequent is still asserted to follow as a result of the antecedent. Thus *If S is not M it is P* is affirmative—'If a swan is not white, it is black.' It is when the connexion of the apodosis with the protasis is denied that the proposition is negative.

The most general form is—*If* A *then not* C.

The most general symbolic form is *If* A *then not* C, and the most expressive *If S is M it is not P*, *e.g.* 'If a man is honest he will not deceive his fellows.'

True hypotheticals are universal,

(ii). **Quantity.** The essence of true hypothetical judgments is their abstract, and necessarily universal character. But cases may arise in which though a connexion is established between *P* and *M*, yet *M* may not be the full ground of *P*, or, though it is the complete ground, may not be universally operative, or may be liable to be counteracted by other conditions. In such cases the appropriate proposition takes the general form *If S is M it may be P*, and negatively *If S is M it need not be P*, which are more explicit expressions of the *Modal Particulars* than were considered in an earlier section [*see* § 71 (ii.)]. The corresponding denotative forms —which in these cases can always be found—are *Sometimes when an S is M, it is* [or *is not*] *P*. As examples we may take 'Sometimes when men are much worried, they commit suicide;' 'If a man is punished for a crime, he, perhaps, will not transgress again'; 'Sometimes when a target is aimed at, it is not hit'; 'Although a man tries his hardest, he may not succeed.' The last three examples are particular negative, the first particular affirmative. The 'Sometimes' in the denotative examples, it must be remembered, is as purely indefinite as is 'Some' in a particular categorical [*see* § 71 (ii.)]; it must not, therefore, be regarded as excluding 'always.'

but the explicit *Modal Particular*—*If S is M it may be P*—has a hypothetical form.

All Modal Particulars have denotative forms.

Particular Propositions represent imperfect judgments.

The great characteristic of all particular propositions is their imperfect and incomplete character. They, on their very face, therefore, challenge completion. They are but stepping stones on the way to that more exact and complete knowledge which finds expression in the true and universal hypothetical.

DISJUNCTIVE PROPOSITIONS.

79. Nature of Disjunctive Propositions.

A Disjunctive Proposition makes an alternative predication —*S is either P or Q*.

A Disjunctive Proposition is one which makes an alternative predication. The most general symbolic form is *Either* **X** *or* **Y**, where **X** and **Y** represent propositions. But as the most expressive form of the hypothetical is that which makes explicit the unity of the judgment, so here the symbolic form *S is either P or Q* representing the prescription to the same subject of an alternative between a definite number of predicates, most truly expresses the nature of the judgment. In the simplest cases these alternative predicates are known to be contained under a wider predicate *M* which can be asserted of *S* (*cf.* § 68). For example 'He is either a doctor, a lawyer, a clergyman, or a teacher' may be expressed in the simple categorical proposition 'He is a member of a learned profession.' So, we may say 'Any swan is white or black' where the wider predicate is the possession of colour. But, though such subsumption is always theoretically possible, in most cases the alternative predicates have never been brought under such a wider predicate; for occasion has not arisen to make such a wide and indefinite assertion about any subject. For instance, we may say 'The election will turn either on the Eight-Hours Question or on the Question of Home Rule,' but we have no word which would exactly cover these two cases, and yet be sufficiently significant to express our meaning if we affirmed it as a predicate of the given subject.

These alternatives may always be theoretically brought under a wider predicate, but this is seldom done in practice.

An alternative increases the indefiniteness of the scope of the predicate.

The effect of an alternative predicate is to increase the indefiniteness of its extent. As all disjunctive propositions are affirmative

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Propositions with alternative subjects are not classed as Disjunctives.

(*see* § 81) the predicate is undistributed, and, therefore, indefinite up to the extent of P (*cf.* § 72); by adding *or* Q this indefiniteness of range is increased, but no fundamental difference is made in the character of the proposition. If the alternation was in the subject —as, if we should say S *or* X *is* P, or S *or* X *is* P *or* Q—then we should change a determinate subject into an indeterminate one, if the original subject, S, was distributed. Such propositions, however, are not usually called Disjunctive.

Logicians differ as to whether or not the disjunctive form necessitates the mutual exclusiveness of the alternative predicates.

In many cases the alternatives are, in fact, exclusive,

The important question as to the interpretation of Disjunctives is whether the alternative form necessitates that the several predicates conjoined in it be mutually exclusive in their application. When it is said S *is* P *or* Q, is it necessarily implied that S cannot be *both* P *and* Q? On this point there has been great difference of opinion amongst logicians. It is granted by all that in a great number, perhaps in the great majority, of cases, the alternatives do, as a matter of fact, exclude each other. Such are: 'He will either pass or fail'; 'This book is to be bound either in calf or in morocco'; 'The rebellion will either succeed or be crushed'; 'Any swan is either white or black'; 'These plays were written either by Shakespeare or by Bacon.' In all these, and in many similar cases, the acceptance of one alternative involves the denial of the other, and it is argued that whenever this does not appear to be the case, or is not meant to be the case, it is because of "our slovenly habits of expression and thought" and that these are "no real evidence against the exclusive character of "disjunction" (Bradley, *Prin. of Log.*, p. 124). But, when such instances are examined more closely there are found to be two possible explanations of this exclusiveness :—(*a*) the terms are, in each case, mutually incompatible in their very nature, so that both cannot possibly be affirmed in the same sense of the same subject; the exclusiveness may be due to this, or (*b*) it may be a necessary consequence of the disjunctive form itself. As the former explanation is manifestly sufficient in itself, such examples will not prove whether or not the disjunctive form, *as a form*, necessarily involves exclusion. To settle this—which is the point really

but this is due to their natural incompatibility.

in dispute—we must examine some examples in which terms are disjunctively predicated which are not incompatible with each other in their very nature. If we still find that the alternative predicates are mutually exclusive we must regard this as due to the disjunctive form alone. Let us take such propositions as 'All candidates must be graduates either of Cambridge, of Oxford, of Dublin, or of London'; 'He is either very timid or very modest'; 'He is either a knave or a fool.' Do we mean, in either case, to deny the possibility of all the predicates being, at the same time, attributes of the subject? May not a candidate be a graduate of more than one of the universities mentioned? or would such plural honours be a bar to his success? Do we deny that the person in question can be both timid and modest? Do we exclude the possibility that the other is both a knave and a fool? In fact, in one sense, may not every knave be said to be necessarily more or less of a fool, in that honesty is the highest wisdom? In each of these examples it seems certain that no exclusion of one predicate by another is involved. If, then, cases can be found—and, of course, many more might be instanced—in which the alternative predicates are not mutually exclusive, it follows that when such exclusiveness does exist it is due to the character of the alternatives themselves and not to the disjunctive form of the proposition in which they happen to occur. That form, as a form, implies no such mutual exclusiveness. If we wish to show formally that our alternatives are intended to be exclusive, we can do so by writing *S is P or Q, but not both*, which is, of course, a compound proposition [*see* § 75 (i.) (*c*)]. By adopting this non-exclusive view of disjunctives, we are, besides, obeying the valuable logical Law of Parsimony—that whenever a choice is offered us between a more and a less determinate meaning, it is safer to choose the latter; for we thus avoid the danger of implying in, or inferring from, any statement more than is justified.

When the alternatives are not incompatible they are not exclusive.

Exclusion is not, therefore, due to the disjunctive form of proposition.

In so far as alternatives are different, they are exclusive.

But though the form does not imply mutual exclusiveness yet there is necessarily some element of difference—that is, of exclusiveness—in all alternatives, as otherwise they could not be alternatives

at all, for they would be identical. In so far as two terms, denoting different species of the same genus, differ from each other they exclude each other. For instance, the being a graduate of Cambridge is not the same as being a graduate of London—the two agree in the fact of graduation, but differ in the place where the graduation occurred. As these places are different, they exclude each other; that is, *one* act of graduation could not take place both at Cambridge and at London. Similarly, the qualities of timidity and modesty differ from each other in some points, and in these points of difference are mutually exclusive, or the two ideas would merge into one. In the same way, the points in which knavery differs from foolishness are points of exclusion. So it is always; in so far as notions differ, they exclude each other; were it not so they would melt into one, for the mentally indistinguishable is a mental unity. It follows that the logical ideal of a disjunctive judgment is one in which the alternative predicates are exhaustive of the denotation of the subject, exclusive of each other, and co-ordinate species under the subject genus. But our treatment must cover cases in which neither in thought nor in language is this ideal realized, and our formal interpretation, therefore, of a disjunctive proposition must be that the alternatives are not necessarily exclusive.

Summary. We reach, then, this result: the alternatives in every disjunctive proposition have something in common; for they are always capable of being subsumed under some wider predicate of the same subject: as species of this genus they are sometimes, in their very nature, incompatible with each other, and are, therefore, exclusive: but, in other cases they are, in their nature, compatible with each other, and are then not exclusive, though they can never be identical: hence the degree of exclusiveness depends on the nature of the alternatives themselves and not on the disjunctive form of proposition; in other words, it is material, not formal.

80. Relation of Disjunctive to Hypothetical and Categorical Propositions.

As every disjunctive proposition prescribes an alternative between a definite number of different predicates, one or

more of which is, therefore, affirmed of the subject, it follows that, if some of these alternatives are denied, the others are affirmed ; either categorically—if only one is left, or disjunctively—if more than one remains. Thus, if we start with the assertion *S is either P or Q*, and then deny that *S is P*, we must necessarily proceed to affirm that *S is Q*. Similarly, if the original assertion is *S is P or Q or R or T*, and this is followed by the denial that *S is either P or Q*, the affirmation that *S is either R or T* is a necessary result. As this affirmation of one, or more, of the alternatives is an inference from the denial of the rest of them, it follows that all Disjunctive Propositions involve a hypothetical judgment of the general form *If S is not P it is Q*, or, *If S is not Q it is P*. These propositions are exactly equivalent to each other, each being, in fact, the Obverted Contrapositive of the other (*see* § 105). But the Disjunctive Judgment makes explicit a categorical element which is wanting to the hypothetical. Were we confined to the latter, thought would be condemned to an endless regress. For though *If S is M it is P*, gives us in *M* the ground of *P*, yet we must go on to similarly ask for the ground of *M*. This regress can only be avoided by assuming that the judgment refers to a more or less self-contained system. It is such a system that the disjunctive judgment in its ideal form makes explicit in its enumeration of the sub-species under the subject genus. It is in the exhaustive character of this enumeration that the sufficiency of the hypothetical as a statement of a condition is found. Hence, we find in the disjunctive the mode of expressing that systematic connexion which is the only form in which we can think reality.

As the affirmation of one alternative in a disjunctive follows from a denial of all the other alternatives, every such proposition involves a hypothetical of the form *If S is not P it is Q.*

Those logicians who adopt the exclusive view of the disjunctive form deny that its full meaning can be expressed in any *one* hypothetical proposition. For, if *P* and *Q* are mutually exclusive, it follows from *S is either P or Q* not only that *If S is not P it is Q* and *If S is not Q it is P*, but also that *If S is P it is not Q*, and *If S is Q it is not P*, and one of these latter forms is required together with one of the former to express the force of the disjunctive, which would then be given either by the pair *If S is not P*

If the alternatives were exclusive, a disjunctive could only be expressed by: *If S is not P it is Q*; and *If S is P it is not Q.*

it is Q, and *If S is P it is not Q*; or by the pair *If S is not Q it is P*, and *If S is Q it is not P*. This extra implication which such an interpretation gives to a disjunctive brings out the force of the remark made near the end of the discussion of exclusiveness, in the last section, on the logical Law of Parsimony. The exclusive view evidently binds us to a greater number of assertions than the non-exclusive view. As we have adopted the latter we cannot, of course, regard the inferentials *If S is P it is not Q* and *If S is Q it is not P* as involved in, or as legitimate expressions of any part of the meaning of, the formal disjunctive *S is either P or Q*.

81. Quality and Quantity of Disjunctive Propositions.

All Disjunctives are Affirmative.

(i.) Quality. It follows from the very nature of Disjunctive Propositions (*see* § 79) that they can only be affirmative; for they must give a choice of predicates, one or other of which must be affirmed of the subject. Propositions of the form *S is neither P nor Q* give no such choice, nor do they increase the scope of the predicate as do propositions of the form *S is either P or Q* (*see* § 79). They are essentially Compound Categorical Propositions [*see* § 75 (i.) (*b*)] It is true we can have a disjunctive proposition involving negative terms—as *S is either P or non-Q*—but the disjunction is as affirmative as if both terms were positive.

(ii.) Quantity. The ideal disjunctive judgment is always both abstract and universal, and expresses relation of content. But like the Generic Judgment it can be expressed in terms of denotation, and in this case we get distinctions of quantity. Thus we get propositions of the form *Every S is either P or Q*; 'Every idle man is either incapable of work or morally blameworthy,' and *Some S's are either P or Q*; 'Some laws are either oppressive or are rendered necessary by an abnormal state of society.' It is evident, however, that such particular propositions are of practically no logical value.

82. Modality of Propositions.

Modality has reference to the degree of certainty, or incertainty, of a judgment, and is concerned with the various ways in which differences in this respect are expressed. De Morgan defines a **Modal**

Proposition as "**one in which the affirmation or negation was "expressed as more or less probable.**" (*Formal Logic*, p. 232.) Some of the scholastic writers on Logic regarded all adverbial modifications of a proposition as a kind of modality. They distinguished, therefore, between *Material Modality*—where the modification belonged either to the subject or to the predicate of the proposition; as, He spoke *angrily;* and *Formal Modality*—where the modification affected the certainty of the relation asserted to exist between the subject and the predicate. This latter, however, is the only kind of modification to which the term Modality is rightly applicable.

Modality is concerned with the degree of certainty, or uncertainty of a judgment.

Extreme Conceptualists [*see* § 8 (ii.) (*b*)]—as Mansel and Hamilton—refuse to admit the discussion of this subject into Logic at all, as it is essentially concerned with the matter of the judgment and not with its mere form. But on the wider view of the science here adopted (*see* § 9) it is necessary to examine it, and to estimate its logical value.

Aristotle divided Modal propositions into:
1. Necessary.
2. Contingent.
3. Possible.
4. Impossible.

Aristotle divided Modal Propositions into four classes—the *Necessary*, the *Contingent*, the *Possible*, and the *Impossible*. This is, evidently, purely objective; the subjective aspect of the mind towards the Necessary and the Impossible is identical—both are cases of full assurance. It is also difficult to see how the amount of belief in a contingent proposition differs from that in a possible one; this latter distinction is, in fact, extremely vague. Only by a reference to the things themselves can it be decided whether the subject S must necessarily possess the predicate P, whether the two are absolutely incompatible, or whether their union is more or less likely. Some scholastic writers, indeed, reduced the forms of Modality to two—the Certain and the Possible. Others, influenced by the analogy with the fourfold scheme of propositions (*cf.* § 72), retained the original four, and connected the Necessary with **A**, the Impossible with **E**, the Contingent with **I**, and the Possible with **O** propositions. The distinction they drew between the two latter modes was that the Contingent was 'what is, but may not be in the future,' and the Possible 'what is not, but may be in the future.' Such a division of Modals, founded as it was upon a purely objective view of the province of Logic, and utterly artificial as it is, has little to recommend it. As Dr. Venn says: "A very slight study "of nature and consequent appreciation of inductive evidence suffice "to convince us that those uniformities upon which all connexions

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"of phenomena, whether called necessary or contingent, depend, "demand extremely profound and extensive enquiry; that they "admit of no such simple division into clearly marked groups; "and that, therefore, the pure logician had better not meddle "with them" (*Logic of Chance*, p. 307).

Kant divided Modal Judgments into: 1. Apodeictic. 2. Assertory. 3. Problematic.

Kant regarded Modality from a standpoint essentially different from that of Aristotle. His view was purely subjective; he considered simply the amount of our belief in a judgment. As he distinguished three degrees of assurance, so he divided Modal Propositions into three classes—the Apodeictic—*S must be P;* the *Assertory—S is P;* the Problematic—*S may be P* (*cf.* § 48). "The "apodeictic judgment is one which we not only accept, but which "we find ourselves unable to reverse in thought; the assertory is "simply accepted; the problematic is one about which we feel in "doubt" (Venn, *Logic of Chance*, p. 310). If we consider this distinction simply from the standpoint of formal logic, we cannot accept it. The apodeictic judgment differs from the assertory only in the emphasis with which it expresses universal connexion. It is, therefore, formally nothing more than an assertory judgment—it only asserts more vigorously. Both judgments claim to be true, and both express complete belief. For if the belief in the assertory judgment were not as strong as in the apodeictic, the former would contain an element of doubt, and would be merely problematic.

The distinctions do not hold from the formal point of view,

But it does not follow that the idea underlying the doctrine of modality is a useless or even an unimportant one. From the point of view of knowledge there is undoubtedly a distinction between truth which is regarded as necessary, and whose overthrow would affect the whole of our mental construction of the world, and propositions which may be accepted as true, but whose overthrow would originate no such mental chaos. And this distinction we find between judgments of connexion of content and judgments based on mere experience. The former—the Generic and Hypothetical Judgments—express the only interpretation we can give of certain aspects of reality, or certain elements of our experience, which is consistent with our conception of the universe as a whole. Such judgments are in the fullest sense, universal, and as universal, they are necessary. But judgments of uncontradicted experience have, as has been already pointed out, no such necessary character (*cf.* § 71). Hence, all Generic and Hypothetical Judgments are apodeictic. Again we have seen that the true ground of the particular proposition is found in the imperfect determination of the con-

but they are important from the standpoint of knowledge.

Generic and Hypothetical Judgments are apodeictic;

tent of S, and that this is most clearly expressed in the Modal Particulars *S may be P*, and *S need not be P*, or in the more explicit forms *If S is M it may be P* and *If S is M it need not be P* which hold the same relation to the universal hypothetical that the modal categorical particulars do to the generic judgment. Hence all particular judgments are in their essence problematic; all truly universal propositions—*i.e.*, hypothetical and generic judgments—are apodeictic, and all propositions based on mere uncontradicted experience are assertory. The whole subject will be clearer after Induction has been discussed, and we shall then return to it (*see* § 160).

Particular Judgments are problematic;

Enumerative Judgments of experience are assertory.

CHAPTER II.

IMPORT OF CATEGORICAL PROPOSITIONS.

83. Predication.

The question of the import of propositions involves: (a) What is related? (b) What is the relation? (c) Is existence implied?

The whole treatment of Logic must depend upon the view held as to the nature of the predication made in a categorical proposition, and the consequent import of that proposition [*cf.* §§ 8 (ii.), 66 *ad fin.*]. The first point to be settled in considering this is whether such a proposition expresses a relation between words only, or between ideas, or between things. The different answers which have been given to this question were stated in § 8 (ii.); and the view here adopted was set forth in § 9—viz., that a proposition interprets an objective fact, by stating a relation which is apprehended in thought and expressed by language. It is not necessary to say more on this fundamental point; we may pass on to consider two other questions:—

(*a*) The nature of the relation expressed in predication, and, as a consequence of this, the aspect in which the terms should be regarded.

(*b*) Whether or not a categorical proposition implies the existence of objects denoted by the terms.

On the first of these subjects several different views have been held, which will be discussed in the next five sections; the question of existence will then be considered in § 89.

84. The Predicative View.

The predicative view regards the relation expressed between the terms of a formal categorical proposition as that between subject and attribute. It makes the element of denotation in the subject, and that of connotation in the predicate, the more prominent. The subject is thought as the name of certain objects, and though it is true they are indicated indirectly—that is, as members of a class to which they belong solely because they possess certain attributes [*cf.* § 71 (i) (*b*)]—yet the attention is fixed on them as things, not on the attributes which their names connote. It is of the *things* which possess the attributes that the assertion is made; the attributes themselves are not definitely before the mind at all, but are merely symbolized by the name. Hence it is that a word which is primarily attributive—such as an adjective—cannot form the subject of a proposition (*cf.* § 25). But in the case of the predicate we are thinking of the attributes which we affirm of the objects denoted by the Subject; for our whole purpose is to predicate such a qualification. We think, not of two sets of objects which we compare, but of one set of which we assert an attribute. This is most obvious when the predicate is a directly attributive word, as when we say 'All metals are fusible,' 'The dog is barking'; but it is equally true when the predicate is a substantive. For instance, in the proposition 'All the candidates for the appointment are graduates,' if we examine the meaning we shall find it to be that the candidates in question possess certain qualifications which are conveniently summed up in the word 'graduates'; we do not think of graduates as individuals, but predicate the connotation of the name. And the same holds in every case; the predicate asserts a qualification of the subject, and this qualification consists of the attributes implied by the predicate.

The *Predicative View* regards the relation between the terms as that between subject and attribute. The *S* is, therefore, read in denotation and the *P* in connotation.

This is the natural interpretation of a categorical proposition whose subject is expressed with a sign of quantity, though it must be borne in mind that its foundation is to be found in the Generic Judgment whose essence is that it deals with content of both subject and predicate. It is also

This interpretation gives rise to the four-fold scheme of propositions.

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fully consistent with the four-fold scheme of propositions; for, if the predicate names an attribute and the subject indicates certain objects, we must either affirm or deny the former of the latter, and in each case the assertion must be made either of a definite or of an indefinite number of individuals (*cf.* § 72).

*85. The Class-inclusion View.

The class view is that the *S* is included in the class denoted by the *P*. Both terms are said to be read in denotation.

On the class view the relation between the subject and predicate is that of inclusion in a class. Both terms are said to be read in denotation, and the proposition is held to assert that the objects denoted by the subject are to be found amongst those denoted by the predicate. Whether the subject is used collectively or distributively [*see* § 27 (ii.)] is of no importance; in each case it forms part of the predicate. The predicate, however, is necessarily regarded as a whole or class—*i.e.*, it is used collectively. 'All owls are birds' means that each owl—and, therefore, the whole class of owls regarded collectively—is to be found within the whole class of birds. This collective interpretation of the predicate is the only permissible one; for to take it distributively would give no real meaning at all;—each owl is certainly not *any* bird. The only possible meaning is that the total class composed of birds contains every individual which can be called an owl; or, what is exactly the same thing, that it contains the whole class of owls. Similarly, a negative proposition means that every individual denoted by the subject is excluded from the whole class of things denoted by the predicate, and that the two classes are, therefore, entirely separate.

This view bases knowledge upon enumeration of instances and neglects the unity of judgment.

With respect to this view it may be pointed out, first, that though it is, of course, possible to attend to the denotation of the predicate (*cf.* § 72), yet in judging it is more natural and common not to do so. No doubt, as every general term can be considered both in denotation and connotation, it is possible so to interpret propositions, and such a mode of interpretation has been common amongst purely formal logicians, for it lends itself readily to the exposition of the

formal aspect of reasoning. But to adopt this interpretation as the fundamental import of judgment is to fall into the error of basing knowledge upon a supposititious possibility of a complete enumeration of instances, instead of upon an investigation directed to establish connexion of content. Moreover, this view of predication neglects the essential unity of the judgment and regards it as stating a relation between two independent objects rather than as expressing an interpretation of one element or aspect of reality.

A class interpretation of both *S* and *P* leads to a five-fold scheme of propositions.

Further, it must be pointed out that if both subject and predicate are regarded as classes—and, as was said above, on this view, the subject *may*, and the predicate *must*, be always so regarded—then the four-fold scheme of propositions is not an exhaustive statement of the relations which may exist between them. We require a five-fold scheme; for if we have two classes, *S* and *P*, it is evident :—

(*a*) They may exactly coincide and so be identical wholes.
(*b*) *S* may be included in but not form the whole of *P*.
(*c*) *S* may include *P* and not be wholly exhausted.
(*d*) *S* and *P* may partially include and partially exclude each other.
(*e*) *S* and *P* may wholly exclude each other.

To express these in ordinary language we must give 'some' the meaning 'some but not all' [*cf.* § 71 (ii.)]. We then have :—

(*a*) All *S* is all *P*.
(*b*) All *S* is some *P*.
(*c*) Some *S* is all *P*.
(*d*) Some *S* is some *P*.
(*e*) No *S* is any *P*.

This scheme expresses relations of fact, not our knowledge of them.

But, it should be remembered that we have here a statement of the actual relations which must hold, in fact, between two classes, not of our knowledge of those relations. This scheme, therefore, furnishes us with no means

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and makes every proposition singular.

of expressing the very common state of doubt, when we know that every S is P, but do not know whether or not any other objects are P as well. Moreover, each of the above propositions is Singular, as each term is necessarily taken collectively. For both these reasons the scheme is inappropriate to the purposes of logic, and any interpretation of the proposition which, when strictly carried out, leads to it is thereby condemned.

*86. Quantification of the Predicate.

Hamilton held that a proposition expresses an equation; that the predicate is always quantified in thought; and that this quantification should be expressed.

As the application of S and P is identical, it follows that, if both the terms of a proposition are read in denotation, the relation between them is reduced to an equation; and it would seem to follow, on this view, that it is necessary to quantify the predicate. Sir W. Hamilton held this to be the true relation between the terms of a proposition, and the only way in which a judgment is really thought. He, therefore, assumed that the predicate is always quantified in thought, and hence—as a consequence of his fundamental postulate, "To state explicitly what is thought implicitly" (*see* § 21)—that it should be always quantified, on demand, in expression. He then took the recognized four forms of proposition **A**, **I**, **E**, **O**, and by making the predicate of each (1) universal and (2) particular he obtained an eight-fold scheme. Thus:—

From the four forms of proposition he obtained eight.

From **A**	*All S is all P*	*afa*	**U**	$S\ u\ P$.
	All S is some P	*afi*	**A**	$S\ a\ P$.
From **I**	*Some S is all P*	*ifa*	**Y**	$S\ y\ P$.
	Some S is some P	*ifi*	**I**	$S\ i\ P$.
From **E**	*No S is any P*	*ana*	**E**	$S\ e\ P$.
	No S is some P	*ani*	η	$S\ \eta\ P$.
From **O**	*Some S is not any P*	*ina*	**O**	$S\ o\ P$.
	Some S is not some P	*ini*	ω	$S\ \omega\ P$.

The symbols *afa*, etc., were employed by Hamilton. In them *f* stands for the affirmative copula, *n* for the negative copula, those letters being the first consonants in the words *affirmo* and *nego*; *a* represents a distributed, and *i* an undis-

tributed term. Of course the subject term is always placed first. The symbols commonly employed, however, are **U**, **A**, **Y**, etc., which were introduced by Archbishop Thomson. Using these in the way adopted in § 72 we obtain the short symbolic expressions *S u P*, *S a P*, etc.

Hamilton urged that the predicate is often quantified, either directly or indirectly.

Hamilton supported his position that the predicate is thought as quantified by urging that it is often quantified in expression, either directly — as when we say 'Sunday, Monday, etc., are *all* the days of the week'—or, more frequently, indirectly, by the use of Exclusive and Exceptive forms of propositions. Thus, he says, 'Of animals man alone is rational' means 'Man is all rational animal,' and 'In man there is nothing great but mind' is equivalent to 'Mind is all humanly great,' that is 'Mind is all that is great in man.'

This scheme was said to simplify logical processes.

From this enlarged scheme of propositions great advantages were said to flow. Amongst the more important results claimed for it were :—that it made evident that the true relation between the subject and predicate of a proposition was an equational one ; that it reduced all forms of the conversion of propositions to simple conversion ; that it replaced all the general laws of syllogism by a single canon ; that it dispensed with Figure in the syllogism, and abrogated all the special laws of syllogism, and the necessity for Reduction ; that it increased the number of valid moods to thirty-six ; that it abolished the Fourth Figure of the syllogism, and made the order of the premises in the Second and Third Figures a matter of indifference, and consequently allowed two conclusions to each syllogism in those figures instead of one. In fact, the adoption of the quantified predicate was to revolutionize Logic. Not only has it not done this, but the whole scheme is now generally and deservedly discredited.

The doctrine is psychologically false,

To begin with, it may be urged against Hamilton's psychological argument that it is wrong to assert that we implicitly quantify the predicate in thought. The predicate is regarded as an attribute, and is not thought mainly in its denotation ; still less is it thought as embracing all or some of its denotation. It is equally wrong to say that the subject

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andis worthless as an analysis of judgments.

is thought collectively, or that a proposition expresses an identity of two groups taken as wholes, as this scheme requires. As the very foundation on which the scheme rests is, thus, unsound, it naturally follows that the scheme itself is worthless as an analysis of the forms of judgment. It may be noted that the supposed psychological foundation of the scheme was always assumed by Hamilton without the slightest attempt at proof.

Quantifying the predicate does not give an identical proposition.

In the next place, it follows from the discussion of the import of the particular proposition that a strictly *formal* statement of identity—that is, a logical equation—cannot be got from a mere quantified predicate, owing to the indefinite reference of 'some' [*cf.* § 71 (ii.)]. To get such an equation, we must *specify*, and not simply *quantify*, the predicate. For it must always be borne in mind that the predicate can only be read in denotation by taking it collectively, as one single group; and the very essence of every equational view of the proposition is that *each* term is thus understood. If, then, the equational doctrine is to be strictly adhered to, the simple conversion of each of these quantified forms, except **U** and **E**, involves the implicit reservation that the *Some S* or *Some P* denotes the same group in the converse proposition that it did in the original one; a limitation which the mere form of the proposition does not, of course, indicate. For instance, the proposition 'All man is some animal' would convert to 'Some animal is all men,' but this latter form is only true when we limit 'Some animal' in a way which the simple form of the proposition does not imply. This formal objection does not hold when we adopt the predicative view of the import of propositions, in which they are not regarded as equations.

Hence, simple conversion on this scheme, involves an implicit specification of our terms.

This leads us to enquire in what sense 'some' is used in this new scheme. On this point there is a great indecision amongst the supporters of the doctrine, and even in the writings of Hamilton himself. Three views have been put forward, and it is necessary, therefore, to see to what result each proposed interpretation of 'some' will lead us.

Three views are put forward as to the meaning of 'some'—

First. If 'some' means *some only*, then each affirmative proposition which contains 'some' implies a negative pro-

position, and *vice versâ*. Thus, from the proposition 'All man is some (only) animal,' it must be inferred that 'No man is some (other) animal,' and the former proposition is equally involved in the latter. "This sort of Inference "Hamilton would call *Integration*, as its effect is, after "determining one part, to reconstitute the whole by bring-"ing into view the remaining part" (Bowen, *Logic*, p. 170). Hence, the assertion of **A** involves that of η, and *vice versâ*, and so with **Y** and **O**. With regard to ω "Mr. Johnson "points out that if *some* means *some but not all*, we are led "to the paradoxical conclusion that ω is equivalent to **U**. "*Some but not all S is not some but not all P* informs us that "certain *S's* are not to be found amongst a certain portion of "the *P's* but that they are to be found amongst the remainder "of the *P's*, while the remaining *S's* are to be found amongst "the first set of *P's*. Hence *all S is P;* and it follows "similarly that *all P is S*. *Some but not all S is not some but* "*not all P* is therefore equivalent to *All S is all P*" (Keynes, *Form. Log.*, 2nd Ed., p. 299). This may be made clearer by the aid of symbols. Let $X\,S$ = *Some (only) S*, then $\overline{X}\,S$ = *the rest of S* ($\overline{X}$ = *non-X*). Similarly, if $V\,P$ = *some (only) P*, then $\overline{V}\,P$ = the rest of P. Now $X\,S$ is not $V\,P$ involves that $X\,S$ is $\overline{V}\,P$; for, if $X\,S$ is excluded from *only a part* of P, it must be included in the remaining part. And $X\,S$ is not $V\,P$ also implies that $\overline{X}\,S$ is $V\,P$; for to say that *only a part* of S is not found in the class $V\,P$ implies that the rest of S is found there. Similarly, $\overline{V}P$ is XS, and VP is $\overline{X}S$. Thus, $(X\,S+\overline{X}S)=(\overline{V}P+VP)$, that is *All S is all P.*

(1) If 'some means *some only*, the eight-fold scheme is redundant, for **A** and η **Y** and **O** are pairs whose members involve each other, and ω is equivalent to **U**.

We are thus reduced to the five forms of proposition—**U**, **A** (*or* η), **Y** (*or* **O**), **I**, **E**—expressive of the relations of quantity between subject and predicate which we gave in § 85 *ad fin.* Had Hamilton, indeed, started with an analysis of the possible relations of quantity between two classes, he would have seen that they can be only these five, and that an eight-fold scheme must, therefore, be redundant. And this redundancy makes it misleading. For every scheme of the forms of propositions professes to give nothing but simple, distinct, and irreducible forms; if, therefore, some of the forms are not distinct and not irreducible, the scheme suggests differences in predication

Thus, we get a five-fold scheme, which expresses all the possible quantitative relations between two classes.

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where none exist. Further objections to the scheme, grounded on the fact that some of the new forms of proposition are not really simple, will be noticed later on.

(2) If 'some' means *some at least*, we are expressing, not the actual relations between classes, but our knowledge of those relations.

Second. If 'some' means *some at least*, then, in addition to the objection that one of the terms of an equation cannot be vague in its application without vitiating the assertion of identity, it must be maintained that we still do not get an analysis of the relations of quantity possible between classes; for it has been seen that these are only five. But it may be urged we are here dealing, not with those real objective relations, but with our knowledge of them, which may be indeterminate. This certainly puts us on more logical ground (*cf.* § 85 *ad fin.*). But if the relation between the terms is merely one of quantity, and they are both classes regarded collectively, then it is evidently immaterial which we regard as subject and which as predicate. Still more is this so if the proposition really states an equation between the terms. But, in this case, **Y** and **A** are not independent forms; for both mean that one class forms an indefinite portion (which may, or may not, be all) of another; and we may write the distributed term as the subject, instead of as the predicate, of **Y**. Similarly η and **O** are really the same; for each excludes the whole of one class from an indeterminate portion of the other. The proposition ω ceases to be significant at all; for it neither denies nor decides anything. It denies nothing; for it is true together with any of the affirmative forms, none of which it contradicts. Even if we affirm *All S is all P*, yet it remains true that *this particular part of S is not that particular part of P.* For example, if we grant that 'All man is all rational animal,' yet we by no means deny that Englishmen (who are 'some man') are not Frenchmen (who are 'some rational animal'). It decides nothing; for it can always be asserted with reference to any two terms, except they happen to be both singular names, one of which belongs, and the other does not belong, to one definite individual; and in that case, of course, the 'some' is altogether out of place. We are, therefore, again reduced to five forms, **U**, **A**, **O**, **I**, **E**, though each of these is

On any class view of propositions the order of terms is immaterial;

hence, **Y** and **A** are really the same,

and so are η and **O**.

ω is devoid of signification.

We are again, then, reduced to a five-fold scheme.

no longer incompatible with each of the others—**U**, **A**, and **I**, may be true together, and so may **O** and **E**. This is, of course, no objection if the propositions are regarded as simply stating our knowledge of the quantitative relations between the terms, and not those relations themselves, but it utterly destroys the position that the proposition is an equation.

(3) If 'some' varies in meaning, we have a combination of the above objections.

Third. If 'some' means *some only* in propositions of the form **A**, **Y**, η and **O**, but *some at least* in these of the form **I** and ω, then we have a combination of the above objections. For **A** and η, **Y** and **O** still form pairs of propositions, the members of each of which are mutually inferrible from each other, and ω is still entirely without real predicative force. The using 'some' in two distinct senses in the same scheme of propositions leads also to the anomalous result that **I** is consistent with either **U**, **A**, or **Y**, but that each of these three is incompatible with both the other two.

In whatever way, then, we interpret 'some,' we find that an analysis of the forms of categorical proposition will not give an eight-fold scheme.

We will now examine the new forms—**U**, **Y**, η, ω—and enquire whether they are ever used in ordinary speech; and, if so, whether they are simple forms of proposition, such as should alone find a place in a logical analysis of elementary forms of predication. Dr. Keynes holds that **U** and **Y** are met with in ordinary discourse. He says: "It must be "admitted that these propositions are met with in ordinary "discourse. We may not indeed find propositions which "are actually written in the form *All S is all P;* but we "have to all intents and purposes **U**, wherever there is an "unmistakeable affirmation that the subject and the pre-"dicate of a proposition are co-extensive. Thus, all defini-"tions are practically **U** propositions; so are all affirmative "propositions of which both the subject and the predicate "are singular terms" (*Formal Logic*, 3rd Ed., p. 176). This is true, in that the denotation of the predicate in a definition is undoubtedly identical with that of the subject. But the main object of definition is not to determine the limits of the

Definitions are U propositions; but to state identity of denotation is not their main function.

denotation, but to make explicit the connotation. Moreover the content asserted by the predicate in a definition is affirmed of every individual denoted by the subject. But in the **U** proposition this distributive reference is lost, and the predication is made of the denotation of the subject as a whole. A definition is then a **U** proposition, but this is its least important aspect. The full predication intended, but not really expressed, by the **U** form can be made by a double employment of the **A** proposition of the traditional logic, and this mode of expression is not open to the objections just urged against the **U** form. Thus, *S a P* and *P a S* together express all that *S u P* is intended to say. Propositions such as 'Mercury, Venus, etc., are all the planets,' which have also been given as examples of **U** propositions, are not so formally; for there is nothing in 'Mercury, Venus,' etc., to show that it is *All S*. We must interpret it as meaning 'The class composed of Mercury, Venus, etc., is all the planets,' an awkward form whose full force is given in the two propositions 'Mercury, Venus, etc., is each a planet' and 'All the planets are amongst those enumerated.' Thus we see that the strict **U** form of proposition is practically never used; and, if it were, it ought not to be admitted into a scheme of simple propositional forms as it would really be exponible [*cf.* § 75 (ii.)].

The real assertion intended by U can only be made by two A propositions.

Hence, U propositions, if ever made, would be exponibles; and this form is, therefore, not simple.

Exclusive and Exceptive Propositions [*see* § 75 (ii.) (*a*) and (*b*)] are usually given as examples of the **Y** form. It may be granted that these propositions can be written in that form—*e.g.*, 'The virtuous alone are happy' may be expressed 'Some virtuous is all happy.' But this does not make them simple propositional forms; they are, as we saw in § 75, compound in their meaning, and may be reduced to two propositions of the form *Some S is P*, *No non-S is P*. They have, then, no place in a scheme of simple propositional forms.

Exclusive and Exceptive Propositions are examples of Y, which is, therefore, not a simple propositional form.

The form η—*No S is some P*—is never used in ordinary speech. Dr. Keynes says: "*Not S alone is P* is practically η "provided we do not regard this proposition as implying "that any *S* is certainly *P*" (*op. cit.*, p. 177). But, if

The form η, if ever used, would also be exponible, and, therefore, not simple.

'some' is used in the sense of 'some only,' then $S\ \eta\ P$ does imply that *All S is some (other) P*. And, whichever way we read 'some' we have here again an exponible proposition; and consequently, this form, too, must be excluded from an analysis of simple forms.

ω is useless.

The uselessness of the form ω has been already sufficiently shown, and it is certain that nobody ever attempts to express a judgment by means of it.

Hence, none of the new forms are admissible in an analysis of simple propositional forms.

Hence we cannot agree with those logicians who advocate the addition of **Y** and η to the four-fold scheme in Formal Logic; for they are not simple propositional forms, and are moreover necessarily based on the class view of predication, against which, as expressive of the real import of predication, objections have been already urged. That the four-fold scheme is formally complete if the predicative view is adopted has already been shown (*see* § 84).

We may conclude our criticism in the words of Prof. Adamson: "To such a scheme the objections are manifold. It is "neither coherent in itself, nor expressive of the nature of "thinking, nor deduced truly from the general principle of "the Hamiltonian logic. For it ought to have been kept in "mind that extension is but an aspect of the notion, not a "separable fact upon which the logical processes of elabora-"tion are to be directed. It is, moreover, sufficiently clear "that the relation of whole and part is far from exhausting "or even adequately representing the relations in which "things become for intelligence matters of cognition, and "it is further evident that the procedure by which types of "judgment are distinguished according to the total or partial "reference to extension contained in them assumes a stage "and amount of knowledge which is really the completed "result of cognition, not that with which it starts, or by "which it proceeds. . . . Hamilton, it may be added, finds "it completely impossible to work out a coherent doctrine "of syllogism from the point of view taken in the treat-"ment of the judgment" (Article *Logic* in *Ency. Brit.*, 9th Ed.).

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*87. The Comprehensive View.

Hamilton read judgments both in extension and in comprehension.

Sir W. Hamilton held that every judgment expresses not only a quantitative relation in extension, or denotation [*see* § 28 (vi.)] between subject and predicate, but also a similar relation in comprehension [*see* § 28 (vi.)]. He says: "We "may . . . articulately define a judgment or proposition to "be the product of that act in which we pronounce that of "two notions thought as subject and predicate, the one does "or does not constitute a part of the other, either in the "quantity of extension, or in the quantity of compre-"hension" (*Lectures on Logic*, vol. i., p. 229). As extension and comprehension vary inversely [*cf.* § 28 (v.)] the notion which is smaller in extent is larger in content, and *vice versâ*. The copula, *is*, has, therefore, two meanings; "In the one "process, that, to wit, in extension, the copula *is* means *is con-"tained under*, whereas in the other, it means *comprehends* "*in*" (*ibid.*, p. 274). For example, 'Man is two-legged' read in extension means that 'man' as a class is contained in the class 'two-legged'; read in comprehension it means that the complex notion 'man' comprehends, as a part of itself, the attribute 'two-legged.' Thus, read in extension the predicate is larger than the subject, but read in comprehension it is less. From this double meaning of all propositions it follows that every reasoning must be considered under a double aspect, and that two kinds of syllogisms are required—the Extensive and the Comprehensive—the latter being derivable from the former by changing the meaning of the copula and transposing the premises.

In extension the copula means *is contained under*; in comprehension it means *contains.*

The part of this doctrine which refers to the extensive reading of propositions has been examined generally in the last two sections, and with special reference to Hamilton's further additions in the last section; and reasons have been given for rejecting it. Nor is the Comprehensive view more tenable. If 'Comprehension' is used correctly, as synonymous with Connotation [*see* § 28 (vi.)] then it is false to say that in every proposition the subject contains the predicate in comprehension; of no synthetic judgment (*see* §§ 40 and 69) is this true. It is, therefore, necessary, if this view of pro-

This view requires comprehension to include all the attributes known to be common to a class,

positions is to stand, to make comprehension include all attributes known to be common to a class [*see* § 28 (ii.)], and, as a necessary consequence, to make all propositions analytic. But here we are met with a difficulty. We surely cannot get out of a notion anything which is not already in it. How, then, can we express *new* information? It is not already part of the subject-notion, and, on this theory, it can never become a part of it. The fundamental notion underlying this reading of the terms of a proposition is that a judgment only expresses a relation of agreement or disagreement between two concepts. Such a view of judgment is, as we have pointed out, one-sided and altogether inadequate (*see* § 8). It errs in the opposite direction to that which regards only the denotation of the terms, and which, consequently, is too objective. Both views equally lose sight of the essential unity of the judgment, and regard it as bringing together elements which were before separate and unrelated in thought.

and makes all propositions analytic.

*88. The Attributive or Connotative View.

J. S. Mill held that every proposition whose subject is not singular is the expression of a relation between attributes. But he did not regard this relation as one of inclusion of the predicate in the subject; on the contrary he submits Hamilton's view to a trenchant criticism (*see Exam. of Ham.*, ch. xviii. and xxii.). Mill consistently maintains that the attributes implied by a class-name, and which form its connotation, are those only which are essential to membership of the class [*see* § 28 (ii.)]; and he attaches great importance to the indirectness by which the members of such a class are indicated by that class-name [*cf.* § 71 (i.) (*b*)]. He says: "Though it is true that we naturally 'construe the "subject of a proposition in its extension,' this extension, or "in other words, the extent of the class denoted by the name, "is not apprehended or indicated directly. It is both "apprehended and indicated solely through the attributes" (*Logic*, Bk. I., ch. v., § 4, note). Hence he argues that all that is really asserted in any proposition whose subject is a

Mill held that every proposition which is not singular expresses a relation between attributes.

General Name is that the attributes connoted by the predicate do, or do not, accompany the attributes connoted by the subject. "Man is mortal" means "Whatever has the attributes of man has the attribute of mortality; mortality "constantly accompanies the attributes of man" (*ibid.*, § 4). This is "the formula . . . which is best adapted to express "the import of the proposition as a portion of our theoretical "knowledge. . . . But when the proposition is considered as "a memorandum for practical use, we shall find a different "mode of expressing the same meaning better adapted to "indicate the office which the proposition performs. . . . In "reference to this purpose, the proposition, All men are "mortal, means that the attributes of man are *evidence of*, "are a *mark* of, mortality; an indication by which the "presence of that attribute is made manifest" (*Logic*, Bk. I., ch. vi., § 5).

Mill is right in holding the ultimate import of judgment to be relation of content, but wrong in deriving this from enumeration of instances.

This analysis of the ultimate import of a proposition is right in so far as it regards the denotative proposition as only an interpretation of a judgment affirming relation of content. So far it is in agreement with the position set forth in § 71. But it is faulty in that it regards connexion of content as established by simple enumeration of instances, and justified by mere uncontradicted experience. This weakness is due to Mill's general empiricist position that sensuous experience is the only possible source of knowledge, a position which led him to the doctrine that reality is nothing but possibility of sensation. The sensuous experience which Mill puts forward as the only basis of knowledge can, obviously, never give certainty, or necessity, or universality in the strict sense of always and everywhere. Even if complete it can only speak definitely of the past and in terms of weaker or stronger expectation of the future. Thus, Mill does not grant that universal judgments express a necessary relation of content, but only a coexistence hitherto invariable. The defect of Mill's view is thus fundamental; he omits the work of thought in constituting reality, he forgets that sensuous experience becomes knowledge only when it is interpreted by being brought under relations conceived by

the mind, and afterwards proved by experience to be true of reality. It is in this recognition of the constitutive action of thought that we find necessity. A judgment is necessary when it expresses the only possible interpretation of the results of experience, that is, when it harmonizes with the total and systematic concept of reality.

89. Implication of Existence.

Logical existence may be in any sphere.

The question we have now to consider is this: Does the assertion of a categorical proposition necessarily imply that its terms are the names of really existing things? This enquiry has no reference to any particular mode of existence; the word is used in its widest sense, and embraces existence in the spheres of fiction, mythology, and imagination, as well as in that of physical reality [*cf.* § 28 (iv.)]. The logical force of the term is well expressed by Prof. W. James: "In "the strict and ultimate sense of the word existence, every-"thing which can be thought of at all exists as *some* sort of "object, whether mythical object, individual thinker's object, "or object in outer space and for intelligence at large" (*Mind*, No. LV., p. 331). Now, every logical term represents something thought of, and, therefore, the mere use of a term implies the existence of some thing, or things, of which it is the name. Thus, "A name always *refers* to "something. . . . That which is named is recognized as having "a significance beyond the infinitesimal moment of the "present, and beyond the knowledge of the individual. . . . "It is, in short, characterized as an object of knowledge" (Bosanquet, *Logic*, vol. i., pp. 18, 19). As this logical existence, this mere 'thisness,' is implied by the simple use of the terms of a proposition, it cannot be specially predicated, nor is it asserted by the copula. But a particular mode of existence can only be asserted by special predication; for it is not involved in the use of either terms or copula (*cf.* § 68). Thus, 'The idea of duty exists' predicates existence in the realm of men's moral thoughts; 'Much misery and crime exist in our large towns' asserts existence in the physically real sphere of man's life. These special

It consists merely in the applicability of the term to the thing named,

and is implied in the mere use of a term.

It cannot, therefore, be specially predicated; but particular modes of existence must be specifically asserted as predicates.

kinds of existence do not, therefore, touch the question we are now considering. They only come under our notice as predicates, and, therefore, as mere special instances of the general rule that *all* predicates and subjects involve the wider logical existence which consists in the applicability of a term to the thing named. The implication of existence follows, therefore, from the very nature of the term. It is equally necessitated by the nature of the proposition. Every proposition expresses a relation between a subject and an attribute (*see* § 84). But we can only conceive a subject as a more or less permanent thing capable of receiving or rejecting more or less transient attributes (*cf.* § 68). Hence, both subject and attribute must have, at least, this much existence—that the latter is capable of affecting the former, and the former of being affected by the latter. This minimum of existence is all that can be implied by *all* propositions, and it is, therefore, all which can be regarded as necessarily involved in any. Moreover, a proposition claims to be true, and, if we disbelieve it, we can only contradict it by another proposition which advances the same claim to truth (*cf.* § 66). But if a proposition is a mere statement in words, with no corresponding reality behind, we cannot say, in any intelligible sense, that it is either true or false. In fact, it ceases to have any real meaning and becomes a mere sham.

The implication of existence follows from the nature of the proposition as well as from that of the term.

We must conclude, then, that every categorical proposition —universal or particular, analytic or synthetic—implies, logically and necessarily, the existence of its terms in some appropriate sphere. It now remains to ask whether both subject and predicate must exist in the *same* sphere. In the case of affirmative propositions this must, necessarily, be the case; for *S a P* and *S i P* assert that in the sphere of existence in which *S* holds a place, certain objects—viz., *All S*, and *Some S* respectively—possess the attribute *P*. But, in the case of negative propositions, the predicate does not, necessarily — though it does usually — exist in the same sphere as the subject. Now, as the subject is the centre of attention, and the point at which the judgment touches

In affirmative propositions the existence of the *S* necessitates that of the *P* in *the same sphere*, but not in negative propositions.

reality, we must regard the sphere of existence to which the proposition refers as that of the subject. Therefore, we must say that in a negative judgment the existence of the predicate *in the sphere of the subject* is problematic. Of course, as a term, it exists in *some* sphere, but not necessarily in the sphere to which the judgment primarily refers. Thus in 'No woman was hanged in England for theft last year,' the sphere of the subject—and, consequently, of the whole judgment—is that of physical existence. But in this sphere the predicate does not exist at all; for nobody—man, woman, or child—received that punishment for theft. The sphere of existence of the predicate is, therefore, that of imagination. Similarly, 'Some mountains are not fifty thousand feet high' does not imply the existence of objects of that height in the realm of physical things in which the subject exists. Our final result, therefore, is this: All categorical propositions necessarily imply the existence of their subjects in the appropriate sphere; in affirmative propositions this involves the existence of the predicate in the same sphere; but in negative propositions the predicate does not necessarily exist in that particular sphere, though it does in some sphere.

The sphere of existence to which the proposition refers must be determined by the subject.

The *P* in a negative proposition need not exist in that sphere, though it must in some sphere

Summary.

This reference to reality is the distinguishing mark of a categorical as distinguished from a hypothetical judgment. The latter, as we have seen, involves a supposition as its very essence, and this supposition may be one which is actually incapable of phenomenal realization, and yet we may be sure that the judgment is exactly and universally true, as it is the result of a valid process of inference, and is the only possible interpretation of experience (*see* § 76). But in the categorical judgment the reference to reality is distinct and direct. In a singular judgment of direct perception, such as 'This table is made of oak,' the existence of the subject seems to be not merely implied but asserted. In the particular enumerative judgment *Some S's are P* the implication of such existence is still very strong, for such judgments on their face claim to be, and often are, the results of direct experience. The implication is, undoubtedly,

The reference to reality distinguishes the categorical from the hypothetical judgment.

It is prominent in proportion as the judgment is based on direct experience.

weaker in the universal judgment *Every S is P*, for that, as we have seen, is not based upon a completed experience of instances, but upon an established connexion of content. But as this connexion can only be established by an analysis of that mode of reality to which the judgment refers, the implication of existence, in the appropriate sphere—*i.e.* in some mode of reality—is not absent.

CHAPTER III.

DIAGRAMMATIC REPRESENTATION OF PROPOSITIONS.

90. Nature and Use of Diagrams.

Diagrams should be self-interpreting, and should correspond exactly with the elementary forms of proposition.

Diagrams are intended to make obvious at a glance the relations between the terms expressed in a proposition. That any scheme may do this satisfactorily, it is essential that :—

(1) The diagrams employed should be self-interpreting immediately the principle on which they are constructed is understood.
(2) Each diagram should be capable of one, and only one, interpretation ; and, conversely,
(3) Each proposition should be representable by one, and only one, diagram.

The value of every scheme of diagrams must, therefore, be estimated by the perfection with which it fulfils these requirements.

Diagrams primarily represent the extension of both terms.

Primarily, the ordinary schemes of diagrammatic representation—especially that considered in the next section—represent the extension, or denotation, of both the terms in a proposition. It has already been pointed out (*see* §§ 72 and 85) that it is always possible thus to attend to both the terms in a formally expressed proposition, though this limited interpretation omits and obscures the most essential aspect of the unity of judgment. But, in some of the schemes, this reference to extension is less prominent—as, for instance, in the scheme explained in § 92. Even in the first scheme, we may interpret the circle which represents the predicate as 'cases

in which the attribute *P* occurs'; when, though we still deal with extension, we have made the connotative element more prominent than when we simply say 'the class *P*.'

Mansel held that Concepts could not be represented by diagrams;

Mansel, from the Conceptualist standpoint [*see* § 8 (ii.) (*b*)] has raised a fundamental objection to the employment of diagrams in Logic. He says: "If Logic is exclusively concerned with Thought, "and Thought is exclusively concerned with Concepts, it is impos- "sible to approve of a practice, sanctioned by some eminent "Logicians, of representing the relation of terms in a syllogism by "that of figures in a diagram." This "is to lose sight of the dis- "tinctive mark of a concept, that it cannot be presented to the "sense, and tends to confuse the mental inclusion of one notion in "the sphere of another, with the local inclusion of a smaller portion "of space in a larger" (*Prolegomena Logica*, p. 55). Those who do not agree that "Logic is exclusively concerned with Thought, and "Thought is exclusively concerned with Concepts" (*cf.* § 9) will regard this objection as based on an inadequate and misleading view of the science. But even those who do accept the Conceptualist position must own that every concept has extension; and it is this extension, and not the concept itself, which the ordinary diagrams aim at representing.

but every concept has extension, and it is this which the diagrams represent.

Diagrams make immediate inferences more obvious.

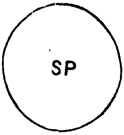
A practical argument for the use of diagrams in Logic is that they afford aid to the beginner in grasping the exact scope of a proposition, and the immediate inferences which can be drawn from it. Even those who are not beginners often find it well to appeal to diagrams when they are dealing with a large number of terms, as in the problems with which Symbolic Logic grapples.

91. Euler's Circles.

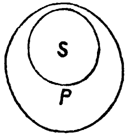
Euler's scheme was based on the actual relations of classes, each of which he represented by a circle.

The best known and most commonly used scheme of diagrams is that of Euler, a distinguished Swiss mathematician and logician of the eighteenth century. He expounded it in his *Lettres à une princesse d'Allemagne* (Lett. 102-5). It is based on the actual relations between two classes, each of which is represented by a circle. This necessitates the following five diagrams to express all the possible relations :—

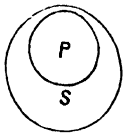
This requires five diagrams,



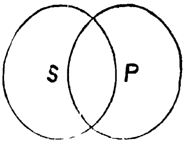
I.



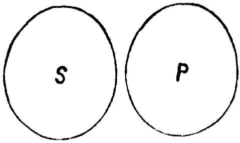
II.



III.



IV.



V.

This scheme admirably satisfies the first criterion of excellence given in the last section—there can be no doubt as to the information given by each of the above diagrams. But, as it is founded, not on the predicative view of propositions, but on an analysis of the possible relations which may subsist between classes, it is not surprising that the diagrams do not satisfactorily represent the four-fold scheme of propositions. They correspond, in fact, to the five elementary forms of proposition which are necessary to express all possible actual class relations (*see* § 85 *ad fin.*). Thus :—

which correspond to the elementary forms of proposition expressive of actual class relations, but do not satisfactorily represent the ordinary four-fold scheme.

I. represents that S and P are coincident—*All S is all P.*

II. that S is included in, but does not form the whole of P—*All S is some (only) P.*

III. that S includes P, but is not wholly exhausted—*Some (only) S is all P.*

IV. that S and P partially include and partially exclude each other—*Some (only) S is some (only) P.*

V. that S and P are mutually exclusive—*No S is any P.*

If, however, we try to fit in this scheme of diagrams with the ordinary four-fold analysis of simple propositional forms, we find that only in the case of **E** have we an adequate

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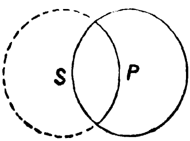
A, I, and O each require a combination of these diagrams to represent them; four of the diagrams are ambiguous in their reference to those propositions;

expression in any one diagram. Bearing in mind the absolute indefiniteness of 'Some' [*see* § 71 (ii.)], it is plain that every other form of proposition can be fully represented only by a combination of diagrams. Thus, for **A** we require I and II; for **I** we need I, II, III, IV; and for **O** we must have III, IV, V. If, on the other hand, we are given either of the Figures I, II, III, or IV, we cannot say with certainty what proposition it is meant to represent. The scheme, then, is of little value for the representation of the ordinary forms of proposition; and when propositions are united into syllogisms, it becomes so complex as to be practically unworkable [*cf.* § 124 (i.)]. Thus, when applied to represent **A**, **E**, **I**, **O** propositions, the scheme does not satisfy either of the two last criteria of excellence given in the last section. To attempt to escape this complexity by representing **A** by II alone and both **I** and **O** by IV alone—as is often done—is misleading, insufficient, and inaccurate. But even were it not open to these objections, we should still have an ambiguous diagram; for IV would represent indifferently **I** and **O**. To attempt to avoid this difficulty, as Euler apparently did, by writing the *S* in the part of the *S*-circle which is excluded from the *P*-circle (as is done above) when the proposition intended is **O**, and in the part of the diagram common to both circles when it is **I**, is not satisfactory; for this assumes that we already know what proposition is intended. The diagram itself still remains ambiguous; and, if it is given in the empty and unlettered form, we do not know what predication is intended.

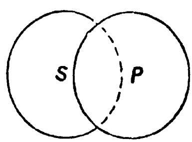
and the same diagram represents both I and O.

Attempts have been made to avoid this ambiguity by the use of dotted lines.

It has been proposed to avoid this ambiguity by using a dotted circumference to denote what is indefinite. Thus, Jevons in his *Primer of Logic* (pp. 46-7) would represent **I** by Fig. VI and **O** by Fig. VII :—



VI.

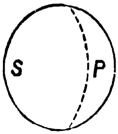


VII.

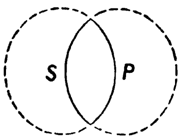
This is not satisfactory; for VI excludes the possibility of *S* either coinciding with, or including the whole of, *P*; and this latter possibility is equally negatived by VII. Moreover, the **A** proposition can still be fully represented only by the combination of Figs. I and II.

Of these attempts Ueberweg's is the most successful.

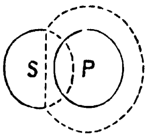
Ueberweg's plan is not open to these objections. He (*Logic*, Eng. trans., pp. 217-218) represents **A** by Fig. VIII, **I** by Fig. IX, and **O** by Fig. X:—



VIII.



IX.



X.

This gives expression to all possible cases, and we have a scheme in which each proposition is represented by one, and only one, diagram, and each diagram can be interpreted by one, and only one, proposition; but Fig. X can scarcely be regarded as sufficiently simple and obvious to be satisfactory. Fig. V is, of course, still retained to express the **E** proposition as it is perfectly unambiguous.

92. Lambert's Scheme.

Lambert represented the extension of terms by lines, unbroken if the term is distributed, and dotted if it is undistributed.

Lambert's plan is to represent the extension of a term by a horizontal straight line, unbroken when the term is distributed and dotted when it is undistributed. If, by the force of the proposition in which it occurs, the extension of a term is partly definite and partly indefinite—as is the predicate of an affirmative proposition—the line is partly unbroken and partly dotted. When two terms are joined in a proposition, the line representing the subject-term is written a little lower than that which stands for the predicate-term. In an affirmative proposition, the unbroken part of the line representing the subject-term is placed under the unbroken part of that which indicates the predicate-term; and in a negative proposition, the lines are so written that their unbroken parts do not overlap. The relative lengths of the lines, whether broken or unbroken, is,

BOOK II, Ch. III.

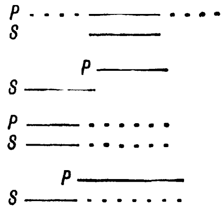
in all cases, immaterial. On this plan, the elementary forms of categorical proposition can be thus expressed :—

A - $S\,a\,P$ . .

E - $S\,e\,P$ . .

I . $S\,i\,P$ . .

O - $S\,o\,P$ . .

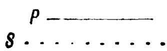


Thus, the diagram for **A** sets forth that S certainly covers part of P—the part marked by the continuous line—and may, or may not, cover the rest, the dots representing our uncertainty.

These diagrams fit in with the four-fold scheme of propositions.

This scheme is not quite as self-interpreting as is the one last described, but it has great advantages over that in every other way. It does fit in with the ordinary four-fold scheme of propositions. Each proposition can be represented by one, and only one, diagram ; and each diagram refers unmistakably to one, and only one, proposition. It can, for this reason, be employed to represent syllogisms more readily than can Euler's circles ; except, indeed, in the modified form which Ueberweg gave them.

The scheme of diagrams given above is a modification of Lambert's scheme so far as the representation of the I proposition is concerned. Lambert employed



which is as appropriate to **O** as it is to **I**. This, probably, accounts to some extent for the neglect with which his scheme has been treated by logicians.

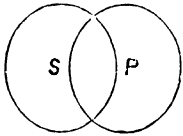
93. Dr. Venn's Diagrams.

Dr. Venn's diagrams are adapted to represent universal propositions.

Dr. Venn, in his *Symbolic Logic*, explains a very ingenious plan which he has invented for representing universal propositions interpreted on the existential or compartmental theory, that is, as denying or affirming the existence of the things denoted by one or more of the complex terms $S\,P$, $S\,\bar{P}$, $\bar{S}\,P$, $\bar{S}\,\bar{P}$. He regards an empty diagram as representing no proposition, but as a

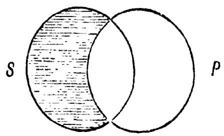
mere framework into which a proposition can be fitted. The framework for any proposition involving two terms is

From the empty diagram the compartments are shaded out which the proposition declares empty.

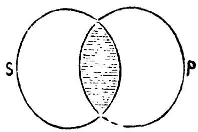


We have here four compartments (one being that which lies outside both the circles)—$S\ P$, $S\ \bar{P}$, $\bar{S}\ P$, $\bar{S}\ \bar{P}$ (where $\bar{S}$ and $\bar{P}$ denote *non-S* and *non-P* respectively), which correspond to the four possible classes which can be obtained from the combinations of S and P and their contradictories.

Every universal proposition denies the existence of one or more of these classes, and this is represented by shading out the compartment. Thus, $S\ a\ P$ is indicated by shading out the compartment $S\ \bar{P}$ (Fig. I), and $S\ e\ P$ by shading out $S\ P$ (Fig. II):—



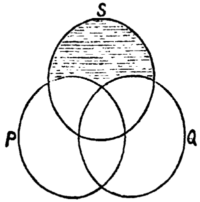
I.



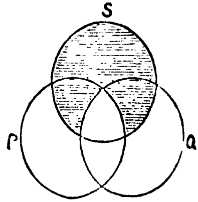
II.

The scheme is adapted to propositions involving as many as five terms,

The scheme is adapted to propositions involving more than two terms, but becomes cumbrous when the number exceeds five. For example S *is* P *or* Q is represented in Fig. III, and S *is both* P *and* Q in Fig. IV:—



III.



IV.

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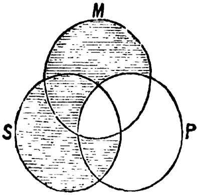
It equally well represents a categorical syllogism, in which both the premises are universal. If, for instance, our premises are $M\ a\ P$ and $S\ a\ M$, the diagram is



which shows at a glance that the relation established between S and P is $S\ a\ P$.

but is not well suited to represent particular propositions.

But the scheme is not well adapted to particular propositions. Dr. Venn proposes that a bar should be drawn across the compartments which the particular proposition declares to be saved, but this is apt to lead to confusion. The scheme, therefore, cannot be used satisfactorily for all the elementary forms of proposition.

94. Scheme Proposed.

It is proposed to base a scheme of diagrams on the implications of existence in a categorical proposition.

We would suggest that a satisfactory scheme of diagrams can be based on the implications of existence contained in a categorical proposition (*see* § 89). Every such proposition combining two terms—S and P—involves reference to the following classes—the S which is P (SP), the S which is not P ($S\bar{P}$), the things beside S which are P ($\bar{S}P$). Of those things outside S which do not possess the attribute P ($\bar{S}\bar{P}$), the proposition tells us nothing directly, and the existence of that class is, therefore, always problematic. We propose that these four classes shall be written along a horizontal line, which shall be unbroken when it represents a class whose existence is implied, and dotted when it stands for a class whose existence, in that sphere to which the proposition refers, is doubtful. The omission of the line representing any one class involving either S or P implies that its existence is implicitly denied by the proposition.

The possible classes are to be written along a line, which, when unbroken, implies certainty of existence, and, when broken, doubt.

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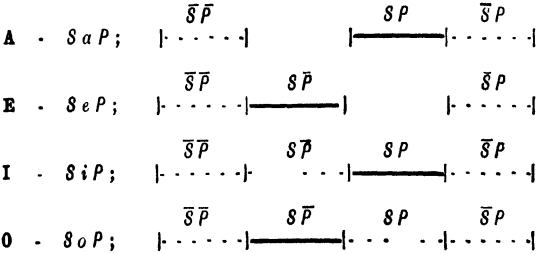
The omission of a class involving $\bar{S}$ or $\bar{P}$ means denial of existence.

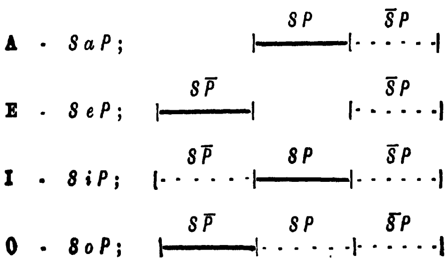
This plan would give the following diagrams :—



The predicates of the negative propositions, **E** and **O**, are represented by dotted lines in accordance with the view advocated in § 89 that their existence is not assured in the same sphere as that of the subject. It will be noticed that a distributed term is confined to one kind of line—unbroken or broken, but that an undistributed term contains both (*cf.* § 72). This is natural, as, in an undistributed term a positive reference is made to some portion of its denotation, but it is left undetermined what that portion is. The division $\bar{S}\bar{P}$ may be practically disregarded except for some purposes of immediate inference, and may, in all other cases, be omitted from the diagrams; but it must be remembered that such omission does not mean that the possibility of that class is denied, for such denial cannot possibly be made by a proposition which contains neither $\bar{S}$ nor $\bar{P}$.

The diagram may be simplified for most purposes by omitting $\bar{S}\bar{P}$

In this simplified form the diagrams will be :—



These diagrams fit in with the four-fold scheme of propositions.

This plan is based on the four-fold scheme of categorical propositions, with which it thoroughly fits in. Each proposition is fully represented by one diagram, and each diagram can be interpreted by only one proposition. A glance at any one of the diagrams will show whether, in that particular case, *S* is, or is not, asserted to possess the attribute *P*—that is, whether it does, or does not, exist in the class *SP*—and whether that statement is, or is not, made definitely about the whole of *S*.

On the same plan disjunctives can be represented, if they involve no more than three terms.

A disjunctive proposition involving no more than three terms can be similarly represented. Thus *S is P or Q* is shown by a dotted line divided into three portions ; for the existence of neither alternative is assured, though it is certain that some one, or two, must exist.

$S P \bar{Q}$ $S P Q$ $S \bar{P} Q$

|- - - - - - -|- - - - - - -|- - - - - - -|

If the disjunction embraces more than three terms, however, it is not possible to represent it simply on this plan. But no plan—except Dr. Venn's—is at all adapted to the representation of such disjunctives, and, it may be added, their representation, if it could be secured, would be of no practical advantage.

Hypotheticals cannot be represented by diagrams.

From their very nature Hypothetical Propositions as such are not capable of diagrammatic representation; for no diagrams can express the relation of dependence which exists between the consequent and the antecedent (*see* § 76).

BOOK III.

IMMEDIATE INFERENCES.

CHAPTER I.

GENERAL REMARKS ON IMMEDIATE INFERENCES.

95. Nature of Immediate Inferences.

Inference is the deriving of one truth from others. It has a formal aspect,

Inference, or Reasoning, is the deriving of one truth from others. By this is meant that the new judgment is accepted as true because, and in so far as, the validity of the judgments from which it is derived is accepted. Hence, every inference has a formal and necessary character, and this is not affected by the truth or falsity of the premises. The premises may be false and yet the inference may be formally valid, *i.e.*, valid in the sense of avoiding contradiction within itself. But in the wider sense of validity, in which the result of the inference must also be consistent with the whole system of knowledge, the truth of the premises is, of course, an essential element (*cf.* § 5). This aspect of inference will be dealt with when we consider the doctrine of Induction; in this and the following Book we shall be primarily concerned with an analysis of the formal aspect of the process.

but involves the derivation of a new judgment.

Inference is not a mental process absolutely distinct in its character from judgment. The essence of the latter is the explanation of some element of reality by reference of it to

some concept already familiar to the mind. In inference there is the same essential feature, but with this difference, that the reference is not made immediately, but indirectly through the medium of some previously accepted truth or truths. In inference, therefore, we pass beyond the judgment, or judgments, from which we start, and attain a new point of view; though, at the same time, the new judgment thus reached must be a necessary consequence of the data from which we set out. Inference thus involves both a process and its result; and to each of these the name is sometimes given. But strictly speaking an inference is the whole mental construction, and sets forth the connexion between the judgment proved and the evidence which proves it. The judgments which express the data or evidence are called *Premises*; the judgment derived from them is termed the *Conclusion*.

An inference includes both premises and conclusion.

An *Immediate Inference* unfolds the implications of a single judgment.

Immediate Inference is the process by which the implications of a single judgment are unfolded. By its immediateness is not meant that no activity of thought is required to reach the new judgment—for then it would not be inference at all—but simply that no datum is necessary besides the one given judgment. Kant called such inferences "Syllogisms of the Understanding" to distinguish them from the "Syllogisms of Reason" in which two premises are required, and to which the name "Syllogism" is commonly restricted. They may, however, most appropriately be styled *Interpretative Inferences*, as distinguished from mediate inferences obtained from a combination of judgments in which thought makes a substantial advance to a new truth.

Many Immediate Inferences are obvious,

It is often questioned whether Immediate Inferences are really inferences at all, as no new truth is reached by them. It may be granted that the majority of them are of comparatively small interest, and that the passage of thought from the premise to the conclusion is a very small and obvious one. But to object as Mill does that "there is in the conclusion no "new truth, nothing but what was already asserted in the "premises, and obvious to whoever apprehends them" (*Logic*, II., i., § 2) would be fatal to all inference; for in every valid inference the conclusion must be a necessary

consequence of the premises, and, therefore, potentially known as soon as these are fully apprehended. The step from premise to conclusion in an Immediate Inference is small; but this does not prove that it is no step at all, or that it is unnecessary to take it. Moreover, the great variety of these steps necessitates a careful and systematic examination of them; for, without such an investigation, it is very doubtful if all the necessary implications contained in a simple proposition are generally grasped. It is only when we have seen in how many new forms we can express what is virtually contained in any single judgment that we, as a rule, fully appreciate the meaning of that judgment.

but their variety demands examination in order that the full force of a judgment may be grasped.

An examination of the forms of reasoning should begin with Immediate Inferences; for we should know what is involved in a single judgment before we go on to enquire what results will follow from a union of several judgments.

An examination of reasoning should begin with Immediate Inferences.

96. Kinds of Immediate Inferences.

There are two main classes of Immediate Inferences:— (i.) *The Opposition of Propositions*, when, from the given truth or falsity of one proposition we infer the truth or falsity of other propositions relating to the same matter—that is, having the same subject and predicate. In other words, an examination of the opposition of propositions means a consideration of the relations as to truth or falsehood which hold between the four forms of propositions, $S\ a\ P$, $S\ e\ P$, $S\ i\ P$, $S\ o\ P$, when S and P have the same signification in every proposition.

There are two kinds of Immediate Inferences: (*a*) *Opposition of Propositions*, or inferences as to truth or falsehood of related judgments.

(ii.) *Eductions*,[1] in which, from a given judgment regarded as true, we derive other judgments which are implied by it; or, in other words, when we look at the same truth from another point of view, and express the same matter in a different verbal form.

(*b*) *Eductions*, or derivation of implied judgments.

We shall consider these two kinds of Immediate Inferences in the next two chapters.

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[1] This name is adopted from Miss Jones' *Elements of Logic*.

CHAPTER II.

OPPOSITION OF PROPOSITIONS.

97. Opposition of Categorical Propositions.

The Opposition of Propositions means the relation between any two propositions of different form with identical *S* and *P*.

By Opposition we infer from the truth, or falsity, of one proposition to the truth, or falsity, of the opposed propositions.

By the Opposition of Propositions is meant the relation which holds between any two propositions which have identically the same subject and predicate. Opposed propositions thus differ in form, but refer to exactly the same matter; that is, to the same things, at the same time, and under the same circumstances. The logical doctrine of Opposition, therefore, sets forth what implications as to the truth or falsehood of each of the other forms of categorical propositions are involved in *positing* (*i.e.*, affirming as true), or *sublating* (*i.e.*, denying the truth of), any one proposition. This is, evidently, an entirely technical and arbitrary use of the word 'opposition.' The natural meaning of the word would be that two opposed propositions could not both be true together; that is, that opposition could exist only between the pairs of incompatible propositions, **A** and **O**, **E** and **I**, **A** and **E**. In this sense the word was originally used. It was, however, found convenient to include under the same head the relations between propositions which are not incompatible, *i.e.*, those between **A** and **I**, **E** and **O**, **I** and **O**. 'Opposition' thus came to include the relation between *any* pair of propositions of different form referring to the same matter, whether that relation were one of incompatibility or of compatibility. When once this technical use of the word 'opposition' is clearly understood, it is unlikely to cause any confusion.

'Opposition' is not restricted to inconsistent propositions;

but includes all relations of propositions differing in form but identical in reference.

As we have universal and particular, affirmative and nega-

tive, propositions, the relations between them will all be included under those subsisting between the following pairs :—

There are four kinds of Opposition: (1) Subalternation. (2) Contradiction. (3) Contrariety. (4) Sub-contrariety.

(1) A universal and the particular of the same quality; **A** and **I**; **E** and **O**.
(2) A universal and the particular of opposite quality; **A** and **O**; **E** and **I**.
(3) A universal and the universal of opposite quality; **A** and **E**.
(4) A particular and the particular of opposite quality; **I** and **O**.

This gives us four kinds of opposition, to which the names (1) *Subalternation*, (2) *Contradiction*, (3) *Contrariety*, and (4) *Sub-contrariety* are respectively given. We will now examine these in order.

Subaltern Opposition exists between a universal and the particular of the same quality.

(i.) **Subalternation.** *Subaltern Opposition exists between a universal and the particular of the same quality;* that is, between **A** and **I**, **E** and **O**. Thus, the propositions differ in quantity but not in quality. This is one of the technical kinds of opposition; for, not only are the two propositions in subaltern opposition not inconsistent with each other, but the truth of the universal necessitates that of the particular. This follows from the Principle of Identity (*see* § 17); for, by that principle any assertion which is true of every member of a class must hold of any number of those individual members, since they must be identical with some of those included under the distributed term. The assertion, when made of an indefinite part, simply repeats an assertion which was contained in the universal proposition.

By the Principle of Identity the truth of the particular follows from that of the universal;

In such a pair of opposites, the universal proposition is called the *Subalternant* or *Subalternans*, and the particular the *Subalternate* or *Subaltern.* Inference from the former to the latter is styled *Consequentia* or *Conclusio ad subalternatam propositionem;* that from the latter to the former, *Conclusio ad subalternantem.*

Hence, the inference of the truth of **I** from that of **A**, and of the truth of **O** from that of **E**, are *ad subalternatam.* The

assertion of 'All metals are fusible' involves that of 'Some metals are fusible'; and, if we posit 'No horses are carnivorous' we equally posit 'Some horses are not carnivorous.' But, if *S a P* is denied, then this denial holds equally if *P* belongs to some only of the *S's*, or to none of them. Hence, from the falsity of **A** we cannot say whether **I** is true or false. Similarly, from the denial of **E** we can neither affirm nor deny **O**. For example, if I deny that 'All metals are malleable' I do not thereby deny that 'Some metals are malleable.' Neither do I affirm the latter proposition (though it happens to be true in fact); for, if I did, then the denial of 'All horses are carnivorous' would involve the assertion of 'Some horses are carnivorous.' The sublating of the universal leaves us quite in the dark as to the truth or falsity of its subalternate.

but from the falsity of the universal we cannot infer either the falsity or the truth of the particular.

If we now examine the inferences *ad subalternantem* (or from particular to universal), we find that the denial of the particular involves the denial of the universal. For what is not true even in some cases cannot be true in all. The denial of *S i P* means 'There are no such things as some *S's* which are *P*,' and this, evidently, negates the assertion that *All S's are P*. Again, if **A** were true, **I** must be true by inference *ad subalternatam;* and hence, if the falsity of **I** did not involve that of **A**, it would follow that **I** could be both true and false at the same time; which is absurd. The same results hold with **E** and **O**. Thus, if we deny the truth of 'Some horses are carnivorous' we thereby deny that of 'All horses are carnivorous'; and if we assert the falsity of 'Some men are not mortal' we equally assert that of 'No men are mortal.' But, to posit the particular cannot justify us in positing the universal; for we can never justify an assertion about *Every S* by asserting that it holds good with regard to *Some S's*. For instance, though it may be true that 'Some men are red-haired,' it does not follow that all men possess that attribute; nor does the truth of 'Some men are not six feet high' imply that no men attain that height.

The denial of the particular involves the denial of the universal;

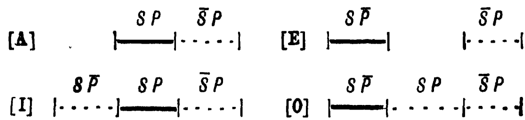
but the truth of the particular does not involve the truth of the universal.

Summary.

Hence, we reach this general result: The truth of the particular follows from that of the universal, but not *vice versâ;* and the falsity of the universal is an inference from that of the particular, but not *vice versâ*.

* These results are illustrated by the diagrams given in § 94.

These results can be illustrated by diagrams.



The length of the lines in these diagrams is, of course, immaterial. There is, therefore, no suggestion that the S is more extensive in the diagrams for **I** and **O** than in those for **A** and **E**. It is exactly the same in extent; for the S referred to must be identical in all cases of opposition. If we now examine the above diagrams we see that, in each case, the assertion of the universal includes that of the particular; for the unbroken line in the former includes the whole of S, and, therefore, covers the unbroken line in the latter, which only necessarily includes a portion of S. Hence, also, to sublate the universal would not sublate the particular; for the denial that the whole of S should be marked by an unbroken line does not necessitate the denial that a portion of it may be correctly so marked. Thus, in **A** the diagram shows that the whole of S is included in $S P$; if the truth of this be denied, then some, at least, of S must be found in $S \bar{P}$, which is shown to be possible by the diagram for **I** without necessarily denying the existence of another part of S in $S P$. On the other hand, if we sublate the particular we necessarily sublate the universal; for if we strike out from the diagram of the particular the unbroken line, we have nothing left corresponding to the unbroken line in the universal, which must, therefore, be also struck out. But to posit the particular will not posit the universal; for the existence of the unbroken line in the particular will not ensure its existence in the universal, as the latter covers the whole of S, but the former only necessarily covers a portion of it.

Similarly, if we use Euler's diagrams (*see* § 91) we see that **A** is represented by Figs. I and II, and I by Figs. I, II, III, IV; **E** by Fig. V, and **O** by Figs. III, IV, V. Hence, in each case,

the diagrams representing the universal include some of those which represent the particular. As the particular is true when any one or more of the diagrams which represent it is secured, it follows that to posit the universal is to posit the particular. But the sublating of the universal only removes some of the diagrams which may represent the particular, and hence does not sublate the particular, which may still be represented by the remaining diagrams. Similarly, the truth of the particular is ensured if those diagrams which do not represent the universal are secured; and, therefore, to posit the particular is not to posit the universal. On the other hand, if the particular is false, all the diagrams which can represent it are removed, and this removal includes all those which can represent the universal, which is thereby also declared false.

Contradictory Propositions differ both in quality and in quantity. A and O; E and I are pairs of Contradictories.

(ii.) **Contradiction.** *Propositions are contradictory to each other when they differ both in quality and in quantity.* Hence, there are two pairs of contradictories—**A** and **O**; **E** and **I**. By the Principle of Contradiction (*see* § 18) both the members of such a pair cannot be true together, and by the Principle of Excluded Middle (*see* § 19) both cannot be false. If 'All metals are fusible' is true, it cannot be true that 'Some metals are not fusible'; and, similarly, if 'No lions are herbivorous' is a true proposition, then it cannot be true that 'Some lions are herbivorous.' And, generally, if we make an assertion about every member of a class, the Principle of Contradiction forbids us to deny that assertion about any member of the same class. Therefore, one of the contradictories in each pair must be *false.* But, by the Principle of Excluded Middle, they cannot both be false. For, by that principle, any given attribute, *P*, must either belong, or not belong, to every individual *S*. It cannot, therefore, be false both to make an assertion of Every *S* and to deny that same assertion of Some of those *S's*. Such propositions as 'All metals are fusible' and 'Some metals are not fusible' cannot both be false together.

By the Principle of Contradiction one of a pair of Contradictory propositions must be false;

and, by the Principle of Excluded Middle, one must be true.

Or, in detail; if we deny the truth of ***Every S is P*** our denial holds whether *P* is denied of the whole of *S*, or of only part of *S*. But the former denial necessarily, by subalternation, includes the latter, which is, therefore, true in any case. Consequently the truth of **O** follows from the denial of **A**. If we deny **O** we really

assert that 'There are no such things as Some *S's* which are not *P*,' and this is the same as the assertion that *All S's are P*. If **E** be denied, then either *all* or *some S's*—in any case the latter—*are P*, and, therefore, **I** is necessarily true. Lastly, if we deny **I**, we really say 'There are no such things as Some *S's* which are *P*,' and this is to assert that *No S's are P*. Therefore, one of the propositions in each pair of contradictories must be *true*.

Hence, contradictories are incompatible with regard to both truth and falsehood.

We see, then, that contradictories are incompatible with regard both to truth and to falsehood. It follows that when two contradictory propositions are given us we infer, by the Principle of Contradiction, that one of them is false, and, by the Principle of Excluded Middle, that one of them is true. Hence, we can deduce the falsity of one from the truth of the other, and the truth of one from the falsity of the other. The relation of contradiction is thus seen to be reciprocal; the positing of one proposition and the sublating of its contradictory are assertions of one and the same fact. It will be seen, as we examine the other forms of Opposition, that in none of them are the propositions thus mutually inferrible, and in none of them is there incompatibility with regard to both truth and falsehood.

Contradiction is the only form of Opposition in which the opposed propositions are mutually inferrible.

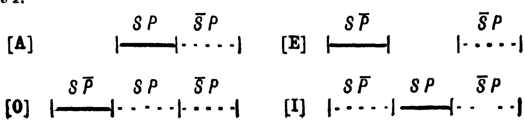
Contradiction is, therefore, the most perfect form of logical opposition.

Contradiction is the minimum of denial.

Whatever we affirm denies something else. The mere asserting of every *S* that it is *P* is, in itself, a denial of any *S* whatever that it is not *P*. To assert, therefore, that *Some S's are not P*, in opposition to *Every S is P*, is the minimum of denial. It is sufficient to destroy the proposition which it contradicts, but it does not affirm the falsity of every part of it. Thus, a pair of contradictory propositions leave no room for an intermediate supposition; one or the other must be accepted as true, as together they exhaust all possible alternatives.

Contradiction can be illustrated by diagrams.

* These results are illustrated by the diagrams given in § 94.



The absence of $S\bar{P}$ from the diagram for **A** shows that the existence of anything which is both S and $\bar{P}$ is denied. This existence is the very thing which is asserted in the diagram for **O**, which, however, does not deny the existence of the class posited by **A**—viz., SP—but merely leaves it a matter of doubt. The same thing holds in the diagrams for **E** and **I**; where **I** posits SP which **E** sublates, but does not sublate anything which is posited by **E**. Hence, one of a pair of contradictory propositions must be false, but they are in conflict with respect to the existence of one class only. In each case, moreover, the diagrams between them show that no other assertion can be made concurrently with these two about any part of S; for together they give full information about the S which is P, and the S which is not P, and every S must belong to one or other of those two classes. Hence, one of the contradictories must be true.

Similarly, a reference to Euler's circles (*see* § 91) will show that a pair of contradictories between them require the whole series of diagrams, thus showing that they are together exhaustive of all possibilities—thus, **A** requires Figs. I and II, **O** takes up III, IV, V; for **I** we need I, II, III, IV, and **E** is fully expressed by V—and no diagram belongs to both members of either pair of contradictories, thus proving their absolute incompatibility.

Every proposition has a contradictory; if the proposition is simple, so is the contradictory, but if the proposition is compound it can be contradicted in more than one way, and its full contradictory is, therefore, compound (*cf.* § 75).

Secondary Contradition exists between Singular Propositions.

Contradiction is the only kind of opposition which can subsist between Singular Propositions [*see* § 71 (i.) (*a*)]; for these can differ only in quality, and, therefore, to posit the one is to sublate the other, and *vice versâ*. This opposition of singular propositions is frequently called *Secondary Contradiction.*

Contrary Propositions are universals of opposite quality.

(iii.) **Contrariety.** *Contrary Opposition exists between a pair of universal propositions of opposite quality;* that is, between **A** and **E**. Thus, contrary propositions differ in quality only, and not in quantity. By the Principle of Contradiction

(*see* § 18) both cannot be *true* together. For, if two contraries were both true, then contradictories would also be true together. For, by subalternation, the truth of **A** would necessitate that of **I**, and the truth of **E** would secure that of **O**. Hence, **A** and **O** would be true together, and so would **E** and **I**. But this is impossible; and, therefore, **A** and **E** cannot be true together. But as a contrary proposition does not simply deny the truth of the opposed universal as a whole, but that of every part of it, and thus asserts its entire falsity, there is a possibility of an intermediate alternative. Hence, the Principle of Excluded Middle (*see* § 19) does not apply, and the propositions may both be *false*. For, whilst the negation of a universal allows inference by Contradiction to the truth of the particular of opposed quality, this latter does not warrant us in deducing the truth of the universal to which it is subaltern. Though by sublating **A** we posit **O**, this will not enable us to posit **E**. Hence, contrary propositions are incompatible with regard to truth, but not with regard to falsity. If one is true, the other must be false, but the falsity of the one does not involve the truth of the other. It may be equally false that 'All men are red-haired' and that 'No men are red-haired'; for the one proposition does not simply negate the other, but makes the opposite assertion with an equal degree of generality. It follows that contrary propositions are not mutually inferrible, and their formal opposition is, therefore, less perfect than is that of contradictories, although, of course, they express a greater degree of material divergence.

By the Principle of Contradiction both cannot be true,

but, as the Principle of Excluded Middle does not apply, both may be false.

Hence, contrary propositions are incompatible with regard to truth only;

and are not mutually inferrible.

From this lesser formal perfection, as well as from the much greater difficulty of establishing the contrary compared with that of merely disproving a given universal proposition, it follows that contrariety is of much less formal importance than contradiction. The bringing forward of one single instance which does not agree with a general proposition is sufficient to disprove it, and the contradiction is secure, as it rests on observed fact. But to establish—not merely that one *S*, or a few *S's*, but—that *every S* disagrees with the general proposition we wish to

Contrariety is more difficult to establish than is Contradiction.

and is less secure.

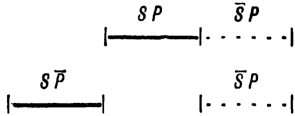
disprove is a task of much greater difficulty, and the result is much less secure against being itself proved false, than is the contradictory. For we can scarcely ever be sure that we have really examined every instance, and one exception is fatal to our general proposition; whilst the simple contradictory, being a particular, can only be overthrown by establishing the opposed general proposition. Thus we see that contradiction is sufficient for disproof, and is, obviously, a more secure position to take up than is the assertion of the contrary. One would deny that 'All men are liars' with much greater strength of conviction than one would assert that 'No men are liars.'

Contrariety can be illustrated by diagrams.

* All these points are illustrated by the diagrams given in § 94.

[A]

[E]



The diagram for **A** sublates $S\bar{P}$ and posits SP; that for **E** not only posits $S\bar{P}$ which **A** sublates, but sublates SP, which **A** posits. These two propositions are, therefore, seen to be in conflict with respect to every possible part of S, and they do not include the possible case that a portion of the class S may be P and, at the same time, a portion of it not be P; that is, that some S's do, and some do not, possess the attribute P. It follows that both may be false though both cannot be true.

Similarly, if we illustrate by Euler's circles (*see* § 91) we find that **A** requires Figs. I and II, and **E** is represented by Fig. V. The two together, therefore, omit Figs. III and IV, thus showing that they do not together exhaust all possible cases. Their thorough-going divergence is shown by the fact that, in the diagrams which represent **A**, S is entirely contained within P; and, in the diagram corresponding to **E**, is wholly excluded from it.

Sub-contrary Propositions are particulars of different quality.

(iv.) **Sub-contrariety.** *Particular propositions stand in sub-contrary opposition to each other;* that is, **I** and **O** are sub-contraries. This opposition depends on the Principle of

Excluded Middle (*see* § 19); for there can be no judgment intermediate between 'Some are' and 'Some are not.' Moreover, to deny the truth of one particular is to assert that of the universal of opposite quality (by Contradiction), and from this follows the truth of the particular which is subaltern to it. Hence, both these propositions cannot be *false*. But the Principle of Contradiction (*see* § 18) does not apply; for the 'some' in the one case is different in its reference from the 'some' in the other. Both propositions may, therefore, be *true*. The truth, for example, of 'Some men are red-haired' does not involve the falsity of 'Some men are not red-haired'; for it is not the same 'some men' who are referred to in both cases. But the form of the propositions does not show this, since the interpretation of 'some' must be purely indefinite. Thus, there is no real contrariety between I and O, and the name 'Sub-contrary' is entirely arbitrary. This is another instance of the technical use of the word 'opposition,' as the two propositions are perfectly compatible with each other; both may be, and often are, true, though both cannot be false. It follows, therefore, that to sublate the one is to posit the other, but not *vice versâ*. Hence sub-contrary propositions are inconsistent with regard to falsity but not with regard to truth.

Both cannot be false;

but both may be true.

Hence, Sub-contraries are inconsistent only with regard to falsity.

* This relation is illustrated in the diagrams given in § 94.

These relations may be illustrated by diagrams.

$$\begin{array}{llll} & S\bar{P} & SP & \bar{S}P \\ [\mathrm{I}] & |\cdots\cdots| & |\rule{3em}{0.4pt}| & |\cdots\cdots| \end{array}$$

$$\begin{array}{llll} & S\bar{P} & SP & \bar{S}P \\ [0] & |\rule{3em}{0.4pt}| & |\cdots\cdots| & |\cdots\cdots| \end{array}$$

Neither diagram denies anything. In each case what is definitely posited by the one is marked as possible by the other—the unbroken line in each coincides with a broken line in the other. Hence, both may be true. But to regard both as false would be to strike out the unbroken line from each, and this would involve the entire denial of the existence of S, which would be inconsistent with the fact that a predication has been made of it (*see* § 89).

Similarly, if reference is made to Euler's diagrams (*see* § 91), it is clear that **I** requires Figs. I, II, III, IV, for its full expression, and **O** is only fully represented by Figs. III, IV, and V; it follows that, as both are partly represented by Figs. III and IV, they can be true together; and, as together they include all the diagrams, they cannot both be false as that would entirely deny the possibility of the existence of *S*, by removing every diagram which can possibly express a relation in which it stands to *P*.

It has been argued that, if 'some' means 'some at least' **I** and **O** may both be false;

It may be well to notice here an argument which has been advanced against the doctrine of opposition by Mr. Stock. He says: "If **I** and **O** were taken as indefinite propositions, meaning "'some, if not all,' the truth of **I** would not exclude the possibility "of the truth of **A**, and, similarly, the truth of **O** would not exclude "the possibility of the truth of **E**. Now **A** and **E** may both be false. "Therefore **I** and **O**, being possibly equivalent to them, may both be "false also. In that case the doctrine of contradiction breaks "down as well. For **I** and **O** may, on this showing, be false, with-"out their contradictories **E** and **A** being thereby rendered true" (*Deductive Logic*, p. 139). But "if **I** and **O** be taken as strictly "particular propositions, which exclude the possibility of the "universal of the same quality being true along with them, we "ought not merely to say that **I** and **O** may both be true, but that "if one be true the other must also be true. For **I** being true, **A** is "false, and therefore **O** is true; and we may argue similarly from "the truth of **O** to the truth of **I**, through the falsity of **E**. Or—to "put the same thing in a less abstract form—since the strictly par-"ticular proposition means 'some, but not all,' it follows that the "truth of one sub-contrary necessarily carries with it the truth of "the other" (*ibid.*, p. 140).

but if 'some' means 'some only' they must both be true.

The latter is true, but is not based on the logical meaning of 'some.'

The latter part of this argument may be granted at once. It has been already pointed out that, if 'some' is used in the sense of 'some only,' each affirmative proposition involves a negative proposition also, and *vice versâ* (*see* § 86). Hence, each of the propositions **I** and **O** involves both **I** and **O**. But this is not the true logical meaning of 'some' [*cf.* § 71 (ii.)]. We may, therefore, leave this objection as being really beside the mark, and address ourselves to the former, which takes 'some' in its ordinary indefinite sense. It is true that **A** may be true when **I** is, and **E** when **O** is. But, when it is argued that, because **A** and **E** may be false together, therefore **I** and **O**, being possibly equivalent to them in fact—but not in statement—may both be false also, the question is begged. For, though

I cannot be equivalent to **A**, and, at the same time, **O** to **E**, when **A** and **E** are both false.

I and **O** are possibly equivalent in fact to **A** and **E** in some cases, they are not possibly so in others; the very use of the word 'possibly' should call attention to this. And when **A** and **E** are both false is just the very case in which **I** and **O** cannot be both respectively equal to them In that case, **I** may be equivalent to **A**, *or* **O** to **E**, but not *both* **I** to **A** *and* **O** to **E**. Mr. Stock's argument really assumes that the 'possibly' which applies to each separately applies to both together. Therefore, **I** and **O** cannot be both false at once, as long as the purely indefinite character of 'some' is preserved. Hence, the doctrine of sub-contrariety does not break down; and, consequently, the argument against the doctrine of contradiction founded on the assumption that it does, will not hold. In fact, Mr. Stock's reasoning would prove too much. For, if **I** and **O** could both be false together, then, by the doctrine of subalternation, **A** and **E** would, likewise, both be false. Thus, we should reach the sufficiently absurd result that every judgment which could possibly be made, either definitely or indefinitely, as to the relation between a given subject and predicate could be false. But, really, the doctrine of contradiction needs no proof, as it rests on two of the fundamental principles of thought—those of Contradiction and Excluded Middle. The only case in which **O** is equivalent to **E** and **I** to **A**, formally and in statement, is when the subject of each proposition is regarded as collective, and the propositions are, thus. really Singular. In that case, contrariety and contradiction merge into one, and the square of opposition (*see* § 98) becomes a straight line, as we are reduced to two alternatives between which there is no third possibility [*cf.* § 97 (ii.)]. In no case, therefore, is it true that "the doctrine of contradiction breaks down."

If I and O could be both false, then A and E would also be both false; thus, no proposition would be true.

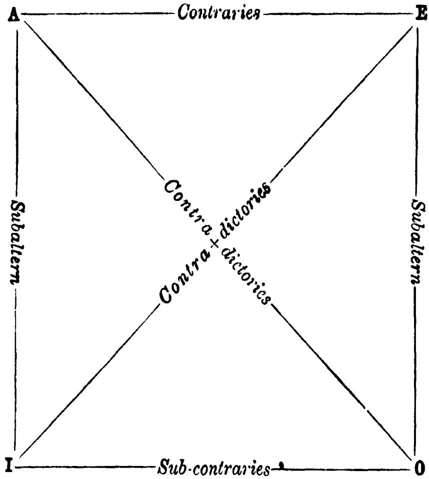
98. The Square of Opposition.

It has long been traditional in Logic to give, as an aid to remembering the doctrine of opposition, the accompanying diagram, called the Square of Opposition.

The Square of Opposition is a diagram which aids the memory in retaining the doctrine of opposition.

If this diagram, with the proper positions of the letters which symbolize the four kinds of propositions, be once firmly stamped on the mind, but little difficulty will be found in retaining in the memory the whole theory of opposition. The universals are placed at the top, the particulars at the bottom, the affirmatives on the left, and the negatives on the

BOOK III. Ch. II. right. The diagonals, as the longest lines, mark Contradiction, which is the most perfect and thoroughgoing form of logical opposition [*see* § 97 (ii.)]. The top line indicates Contrariety, and the bottom line, parallel to it, Sub-contrariety. The fact that both are horizontal naturally suggests that each connects propositions of the same quantity. The per-



pendicular lines appropriately represent Subalternation. As the diagonals run from the one top corner to the opposite bottom corner they indicate that contradictory propositions differ both in quality and quantity. Similarly, the top and bottom lines suggest a difference in quality only, and the side lines a difference in quantity only.

99. Summary of Inferences from Opposition.

Summary— We will now summarize the inferences which the doctrine of opposition enables us to draw, when we consider the results which flow from positing and sublating in turn each of the four forms of propositions :—

(1) **Posit A.** By contradiction **O** is *sublated.*

To posit **A**, sublates **O** and **E**, and posits **I**.

E is *sublated* directly by contrariety to the given proposition (**A**), and indirectly by subalternation from its contradictory (**O**).

I is *posited* directly by subalternation to the given proposition (**A**), and indirectly by sub-contrariety to its contradictory (**O**).

(2) **Sublate A.** By contradiction **O** is *posited.*

To sublate **A**, posits **O**, and leaves **E** and **I** doubtful.

E is left *doubtful;* for it is neither posited nor sublated, either directly by contrariety or indirectly by subalternation.

I is also left *doubtful;* for it is neither posited nor sublated, either directly by subalternation or indirectly by sub-contrariety.

(3) **Posit E.** By contradiction **I** is *sublated.*

To posit **E**, sublates **I** and **A**, and posits **O**.

A is *sublated*, directly by contrariety to the given proposition (**E**), and indirectly by subalternation from its contradictory (**I**).

O is *posited*, directly by subalternation to the given proposition (**E**), and indirectly by sub-contrariety to its contradictory (**I**).

(4) **Sublate E.** By contradiction **I** is *posited.*

To sublate **E**, posits **I**, and leaves **A** and **O** doubtful.

A is left *doubtful;* for it is neither posited nor sublated, either directly by contrariety or indirectly by subalternation.

O is also left *doubtful;* for it is neither posited nor sublated, either directly by subalternation or indirectly by sub-contrariety.

(5) **Posit I.** By contradiction **E** is *sublated.*

To posit **I**, sublates **E**, and leaves **A** and **O** doubtful.

A is left *doubtful;* for it is neither posited nor sublated, either directly by subalternation from the given proposition (**I**), or indirectly by contrariety to its contradictory (**E**).

O is also left *doubtful;* for it is neither posited nor sublated, either directly by sub-contrariety to the given proposition (**I**), or indirectly by sub alternation to its contradictory (**E**).

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To sublate I, posits E and O, and sublates A.

(6) **Sublate I.** By contradiction **E** is *posited.*

O is *posited*, directly by sub-contrariety to the given proposition (**I**), and indirectly by subalternation to its contradictory (**E**).

A is *sublated*, directly by subalternation from the given proposition (**I**), and indirectly by contrariety to its contradictory (**E**).

To posit O, sublates A, and leaves I and E doubtful.

(7) **Posit O.** By contradiction **A** is *sublated.*

I is left *doubtful*: for it is neither posited nor sublated, either directly by sub-contrariety or indirectly by subalternation.

E is also left *doubtful;* for it is neither posited nor sublated, either directly by subalternation or indirectly by contrariety.

To sublate O, posits A and I, and sublates E.

(8) **Sublate O.** By contradiction **A** is *posited.*

I is *posited*, directly by sub-contrariety to the given proposition (**O**), and indirectly by subalternation to its contradictory (**A**).

E is *sublated*, directly by subalternation from the given proposition (**O**), and indirectly by contrariety to its contradictory (**A**).

In the above, every result has been given twice over; for the positing a universal is the same as sublating the contradictory particular, and the sublating a universal is identical with positing its contradictory. Hence (1) and (8), (2) and (7), (3) and (6), (4) and (5) give exactly the same inferences. We see from this detailed examination that from the truth of a universal, or from the falsity of a particular, we can make definite inferences to the truth or falsity of each of the three other opposed propositions. But, from the falsity of a universal, or the truth of a particular, the only inference we can make is to the truth or falsity of the contradictory; about the other two opposed propositions we can assert nothing.

We can draw most inferences when a universal is true or a articular

It should also be noted that all the above results can be reached by a consideration of contradiction and subalternation alone, either singly or in combination.

Table of inferences from opposition.

The following Table exhibits at a glance all the above results. The kind of opposition through which they are reached is given by the letter, or letters, in brackets under each result. C means by contradiction; S, by subalternation; Cy, by Contrariety; Scy, by sub-contrariety. If any of these letters is printed in italics it means that the process it represents is indirect; that is, the result is obtained, not immediately from the given proposition, but indirectly through its contradictory.

	Given	A	O	E	I
1	**A true**		false (C)	false (Cy, *S*)	true (S, *Scy*)
2	**A false**		true (C)	doubtful	doubtful
3	**E true**	false (Cy, *S*)	true (S, *Scy*)		false (C)
4	**E false**	doubtful	doubtful		true (C)
5	**I true**	doubtful	doubtful	false (C)	
6	**I false**	false (S, *Cy*)	true (Scy, *S*)	true (C)	
7	**O true**	false (C)		doubtful	doubtful
8	**O false**	true (C)		false (S, *Cy*)	true (Scy, *S*)

The doctrine of Opposition applies to propositions in which connexion of content is prominent.

In the above treatment the traditional logic has been followed in considering the propositions under the quantified form in which denotation is the prominent aspect under which the subject is regarded. But the doctrine of opposition applies in every particular to propositions which make connexion of content the prominent element and which are the more fundamental forms of judgment in which the justification of the denotative proposition must be sought (*cf.* § 71). Thus, the Generic Judgment *S is P* is contradicted by the Modal Particular *S need not be P*, whilst it has for its contrary the Generic Judgment *S is not P*, and for its subaltern the Modal Particular *S may be P*.

100. Opposition of Hypothetical Propositions.

Hypothetical and Modal Particular Propositions stand to each other in all the relations of opposition.

The remarks made at the end of the last section apply equally well to those more definite judgments of connexion of content which are expressed in hypothetical form. The true hypotheticals : *If S is M it is P*, and *If S is M it is not P*—or expressed in the more general but less definite symbolism *If* **A** *then* **X**, and *If* **A** *then not* **X**—are universals, and correspond to the **A** and **E** categorical forms respectively; whilst the more explicit forms of the Modal Particulars : *If S is M it may be P*, and *If S is M it need not be P*—or in the wider symbolic form, *If* **A** *then perhaps* **X**, and *If* **A** *then not necessarily* **X**—correspond to the **I** and **O** categorical forms. Having thus all the four necessary forms, the whole doctrine of opposition is applicable.

Similarly with the denotative forms—or conditionals as we have ventured to call them—which give more concrete expression to the content of these abstract judgments (*see* § 76). Here the four forms are :—

If any S is M that S is always P—	corresponding	to	**A.**
If any S is M that S is never P—	"	"	**E.**
If an S is M that S is sometimes P—	"	"	**I.**
If an S is M that S is sometimes not P	"	"	**O**

As the last two forms do not imply more than that *S* being *P* is a possible consequence of its being *M*, but not that *S* actually is *P* in any one case in which it is *M*, they may be often better expressed by :—

If an S is M that S may be P—corresponding to **I**.
If an S is M that S need not be P— „ „ **O**.

'Sometimes' in the particular conditional is as indefinite as 'some' in the particular categorical.

* In *Subalternation*, the first statement of the particular conditionals, perhaps, makes the opposition more apparent, as it is evident that the statement of a universal connection between *S* being *M* and its being *P* involves the less definite particular statement. But, it must be remembered that 'sometimes' has here the same absolute indefiniteness as the ordinary logical 'some.'

The Contradictory and Contrary of a Conditional are apt to be confused.

* The *Contradictory* and *Contrary* of a conditional are more easily confused than are those of a categorical. At first sight it might seem that *If any S is M that S is P*, and *If any S is M that S is not P* are contradictories. But they are not, as they do not exhaust all possible alternatives. A reference to the table given above will show, indeed, that they are both universals, and are, therefore, contraries. The true contradictory of *If any S is M that S is P* is *If an S is M that S need not be P*, or *If an S is M that S is sometimes not P;* as 'If any country is well governed, its people are happy' is contradicted by 'If a country is well governed, its people need not be happy,' or by 'Though a country is well governed, its people are sometimes not happy.' These propositions are mutually inferrible, and fulfil all the other requirements of contradictory opposition [*see* § 97 (ii.)]. The contrary of the above proposition would, of course, be 'If any country is well governed, its people are not happy.'

In Sub-contrariety it is better to use those forms which merely imply that the consequent is a possible result from the antecedent.

* In *Sub-contrariety*, the second statement of the particular conditional is preferable, as not suggesting that *S* is actually *P* in any one case. If it be granted that *S* being *P* is a possible, though not known to be a universal or necessary, consequence of its being *M*, then it is evident that the two propositions *If an S is M that S may be P*, and *If an S is M that S need not be P* may both be true together. But they cannot

both be false; for to deny that P may be affirmed of S is to assert that it must not; and to deny that it need not is to assert that it must; and these assertions, being contradictories, cannot be true together. Therefore, the original propositions cannot both be false together.

101. Opposition of Disjunctive Propositions.

The most general symbolic form of the disjunctive proposition—*Either* **X** *or* **Y**—is most suitable to those cases in which the alternative judgments have not the same subject. A disjunctive in this form must be regarded as singular, and as, consequently, only capable of contradiction. The contradictory proposition is *Neither* **X** *nor* **Y**, and this is not itself a disjunctive judgment. But the more perfectly stated disjunctive judgments, in which several predicates are alternatively affirmed of the same subject, admit of distinctions of quantity, and propositions of opposite quality can be found which stand to them in the relations of contradiction and contrariety. Thus, with the judgment of content, *S is either P or Q,* the square of opposition can be completed by the propositions *S is neither P nor Q* (contrary); *S may be either P or Q* (subaltern); *S need not be either P or Q* (contradictory). These distinctions—as in the case of categorical judgments—stand out yet more clearly in the denotative forms of the propositions. Here we have the universal affirmative *Every S is either P or Q;* the universal negative *No S is either P or Q;* the particular affirmative *Some S's are either P or Q;* and the particular negative *Some S's are neither P nor Q.* But it will be noticed that none of the negative forms are disjunctive propositions. *S is neither P nor Q* is equally well expressed in the copulative categorical form *S is both* $\overline{P}$ *and* $\overline{Q}$ [*cf.* § 75 (i.) (*a*)], and similar propositions express the negative denotative forms. Hence, the full doctrine of opposition cannot be said to be applicable to disjunctive propositions. It may be further pointed out that the particular affirmative form is of but little value as an expression of knowledge; it is far removed from the ideal of a logical disjunctive judgment, which unfolds the differences within a system, or,

in other words, predicates a choice between the various species of one and the same genus (*cf.* § 79).

As a concrete example we may take the proposition 'Every swan is either white or black.' Its subaltern is 'Some swans are either white or black'; its contradictory 'Some swans are neither white nor black,' and its contrary 'No swan is either white or black.'

CHAPTER III.

EDUCTIONS.

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102. Chief Eductions of Categorical Propositions.

Eductions are immediate inferences of propositions whose truth is implied by a proposition accepted as true;

Eductions are those forms of Immediate Inference by which, from a given proposition, accepted as true, we educe other propositions, differing from it in subject, in predicate, or in both, whose truth is implied by it. Every Categorical Proposition gives us information of a certain subject, in terms of a certain predicate. But, each of these terms has a conceivable negative; and every categorical proposition, therefore, suggests to our minds, directly or indirectly, four terms—*S*, *P*, *non-S*, *non-P*. The problem before us is to enquire what predications about each, or any, of these possible terms are implied when *S* and *P* are connected in any given categorical judgment. In other words, whether, if we take each of these terms in turn as subject, the given proposition justifies us in predicating of it any of the other terms. We need not, of course, consider any forms of proposition in which the predicate is either the same term as the subject, or its negative—as *S is S*, *S is non-S*, *P is not P*, etc.—which are either mere tautology, or are self-contradictory and, therefore, self-destructive (*cf.* §§ 17 and 18). Our enquiry is limited to those propositions in which one term is *S* or *non-S*, and the other *P* or *non-P*.

i.e., of the predications which can be made of each of the terms S, $\bar{s}$, P, $\bar{P}$.

Each predication can be made with either a positive or a negative predicate.

Now, when any one of these four terms is taken as subject, we have two possible predicates offered to us; thus, we can predicate either *P* or *non-P* of *S*, and either *S* or *non-S* of *P*. This leads us to the kind of Eduction called *Obversion*, in which we retain the same subject but negative the predicate

of the original proposition. Again, if S is the subject, and P the predicate, of the given proposition, we can form other propositions whose subjects are respectively, P, *non-P*, and *non-S*, and each of these propositions can take two forms, one of which is derived from the other by obversion. Thus we get the following possible modes of inference, most of which involves a change, not only in the verbal expression but, in the form of the judgment as thought:—

The four kinds of Eductions are: (1) *Obversion.* (2) *Conversion.* (3) *Contraposition.* (4) *Inversion.*

(1) *Obversion*—when the subject of the original proposition is unchanged, but the predicate is negatived.
(2) *Conversion*—when the subject of the inferred proposition is P, and its predicate S or *non-S*.
(3) *Contraposition*—when the subject of the inferred proposition is *non-P* and its predicate S or *non-S*.
(4) *Inversion*—when the subject of the inferred proposition is *non-S*, and its predicate P or *non-P*.

None of these can be valid inferences from any given proposition, unless the inferred proposition is involved in, and expresses the same truth as, that proposition itself expresses. We must, therefore, by careful examination, see which of them are justified by propositions of each of the four forms, **A**, **E**, **I**, **O**.

Each of the three last has two forms, each of which is the obverse of the other.

Each of the inferences (2), (3) and (4) in the above list can take two forms, one with a positive, and the other with a negative predicate. Each of these forms is obtainable from the other by the process of obversion. As, however, the simplest forms are those which have the positive predicates, the simple names, *Conversion*, *Contraposition*, and *Inversion*, are applied to the processes by which they are arrived at. Those propositions themselves are called the *Converse*, *Contrapositive*, and *Inverse*, of the original proposition; whilst the corresponding forms with negative predicates are termed the *Obverted Converse*, the *Obverted Contrapositive*, and the *Obverted Inverse*, respectively, of that proposition. Thus, each of these names expresses the relation in which that derived proposition stands to the given one.

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If we now use $\bar{S}$ and $\bar{P}$ to denote *non-S* and *non-P* respectively, we have the following empty schema of possible Eductions from categorical propositions :—

Table of possible Eductions.

i.		1	Original Proposition . . .	$S—P$
		2	Obverse of (1)	$S—\bar{P}$
ii.	Converses of (1) . .	3	Converse of (1)	$P—S$
		4	Obverted Converse of (1) . .	$P—\bar{S}$
iii.	Contrapositives of (1)	5	Contrapositive of (1) . . .	$\bar{P}—S$
		6	Obverted Contrapositive of (1)	$\bar{P}—\bar{S}$
iv.	Inverses of (1) . .	7	Inverse of (1)	$\bar{S}—P$
		8	Obverted Inverse of (1) . .	$\bar{S}—\bar{P}$

We have now to see to what extent this empty schema can be filled out by either of the four kinds of categorical predication—**A**, **E**, **I**, **O**,—when the original proposition itself is of either of those forms.

These inferences are useful as bringing out all the implications of the original predication.

Many of the inferred forms are unusual and unnatural modes of expressing the truth which is stated most simply in the original proposition. Those of them, too, which contain negative terms are open, as primary modes of statement, to the objections made to propositions containing those terms in § 29. But, when they are regarded as simply secondary modes of expressing the content of the original judgment, they are useful; as they make prominent a fresh side of the truth there enunciated. And the whole of them together, by placing that assertion in every possible light, make its implications much clearer and more definite than a mere consideration of the proposition itself would do.

As Obversion and Conversion are the primary modes by

which these eductions are made—for all the other inferences are obtainable by combinations of these—a detailed consideration of them should precede that of the other forms.

Obversion and Conversion are the primary modes of Eduction.

(i.) **Obversion is a change in the quality of a predication made of any given subject, whilst the import of the judgment remains unchanged.** The original proposition is called the *Obvertend*, and that which is inferred from it is termed the *Obverse.*

Obversion is the changing the quality of a proposition, but neither its subject nor its import.

Obvertend—the original proposition. *Obverse*—the inferred proposition.

Whenever we assert anything we, by implication, deny the opposite. That is, the affirmation of any predicate of a certain subject implies the denial of its negative; and the denial of any predicate implies the affirmation of its negative. The former of these follows from the Principle of Contradiction—for, if any S is P it cannot be *non-P* (*see* § 18); and the latter from that of Excluded Middle—for, if any S is not P it must be *non-P* (*see* § 19). All obversions of affirmative propositions, therefore, depend on the former of these two principles; and all obversions of negative propositions on the latter. But, to deny a negative is to affirm, for two negatives destroy each other; and to affirm a negative is to deny; and, thus, obversion involves no change of meaning. The matter, therefore, which is expressed by an affirmative proposition can always be re-expressed by a negative, and *vice versâ.* This is, however, a mere change in the mode of expression; it involves no process of thought, and consequently is not a real inference. It is, however, useful as a first step in contraposition.

Obversion of Affirmatives rests on Principle of Contradiction; of Negatives on that of Excluded Middle.

From this it follows that the subject of the obverse is the same as the subject of the obvertend in every respect, as, otherwise, we should not have a true denial of the opposite of that obvertend. The *quantity* of the two propositions is, therefore, the same. The predicate of the obverse is the negative of that of the obvertend, and this, to avoid alteration in meaning, necessitates a change in the *quality* of the proposition. This gives us the one simple rule for obverting any proposition :—

Rule for Obversion—Negative the predicate, change quality.

Negative the predicate and change the quality, but leave the quantity unaltered.

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A and E, I and O are pairs of mutual obverses.

Applying this rule to the four forms of categorical propositions, we find that

A *obverts to* **E**, **E** *to* **A**, **I** *to* **O**, *and* **O** *to* **I**;

or, expressed symbolically

Table of Obversions.

Original Proposition	$S\ a\ P$	$S\ e\ P$	$S\ i\ P$	$S\ o\ P$
Obverse	$S\ e\ \bar{P}$	$S\ a\ \bar{P}$	$S\ o\ \bar{P}$	$S\ i\ \bar{P}$

It must be remembered that obversion is a reciprocal process, and, thus, that $S\ a\ P$ is as much the obverse of $S\ e\ \bar{P}$, as the latter is the obverse of the former.

Examples of Obversion.

As material examples we may give the following pairs of propositions, each member of every pair being the obverse of the other member :—

{**A**. *All men are mortal.*
{**E**. *No men are not-mortal.*
{**E**. *No thoughtful men are superstitious.*
{**A**. *All thoughtful men are non-superstitious.*
{**I**. *Some men are happy.*
{**O**. *Some men are not not-happy.*
{**O**. *Some men are not rich.*
{**I**. *Some men are not-rich.*

The formal negative term may be replaced in an obverse by a material negative, or privative, term, only when that term is exactly equivalent to the formal negative.

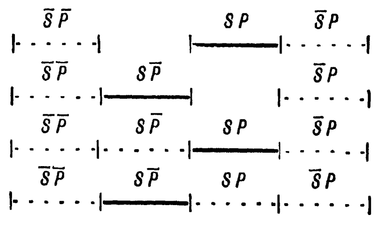
* We may often write the obverse in a form more in accordance with the usages of ordinary speech by using a material contradictory, or a privative, term [*see* § 29 (i.) (ii.)], instead of the formal negative, for the new predicate. But, unless this term is exactly equivalent in meaning to the formal negative, we do not make a true obversion by its use. For instance, in obverting **A** as given above, we could say 'No men are immortal,' for 'immortal' and 'not-mortal' exactly correspond. But we could not give 'Some men are not unhappy' as the obverse of 'Some men are happy'; for 'happy' and 'unhappy' do not exhaust all possibilities, and, thus, the principle of contradiction does not apply to them. It is true that this proposition is justified by the given one, for

'not-happy' includes unhappy, as well as all other shades of departure from 'happy.' But it is not the obverse; for we cannot get back from it, by obversion, to our original proposition. The same holds in the case of all affirmative propositions; the obverse justifies the denial of all terms which can be brought under the formal negative. But even this is not justifiable in the case of the obversion of negative propositions. From 'Some men are not happy' we cannot conclude that 'Some men are unhappy,' for this latter proposition asserts, not merely the absence of happiness, but the presence of a certain amount of positive misery [*cf.* § 29 (i.)]. Still less can we infer from 'Some men are not rich' that 'Some men are poor'; for 'rich' and 'poor' are contraries, and there are many intermediate stages between them. Obversion is, in short, a formal process; and, therefore, if we do not use a formal negative term for our new predicate, we must make sure that the term we do use is the exact equivalent of that formal negative.

The obverse of a proposition can be gathered immediately from its diagram.

The results of obversion can be immediately gathered from an inspection of the diagrams given in § 94. As we are dealing with propositions involving *non-P*, it is better to use the fuller set of diagrams given first in that section :—

[A]
[E]
[I]
[O]



In the diagram for **A**, $\bar{P}$ only occurs in combination with $\bar{S}$, therefore *No S is* $\bar{P}$. In that for **E**, S occurs only in the class $S\bar{P}$; hence, *All S is* $\bar{P}$. In that for **I**, *Some S*, at least, *is not* $\bar{P}$, for it is P. Lastly, in that for **O**, *Some S is* $\bar{P}$ is given immediately.

This gives a fresh illustration of the implications of existence.

* The same diagrams also, when thus applied to obversion, illustrate afresh the view of existential import adopted in § 89. For, in the diagrams for **A** and **I**, wherever $\bar{P}$ occurs its existence is marked by the dotted line which implies doubt; therefore in the negative propositions, $S\ e\ \bar{P}$ and $S\ o\ \bar{P}$, which are the respective obverses, the existence of the predicate in the same sphere as the subject is not assured. But when we affirm $S\ a\ \bar{P}$ and $S\ i\ \bar{P}$ as the respective obverses of **E** and **O**, our diagrams show us that the existence of these classes is certain. And this must needs be so; for, if the negative propositions, $S\ e\ P$ and $S\ o\ P$, do not ensure P in the same universe as S, then, as, by the principle of Excluded Middle (*see* § 19), every S must be either P or $\bar{P}$, even if P does not exist at all in that universe, yet $\bar{P}$ must.

An examination of Euler's circles (*see* § 91) will also give the obverses of **A**, **E**, **I**, **O**, though, as each of those propositions, except **E**, requires more than one diagram, the results are not so immediately manifest.

Several other names have been given to obversion.

Obversion has been called Permutation (by Fowler, Ray, and Stock); Æquipollence (by Ueberweg, Bowen, and Ray); Infinitation (by Bowen); Immediate Inference by Privative Conception (by Jevons); Contraversion (by De Morgan); and Contraposition (by Spalding). But Obversion is the most usual name, and is adopted by the majority of writers either by itself, or (as in the case of Ueberweg, Ray, Stock, and Jevons) as synonymous with one of the other names.

The so-called *Material Obversion* requires a reference to the matter of the obverse proposition,

* **Material Obversion**. Professor Bain considers that, in addition to the formal process we have been considering, "there are Obverse Inferences justified only on an examina-"tion of the matter of the proposition. From 'warmth is "agreeable' we can affirm, by formal obversion, 'warmth is "not disagreeable, and not indifferent.' We cannot affirm, "without an examination of the subject matter, 'cold is dis-"agreeable.' . . . Experience teaches us that in an actual "state of pleasurable warmth, the sudden change to cold is "also a change to the disagreeable. Whenever an agent is "giving us pleasure in act, the abrupt withdrawal of that

"agent is a positive cause of pain. On the faith of this in-"duction, we can obvert materially a large number of propo-"sitions regarding pleasure and pain, good and evil" (*Ded. Log.*, pp. 111-2).

But to call this obversion is unusual. The new proposition has not the same subject as the old, but a negative of that subject. It is not derived in any way from the original proposition, but, as Prof. Bain himself says, rests on the strength of an induction quite outside it. A proposition may point out to us what to examine; it may suggest a possible result, and this result may be found to agree with reality. Thus 'warmth is agreeable' may suggest that 'the opposite of warmth is the opposite of agreeable,' but we cannot infer the latter proposition from the former. In fact, it is quite conceivable that two opposite subjects should yet have the same predicate; for two opposite states may both be agreeable, or the reverse. For example, because 'Light is beneficial' it does not follow that 'Darkness is harmful,' nor does the agreeableness of exercise postulate the painfulness of rest.

changes the subject of the obvertend, and is not an inference from it.

(ii.) **Conversion is the eduction of one proposition from another by transposing the terms.** The original proposition is called the *Convertend*, and that which is derived from it is named the *Converse*.

Conversion is the inference of one proposition from another, by transposing the terms. *Convertend*—the original proposition. *Converse*—the inferred proposition.

We have, evidently, here a complete alteration of standpoint, as we have changed the subject or nucleus of our proposition. The predication is now made of P in terms of S, whereas the original proposition contained an assertion about S in terms of P. Moreover, the truth of the converse follows directly from that of the convertend. Hence, the process is a real interpretative inference. Every proposition before being converted—or, indeed, used in any kind of formal inference—must be reduced to the strict logical form, *S is P* or *S is not P* (*cf.* § 68), and the *whole* predicate must change places with the *whole* subject. For instance, the converse of 'Every old man has been a boy' is not 'Every boy has been an old man,' but 'Some who have been boys are old men'; for the original proposition, in its logical form, is 'Every old

man is a person who has been a boy.' As the converse simply makes the same assertion as the convertend, looked at, as it were, from the other side, it is clear that the *quality* of both propositions will be the same.

Conversion does not change the quality of the proposition,

Every act of conversion involves reading the original predicate in its denotation, in order that it may be made a subject-term. That we really do make this change from a connotative to a denotative view is shown by the fact that, if the predicate of the convertend is an adjective—as in 'No crows are white'—a substantive must be supplied before we can use that term as the subject of the converse—as 'No white things are crows.' This involves a consideration of the distribution of the predicate (*see* § 72) in order that the converse may not assert more than is justified by the convertend; and may necessitate a change of *quantity*. In other words, a mere transposition of terms is not always permissible; we cannot go from 'All cats are animals' to 'All animals are cats.' The only conversion we are concerned with is *Illative Conversion*; that is, conversion which is a valid inference, and in which either both convertend and converse are true, or both are false. Such conversion must obey these two rules :—

but may change its quantity.

Conversion which is a valid inference is called *Illative*.

Rules for Conversion—Retain quality; distribute no term not given as distributed.

1. *The quality of the proposition must remain unchanged.*
2. *No term may be distributed in the converse which is not distributed in the convertend.*

We must now apply these rules to the conversion of each kind of categorical proposition.

(*a*) *Conversion of* **A**. In the proposition $S\ a\ P$, whilst S is distributed, P is not. We cannot, therefore, convert to $P\ a\ S$ —for that would break Rule 2—but we must retain P in its undistributed condition, and write the converse $P\ i\ S$. Hence **A** converts to **I**, and the conversion involves a change of quantity from universal to particular. Such conversion was called by Aristotle κατὰ μέρος or *partitive* conversion. This name has, however, given place to the less descriptive one of *conversio per accidens* or *conversion by limitation.* Though the necessity for this mode of converting **A** propositions is obvious

A converts to **I**.

This conversion is called *per accidens*.

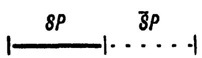
A is often improperly converted.

enough when the rules for conversion are kept in mind, yet the improper conversion of **A** propositions is one of the most frequent causes of fallacy. Because it is a fairly well established fact that all very clever persons have large brains, an abnormally large cranium is often held to be a sign of great ability. As lazy persons are often out of work, people jump to the conclusion that if a man is often out of work he is necessarily lazy. Since the wages of unskilled labour in England are low, it is frequently assumed that all badly paid persons are unskilful. Because all pious people go regularly to church, regular church-going is commonly regarded as a sure sign of piety. Such mistakes are continually made, yet they are on a par with arguing that every animal is a monkey because every monkey is an animal. No doubt, in some cases—as tautologous propositions and definitions, or when both subject and predicate are singular names—the simple converse, *i.e.*, converse without change of quantity, of **A** would give a true proposition. 'Every equiangular triangle is equilateral' is as true a proposition as is 'Every equilateral triangle is equiangular.' But its truth has to be established by a separate and independent demonstration; it cannot be inferred from the latter proposition by conversion. For conversion, as a formal process of inference, must be applicable to every proposition of the same kind; there cannot be two modes of formally converting **A** propositions. When the simple converse would be true in fact, it is because of special circumstances which do not appear in the statement of the convertend. Hence, as $P\ i\ S$ is the only converse which is materially true in all cases, and is formally true in any, that is the logical converse of $S\ a\ P$. For, whilst $S\ a\ P$ asserts positively that the attribute which P denotes is found in every S, it is not stated whether, or not, it is found in other cases.

The simple converse of A would sometimes be a true proposition, but its truth must be established independently.

That A must be converted per accidens is evident from its diagram.

This is obvious from an inspection of the diagram for an **A** proposition



where it is plain that we cannot say of *All P* that it is S; for the existence of the class $\bar{S}P$ is shown to be possible. The

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figure also shows that the converse is as real a proposition as the convertend; if the subject and predicate exist in the latter, they equally exist in the former.

E converts *simply* to E.

(*b*) *Conversion of* **E**. An **E** proposition can be converted *simply*; that is, without change of quantity. For, $S\ e\ P$ asserts that the attributes connoted by P are found in none of the objects which S denotes, but only in other objects. Hence, none of the objects in which P is found, and which are all denoted by P used as a substantive name, possess the attributes which are connoted by S. The separation between the things which are S and those which possess the attribute P is total and absolute; and is, therefore, reciprocal. Whether we regard it from the side of S or of P, each individual S differs from each individual P.

Thus, we can convert $S\ e\ P$ to $P\ e\ S$. If 'No horses are carnivorous,' it follows that 'No carnivorous animals are horses.'

This is obvious from the diagram.

This is plain from the diagram for an **E** proposition.

$S\bar{P}$ $\bar{S}P$

* This diagram draws attention to another point—that we have no assurance of the existence of the subject of the converse in the universe of discourse fixed by the convertend. This is a necessary outcome of the view that the predicate of a negative proposition is not necessarily existent in the sphere to which that proposition refers (*see* § 89). But, if an inference is valid, the inferred proposition must refer to the same sphere as the original proposition, and must be true in that sphere if the proposition from which it is deduced is true. Moreover, its truth must be justified solely by the given proposition itself, without any information external to that proposition, or it ceases to be a formal inference from that proposition. But, in the case of the conversion of **E** we cannot be sure that, when the convertend is true, the converse is also true, unless we know from other and material considerations that the predicate of

The truth of the converse of E is conditional upon the existence of P; if stated as a categorical proposition it is, therefore, an invalid inference;

the convertend belongs to, and exists in, the same sphere as the subject. Of course, in most material examples, both the terms of the convertend are known to refer to the same sphere, and then simple conversion gives a true proposition. But this is not an inference from the convertend alone, but from the convertend interpreted in a particular way by information external to itself. It follows, as the formal process must apply in all cases and must not travel outside the given proposition, that the conversion of **E** is an invalid process, if the converse is stated as a categorical proposition. The formally correct statement of the converse is, therefore, conditional as regards the existence of *P*—*If any P exists, it is not S.* For example, 'No woman is now hanged for theft in England' converts simply to 'Nobody now hanged for theft in England is a woman.' But, as a matter of fact, nobody—man, woman or child—is now hanged in England for that crime; though the converse, thus stated, must be regarded as asserting that some thieves are so punished, for it implies the existence of its subject in the sphere to which the convertend belongs—that is, the sphere of actual physical reality. The true statement would be 'If any person is now hanged for theft in England, that person is not a woman.' And, as a statement in this conditional form is the only one which is true in all cases, it is the only formal inference which can be drawn, by conversion, from an **E** proposition.

Its formally correct statement is ***If any P exists, it is not S.***

(*c*) *Conversion of* **I**. As neither term in an **I** proposition is distributed, it is clear that, by converting it simply, we shall break neither of the rules of conversion. Thus, *S i P* converts to *P i S*, and the proposition remains particular. 'Some herbs are poisonous' gives as a converse 'Some poisonous things are herbs.' The 'some' remains, of course, purely indefinite; and when we speak of the simple conversion of **I** we do not mean that 'some' denotes the same proportion of the total denotation of the subjects of both convertend and converse. When the subject of the convertend is a genus of which the predicate is a species, the simple converse reads somewhat awkwardly. Thus, 'Some human beings are boys' converts

I converts simply to I.

to 'Some boys are human beings,' which, we feel, is not so definite an assertion as our knowledge of the matter would warrant us in making. This is particularly noticeable when we reconvert the converse of an **A** proposition. The converse of $S\ a\ P$ is $P\ i\ S$, and we can only convert this again to $S\ i\ P$, where the double logical process has led to a loss of fulness in the statement. For example, 'All monkeys are animals' converts to 'Some animals are monkeys, and the simple converse of this is 'Some monkeys are animals.' This shows that conversion *per accidens* is not a reciprocal process, as simple conversion is. But, no matter what the **I** proposition is, or whence it is derived, it can, by itself, only justify us in deducing another **I** proposition as its converse.

Simple conversion is reciprocal, but conversion *per accidens* is not.

The diagram for **I** shows that it is convertible simply.

That **I** is simply convertible is immediately evident on an inspection of the diagram

$$\begin{array}{ccc} S\bar{P} & SP & \bar{S}P \end{array}$$

|- - - - - -|————|- - - - - -|

It is plain that the assertion about P is indefinite in the same way as is the assertion about S. *Some* P, at least, is S, but the diagram can say nothing positive about *All* P.

O cannot be converted.

(*d*) *Conversion of* **O**. As the predicate of an **O** proposition is distributed, but the subject undistributed (*see* § 72), we cannot convert a proposition of that form at all. For, by Rule 1, $S\ o\ P$ must convert to a negative proposition with S for its predicate. This would distribute S; but Rule 2 forbids this distribution, as S is not distributed in the convertend. $S\ o\ P$ asserts that Some S's have not the attribute P, but it says nothing about the other possible S's. Hence, though the *Some* S's which form the subject are entirely separated from all those things which possess the attribute P, it does not follow that these latter are excluded from *all* the S's. It is possible that every P is S, though there are other instances of S as well (*cf.* Fig. III, § 91) which are not P. For example, 'Some men are not honest' will not justify us in inferring that 'Some honest beings are not men'; nor can we say that some who pass an examination

do not sit for it, because it is true that some who sit for an examination do not pass. In many cases, no doubt, the simple converse of an **O** proposition would be materially true ; thus 'Some men are not black' and 'Some black things are not men' are both true propositions, but neither can be inferred by formal conversion from the other, for neither statement is justified by the other.

The diagram for **O** shows its inconvertibility.

The inconvertibility of **O** is evident from the diagram

$S\bar{P}$ SP $\bar{S}P$

|————|- - - - - -|- - - - - -|

This shows clearly that we can make no definite assertion about any part of P in terms of S ; for P is entirely represented by the dotted line which signifies uncertainty.

The doctrine of conversion can also be traced out from Euler's circles (*see* § 91), though the plurality of diagrams required for every proposition, except **E**, makes the process somewhat complex.

Summary.

To sum up the results we have obtained :—

A *converts per accidens ;* **E** *and* **I**, *simply ;* **O**, *not at all.*

Several logicians have attempted to furnish proofs of the validity of conversion. These have all taken the indirect form of a *reductio ad absurdum*, that is of showing that the assumption of the contradictory of the converse leads to results inconsistent with the convertend. But as the process is an immediate application of the formal laws of thought (*see* §§ 17-19), it is really a primary one, and as such does not require proof.

By obverting the converse we get a new inference from a proposition.

(*c*) *Obverted Conversion.* As any categorical proposition whatever can be obverted, we can get a new inference from the original proposition by obverting the converse, according to the rules given in sub-section (i.). Thus, expressed symbolically, we get :—

Table of Conversions.

1.	Original Proposition ...	$S\ a\ P$	$S\ e\ P$	$S\ i\ P$	$S\ o\ P$
2.	Converse of (1) ...	$P\ i\ S$	$P\ e\ S$	$P\ i\ S$	(None)
3.	Obverted Converse of (1)	$P\ o\ \bar{S}$	$P\ a\ \bar{S}$	$P\ o\ \bar{S}$	(None)

Examples of Conversion.

As material examples we may give :—

Original Proposition	A	*Every truthful man is trusted.*
Converse	I	*Some trusted men are truthful.*
Obverted Converse	O	*Some trusted men are not untruthful.*

Original Proposition	E	*No cultivated district is uninhabited.*
Converse	E	*No uninhabited district is cultivated.*
Obverted Converse	A	*All uninhabited districts are uncultivated.*

Original Proposition	I	*Some British subjects are dishonest.*
Converse	I	*Some dishonest people are British subjects.*
Obverted Converse	O	*Some dishonest people are not aliens.*

We have purposely chosen examples in which the negative predicate of the obverted converse can be expressed by a material negative, or by a privative word. This, of course, cannot always be done ; and then the derived proposition is frequently awkward in expression.

Contraposition is the inferring a proposition with $\bar{P}$ for subject.

(iii.) **Contraposition is the inferring, from a given proposition, another proposition whose subject is the contradictory of the predicate of the original proposition.** The derived proposition is called the *Contrapositive;* there is no corresponding distinctive name for the original proposition.

Contrapositive—the inferred proposition.

The contrapositive of any given proposition is most easily arrived at indirectly. It makes a predication about the contradictory of the predicate of the given proposition. Now, this contradictory appears as the predicate of the obverse of that proposition. If, then, this obverse can be converted it gives a proposition of the form required, in which the negative of the original predicate is the subject, and the subject of the original proposition is the predicate. Hence, the simple rule for contraposition is :—

Rule for Contraposition—First obvert, then convert.

First obvert, then convert.

Contraposition changes quality,

This will give, in every case, a proposition differing in *quality* from the original one; for obversion changes the quality, and conversion does not change it back again. But the *quantity* remains unchanged, except in the case of the contraposition of **E**; for, obversion does not change quantity, and, therefore, any change in quantity must be due to the subsequent conversion. Now, as **A** and **O** obvert to **E** and **I**

but leaves quantity unaltered, except in the case of E.

respectively, and both of these convert simply, the quantity will remain unaltered. But **E** obverts to **A**, which can only be converted *per accidens*, and, hence the contrapositive of the universal negative is a particular affirmative. Thus, comparing the contraposition with the conversion of universal propositions in respect to quantity, it is seen that when the one inference causes a change in quantity, the other does not, and *vice versâ*. As **I** obverts to **O**, which cannot be converted, there can be no contrapositive of **I**.

I cannot be contraposited.

Contraposition is sometimes called *Conversion by Negation*, and, as we see, it can be applied to **O** propositions, and is the only form of 'conversion' which can be so applied. But, it is better not to use 'conversion' in this sense, as the contrapositive has not the same subject as the converse, and also differs from it in quality.

Contraposition is sometimes called *Conversion by Negation*, but the name is not appropriate.

Obverted Contraposition.—Having obtained the contrapositive of any proposition we can obvert it, and thus get a proposition of the same quality as the original one. This *Obverted Contrapositive* has for each of its terms the contradictory of a term in the given proposition—its subject is the negative of the original predicate, and its predicate the negative of the original subject. Some writers have confined the name Contrapositive to this form. The older logicians all did this, as they held that contraposition, being a kind of conversion, should not change the quality of the given proposition. There seems, however, to be no reason for thus restricting the application of the name. Both forms are contrapositives, and, when we wish to distinguish them, we call the simpler—that is, the one which retains one of the original terms—the contrapositive, whilst the proposition derived from that by obversion is fitly named the obverted contrapositive.

The *Obverted Contrapositive* has the same quality as the original proposition. The name Contrapositive has sometimes been needlessly confined to this form.

We get, then, the following results, expressed symbolically:—

Table of Contrapositives.

1	Original Proposition	$S\ a\ P$	$S\ e\ P$	$S\ i\ P$	$S\ o\ P$
2	[Obverse of (1)]	$[S\ e\ \bar{P}]$	$[S\ a\ \bar{P}]$	$[S\ o\ \bar{P}]$	$[S\ i\ \bar{P}]$
3	Contrapositive of (1)	$\bar{P}\ e\ S$	$\bar{P}\ i\ S$	(None)	$\bar{P}\ i\ S$
4	Obverted Contrapositive of (1)	$\bar{P}\ a\ \bar{S}$	$\bar{P}\ o\ \bar{S}$	(None)	$\bar{P}\ o\ \bar{S}$

Examples of Contraposition.

As material examples we may give :—

Original Proposition	A	*Every poison is capable of destroying life.*
[Obverse]	[E]	[*No poison is incapable of destroying life.*]
Contrapositive	E	*Nothing incapable of destroying life is poisonous.*
Obvd. Contrapositive	A	*Everything incapable of destroying life is non-poisonous.*

Original Proposition	E	*No lazy person is deserving of success.*
[Obverse]	[A]	[*Every lazy person is undeserving of success.*]
Contrapositive	I	*Some people undeserving of success are lazy.*
Obvd. Contrapositive	O	*Some people undeserving of success are not not-lazy.*

Original Proposition	O	*Some unjust laws are not repealed.*
[Obverse]	[I]	[*Some unjust laws are unrepealed.*]
Contrapositive	I	*Some unrepealed laws are unjust.*
Obvd. Contrapositive	O	*Some unrepealed laws are not just.*

We have carefully chosen instances where we can use terms equivalent in meaning to the formal negatives, in order that the resultant propositions might not be too far removed from the usages of ordinary speech. When we have to use formal negative terms these eductions often result in strained and unnatural modes of expression. For example, the obverted contrapositive of 'No plants feed' is 'Some non-feeders are not non-plants.'

The great value of contraposition is this. The aim of science is to teach propositions which are in fact reciprocal. In such propositions the predicate is stated so definitely that it is strictly characteristic of the subject, that is, it belongs in exactly that form to nothing else, and the knowledge expressed by the proposition is, therefore, of the most precise form attainable. When then $S\ a\ P$ is established, we want to know if $P\ a\ S$ is also true; and the readiest way to establish this is generally to examine cases of $\bar{S}$ and endeavour to establish the proposition $\bar{S}\ e\ P$ which is the contrapositive of $P\ a\ S$. The importance of this will appear more clearly in the discussion of Induction in Book V.

(iv.) **Inversion is the inferring, from a given proposition, another proposition whose subject is the contradictory of the subject of the original proposition.** The given proposition is called the *Invertend*, that which is inferred from it is termed the *Inverse.*

Inversion is the inferring a proposition with $\bar{S}$ for its subject. *Invertend*—the given proposition. *Inverse*—the inferred proposition.

The inverse of any given proposition is most easily arrived at indirectly, through some of the forms of eduction we have already considered. We can only obtain the contradictory of a term by obverting the proposition of which that term forms the predicate. S must, therefore, have been made the predicate of a proposition, and then that proposition must have been obverted for us to get *non*-S. Two eductions—the obverted converse and the obverted contrapositive—satisfy these conditions. If, then, we can convert either of these we have an Inverse. Hence the rule for Inversion is :—

The inverse is most easily reached indirectly.

Convert either the Obverted Converse or the Obverted Contrapositive.

Rule for Inversion :— Convert either the Obverted Converse or the Obverted Contrapositive.

In the case of **A** the obverted converse is $P\ o\ \bar{S}$ [*see* sub-§ (ii.) (*e*)], and this is inconvertible. But the obverted contrapositive is $\bar{P}\ a\ \bar{S}$ [*see* sub-§ (iii.)], which can be converted to $\bar{S}\ i\ \bar{P}$. As both the terms of this proposition are contradictories of those which appear in the original proposition, it is not the simple, but the obverted, inverse. As, however, obversion is a reciprocal process, we can obvert this to $\bar{S}\ o\ P$ which gives the simple inverse.

$S\ a\ P$ inverts to $\bar{S}\ o\ P$ by converting the Obvd. Contrap. and obverting the result.

In the case of **E**, the obverted converse is $P\ a\ \bar{S}$ [*see* sub-§ (ii.) (*e*)], which, by conversion, gives the inverse $\bar{S}\ i\ P$; this we can obvert to $\bar{S}\ o\ \bar{P}$, which is the obverted inverse.

$S\ e\ P$ inverts to $\bar{S}\ i\ P$ by converting the Obvd. Converse.

In the case of **I**, the obverted converse is $P\ o\ \bar{S}$ [*see* sub-§ (ii.) (*e*)], which cannot be converted ; and it has no obverted contrapositive [*see* sub-§ (iii.)], and, therefore, it can have no inverse.

$S\ i\ P$ has no inverse.

In the case of **O**, there is no obverted converse [*see* sub-§ (ii.) (*e*)], and the obverted contrapositive is $\bar{P}\ o\ \bar{S}$ [*see* sub-§ (iii.)], which cannot be converted ; **O** has, therefore, no inverse.

$S\ o\ P$ has no inverse.

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So we may sum up the possible inverses symbolically thus :—

Table of Inverses.

1	Original Proposition - -	$S\ a\ P$	$S\ e\ P$	$S\ i\ P$	$S\ o\ P$
2	[Obverted Converse of (1)] -		$[P\ a\ \bar{S}]$		
3	[Obverted Contrapositive of (1)]	$[\bar{P}\ a\ \bar{S}]$			
4	Inverse of (1) - - - -	$\bar{S}\ o\ P$	$\bar{S}\ i\ P$	(None)	(None)
5	Obverted Inverse of (1) - -	$\bar{S}\ i\ \bar{P}$	$\bar{S}\ o\ \bar{P}$	(None)	(None)

N.B.—Only the eduction from which each inverse is immediately obtained is given. The blank spaces, therefore, simply mean that, if that particular eduction exists, the inverse cannot be derived from it.

Examples of Inversion.

As material examples we may give :—

Original Proposition - **A** - *Every truthful man is trusted.*
[Obvd. Contrapositive] - [**A**] - [*Every not-trusted man is untruthful.*]
Inverse - - - - **O** - *Some untruthful men are not trusted.*
Obvd. Inverse - - **I** - *Some untruthful men are not-trusted.*

Original Proposition - **E** - *No unjust act is worthy of praise.*
[Obvd. Converse] - [**A**] - [*Every act worthy of praise is just.*]
Inverse - - - **I** - *Some just acts are worthy of praise.*
Obvd. Inverse - - **O** - *Some just acts are not unworthy of praise.*

The truth of the inverse of **A** is conditional upon the existence of $\bar{S}$ and $\bar{P}$; that of the inverse of **E** upon $\bar{S}$ and P; if the inferences are stated in a categorical form, they are invalid.

* As the inverse of **A** in its affirmative form—*i.e.* the obverted inverse—involves both the terms $\bar{S}$ and $\bar{P}$, and as the inverse of **E** involves $\bar{S}$ and P, none of which terms are guaranteed to exist by the original propositions, it follows that these eductions must be merely conditional as regards the existence of both their terms. Thus, the only meaning of the inverse of **A** is, *If $\bar{S}$ and $\bar{P}$ both exist, then some $\bar{S}$ is $\bar{P}$*, and that of **E** is, *If $\bar{S}$ and P both exist, then some $\bar{S}$ is P.* Hence, an inverse from a true proposition is not necessarily true when stated categorically, but only when stated as conditional upon the existence of both the subject and the predicate of the inferred proposition, and this conditional form must be regarded as the true formal inverse.

103. Summary of Chief Eductions.

Summary of Eductions.

In the last section we examined in detail how far, from a given proposition of one of the forms **A, E, I, O,** we can infer propositions in which predications are made about each of the terms of the original proposition and the contradictories of those terms. In other words, we have investigated to what extent it is possible to fill up the empty schema given in the early part of that section. We have also discussed to what extent the truth of the derived propositions is conditional upon the existence of classes denoted by certain terms whose existence is not necessarily implied by the original propositions.

We see that from any universal proposition a predication, categorical or conditional, can be deduced about each of the terms $S, \bar{S}, P, \bar{P}$; but from a particular we can only infer predications about S and one of the two terms P and $\bar{P}$. It may be noted that the converses of **A** are the same as those of **I,** and the contrapositives of **E** the same as those of **O.**

These results may be thus tabulated :—

Complete Table of Eductions of Categorical Propositions

			A	E	I	O
i.	1	Original Proposition - -	$S\ a\ P$	$S\ e\ P$	$S\ i\ P$	$S\ o\ P$
	2	Obverse of (1) - - -	$S\ e\ \bar{P}$	$S\ a\ \bar{P}$	$S\ o\ \bar{P}$	$S\ i\ \bar{P}$
ii.	3	Converse of (1) - - -	$P\ i\ S$	$P\ e\ S$[1]	$P\ i\ S$	
	4	Obverted Converse of (1) -	$P\ o\ \bar{S}$	$P\ a\ \bar{S}$[1]	$P\ o\ \bar{S}$	
iii.	5	Contrapositive of (1) - -	$\bar{P}\ e\ S$[2]	$\bar{P}\ i\ S$		$\bar{P}\ i\ S$
	6	Obverted Contrapositive of (1)	$\bar{P}\ a\ \bar{S}$[2]	$\bar{P}\ o\ \bar{S}$		$\bar{P}\ o\ \bar{S}$
iv.	7	Inverse of (1) - - -	$\bar{S}\ o\ P$[3]	$\bar{S}\ i\ P$[4]		
	8	Obverted Inverse of (1) -	$\bar{S}\ i\ \bar{P}$[3]	$\bar{S}\ o\ \bar{P}$[4]		

[1] *Conditional upon the existence of* P.
[2] *Conditional upon the existence of* $\bar{P}$.
[3] *Conditional upon the existence of* $\bar{S}$ *and* $\bar{P}$.
[4] *Conditional upon the existence of* $\bar{S}$ *and* P.

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104. Less Important Eductions.

Some eductions are material and not of universal validity.

The eductions we are to deal with in this section are not purely formal inferences. They do not hold in the case of all categorical propositions, and their validity, or invalidity, must, at all times, be decided by a consideration of the matter of the propositions concerned. They are, thus, not necessary inferences, and are of considerably less generality, and, consequently, of less importance, than are the eductions we have already considered in this chapter. They may be classed under two heads :—

In *Inference by Added Determinants* both subject and predicate are limited in the same way.

This limitation is *Determination*,

and the limiting word is a *Determinant*.

(i.) **Inference by Added Determinants is the deducing, from a given proposition, another proposition of a narrower application, by limiting both the subject and the predicate of the original proposition in an identical manner.** Such limitation is called *Determination*, and is effected by adding the same qualification to each term. Both subject and predicate are thus made complex (*cf.* § 74); and each element of the complex term is really a determinant of the other. But, in speaking of this kind of inference, the name Determinant is usually restricted to the freshly added qualification. *A Determinant* may, therefore, be defined as *a qualification added to a term, which, as it does not belong to that term in its whole denotation, limits, or determines, its application in this special case* [*cf.* § 74 (ii.)]. Hence, if the same determinant is added to both subject and predicate, the extent of each is limited, but each is made more definite, and the more limited proposition is a true inference from the wider one. For example, 'All negroes are men,' therefore 'Every honest negro is an honest man'; 'Wrongdoers are deserving of punishment,' consequently 'Female wrongdoers are females who deserve punishment'; 'Poetry is food for the imagination,' hence 'Good poetry is good food for the imagination.' But it must be *precisely the same* determinant in each case, and this will not always be secured by using the same word, for the meaning of words is constantly modified by the context [*cf.* §§ 3; 171 (ii.)]. And this modification is very various, and often of so subtle a character that it escapes

The determinant of the subject must be identical with that of the predicate.

The meaning of a word being modified by the context,

notice. Thus, the employment of this kind of inference is very liable to lead to fallacy, which must be guarded against by a careful reference to the special meaning in each case. If the attributes added as determinants imply any kind of comparison, the liability to fallacy is enormously increased. For instance, because it is true that 'An ant is an animal,' it does not follow that 'A large ant is a large animal,' for 'large' is a comparative term ; we can only deduce the tautologous proposition that 'A large ant is an animal large for an ant.' Nor can we infer from 'A bass singer is a man' that 'A bad bass singer is a bad man,' but only that he is a man who sings bass badly, which is a very different thing. If the added attributes imply quite definite qualities, the inferred proposition is more likely to be true, but this greatly limits the range of this kind of inference. We can infer from 'A prison is a place of detention' that 'A stone prison is a stone place of detention'; and from 'A ball is a plaything' that 'A leather ball is a leather plaything.' But from 'The unemployed are deserving of help,' we are not likely to draw the inference that 'The unemployed when rioting are deserving of help in rioting.' In all cases, too, the predicate must either be a substantive or equivalent in force to a substantive. From 'The army is worn out by the long march' we cannot infer that 'Half the army is half worn out by the long march'; the true inference is 'Half the army is half the body which is worn out by the long march.' If the original proposition is negative, then the limiting the application of subject and predicate makes no difference in the information conveyed ; for the exclusion is complete at first, and that wider exclusion necessarily includes the narrower.

this kind of inference is often fallacious;

especially if the determinant implies comparison.

The predicate must be a substantive term.

Occasionally a valid inference can be made when the determinants of the subject and predicate are not the same. In this case, the determinant of the subject is itself the subject of a proposition of which the determinant of the predicate is the predicate. Thus, from 'Theft is deserving of punishment' and 'Unemployed workmen are poor' we can infer 'Unemployed workmen who steal are poor men who deserve

Two propositions can be sometimes combined so that the terms of one are determinants of those of the other

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punishment.' Leibniz thus symbolized such inferences: 'If $A=B$ and $L=M$, then $A+L=B+M$,' where $=$ does not signify equality, but merely denotes the logical copula 'is,' and $+$ simply implies the addition of elements to each other to form a complex term. The formula does not imply that any two propositions can be thus combined. From 'Lions are carnivorous' and 'Oxen are herbivorous,' we cannot deduce the statement that 'Lions and oxen are carnivorous and herbivorous'; for that would mean that each class of animals consume both flesh and vegetable food. The combination can only be made when the terms in the one proposition limit those in the other [*cf.* § 74 (ii.)].

In *Inference by Complex Conception* the subject and predicate are made determinants of a third term.

(ii.) **Inference by Complex Conception is the deducing, from a given proposition, another proposition of narrower application by combining both the subject and the predicate of the original proposition with the same name, whose denotation is thereby limited.** This mode of inference is very similar to the last. It differs from it in that, instead of the original subject and predicate being determined by the addition made to them, they themselves determine that added element. Thus, from 'A horse is a quadruped' we infer 'The head of a horse is the head of a quadruped,' from 'Arsenic is a poison' that 'A dose of arsenic is a dose of poison,' and from 'Poverty is a temptation to crime' that 'The removal of poverty is the removal of a temptation to crime.' In these examples, it is the words 'head,' 'dose,' and 'removal' which are respectively determined and limited in their application. The same precautions to avoid fallacy are necessary in employing this mode of inference as in the case of Added Determinants. Because 'All judges are lawyers' it does not follow that 'A majority of judges is a majority of lawyers,' nor can we infer from 'All great poets are writers of verse' that 'A large number of great poets is a large number of verse-writers'; for what would be considered a large number in the one case would not be so regarded in the other.

This mode of inference is as liable to fallacy as is that by Added Determinants.

105. Eductions of Hypothetical Propositions.

Though true hypotheticals are universal, yet we have seen that modal particulars take the same general form, and may be regarded us imperfectly developed hypotheticals (*see* §§ 76, 100). Embracing these propositions we have forms corresponding to each of the four forms of categorical propositions, and the full table of eductions given in § 103 is applicable to them. These inferences are seen, perhaps, more clearly when the propositions are not written in the abstract form directly expressive of connexion of content, but in the following more concrete and denotative forms which are justified by, and correspond to them, and which we have called conditional (*cf.* §§ 76, 100)—

A. *If any S is M, then always, that S is P.*
E. *If any S is M, then never, that S is P.*
I. *If an S is M, then sometimes, that S is P.*
O. *If an S is M, then sometimes not, that S is P.*

It must be remembered that 'sometimes' is purely indefinite, like 'some,' and moreover it does not necessarily imply the actual occurrence of the consequent in any one instance; its force is really 'it may be,' whilst 'sometimes not' simply means 'it need not be.'

(i) The eductions from **A**, expressed symbolically, will be as follows:—

Table of Eductions from A conditionals.

Orig. Prop.	**A**	*If any S is M, then always, that S is P.*
Obverse	**E**	*If any S is M, then never, that S is not P.*
Converse	**I**	*If an S is P, then sometimes, that S is M.*
Ob. Conv.	**O**	*If an S is P, then sometimes not, that S is not M.*
Contrap.	**E**	*If any S is not P, then never, that S is M.*
Ob. Contr.	**A**	*If any S is not P, then always, that S is not M.*
Inverse	**O**	*If an S is not M, then sometimes not, that S is P.*
Ob. Inv.	**I**	*If an S is not M, then sometimes, that S is not P.*

Examples of Eductions from A conditionals.

As material examples we may give the following, writing the propositions as nearly as possible in the usage of common speech :—

Orig. Prop.	A	*If any man is honest, he is trusted.*
Obverse	E	*If any man is honest, then never is he not trusted.*
Converse	I	*If a man is trusted, he is sometimes honest.*
Ob. Conv.	O	*If a man is trusted, he is sometimes not dishonest.*
Contrap.	E	*If any man is not trusted, he is not honest.*
Ob. Contr.	A	*If any man is not trusted, he is dishonest.*
Inverse	O	*If a man is not honest, he is sometimes not trusted.*
Ob. Inv.	I	*If a man is not honest, he is sometimes distrusted.*

Table of Eductions from E conditionals.

(ii) The eductions from **E** may be thus symbolically stated :—

Orig. Prop	E	*If any S is M, then never, that S is P.*
Obverse	A	*If any S is M, then always, that S is not P.*
Converse	E	*If any S is P, then never, that S is M.*
Ob. Conv.	A	*If any S is P, then always, that S is not M.*
Contrap.	I	*If an S is not P, then sometimes, that S is M.*
Ob. Contr.	O	*If an S is not P, then sometimes not, that S is not M.*
Inverse	I	*If an S is not M, then sometimes, that S is P.*
Ob. Inv.	O	*If an S is not M, then sometimes not, that S is not P.*

Examples of Eductions from E conditionals.

The following are material examples :—

Orig. Prop.	E	*If any man is happy, he is not vicious.*
Obverse	A	*If any man is happy, he is non-vicious.*
Converse	E	*If any man is vicious, he is not happy.*
Ob. Conv.	A	*If any man is vicious, he is not-happy.*
Contrap.	I	*If a man is not vicious, he is sometimes happy.*
Ob. Contr.	O	*If a man is not vicious, he is sometimes not not-happy.*
Inverse	I	*If a man is not happy, he is sometimes vicious.*
Ob. Inv.	O	*If a man is not happy, he is sometimes not non-vicious.*

Table of Eductions from I conditionals.

(iii) The eductions from **I** are thus expressed in symbols :—

Orig. Prop.	I	*If an S is M, then sometimes, that S is P.*
Obverse	O	*If an S is M, then sometimes not, that S is not P.*
Converse	I	*If an S is P, then sometimes, that S is M.*
Ob. Conv.	O	*If an S is P, then sometimes not, that S is not M.*

Examples of Eductions from I conditionals.

As material examples may be given :—

Orig. Prop.	I	*If a story is believed, it may be true.*
Obverse	O	*If a story is believed, it need not be untrue.*
Converse	I	*If a story is true, it may be believed.*
Ob. Conv.	O	*If a story is true, it need not be disbelieved.*

Care must be taken to ensure that the proposition is really **I**, and not **A** in disguise. Whenever 'sometimes' implies actual occurrence, the proposition is really **A**; it is only **I** when the consequent does not necessarily result at all from the antecedent. For instance, 'If a man plays recklessly, he sometimes loses' is really **A**; for it means 'If any man plays recklessly, it always follows that he has some losses.' Such a proposition can, of course, be contraposited and inverted, processes which the real **I** propositions cannot undergo.

Table of Eductions from O conditionals.

(iv) The symbolic expressions of the eductions from **O** are :—

Orig. Prop.	O	*If an S is M, then sometimes not, that S is P.*
Obverse	I	*If an S is M, then sometimes, that S is not P.*
Contrap.	I	*If an S is not P, then sometimes, that S is M.*
Ob. Contr.	O	*If an S is not P, then sometimes not, that S is not M.*

Examples of Eductions from O conditionals.

Or, illustrating by material examples :—

Orig. Prop.	O	*If a man is impulsive, he sometimes is not prudent.*
Obverse	I	*If a man is impulsive, he sometimes is non-prudent.*
Contrap.	I	*If a man is not prudent, he is sometimes impulsive.*
Ob. Contr.	O	*If a man is not prudent, he is sometimes not unimpulsive.*

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106. Eductions of Disjunctive Propositions.

The eductions from disjunctive propositions are not disjunctive.

Eductions can only be drawn from disjunctive propositions in which alternative predicates are affirmed of one subject. They are more clearly seen if we take the denotative forms of proposition, corresponding to the categorical **A** and **I** (*cf.* § 81), and the same eductions can be drawn from the former as from the latter (*see* § 103). The derived propositions, however, are not themselves disjunctive.

Table of eductions from a universal disjunctive.

(i.) The symbolic expressions of the eductions from a universal disjunctive are :—

Orig. Prop.	*Every S is either P or Q.*
Obverse.	*No S is both $\bar{P}$ and $\bar{Q}$.*
Converse.	*Some things that are either P or Q are S.*
Ob. Conv.	*Some things that are either P or Q are not $\bar{S}$.*
Contrap.	*Nothing that is both $\bar{P}$ and $\bar{Q}$ is S.*
Obv. Contr.	*Everything that is both $\bar{P}$ and $\bar{Q}$ is $\bar{S}$.*
Inverse.	*Some $\bar{S}$'s are neither P nor Q.*
Obv. Inv.	*Some $\bar{S}$'s are both $\bar{P}$ and $\bar{Q}$.*

As material examples we may give :—

Examples of eductions from a universal disjunctive.

Orig. Prop.	*Every duty on imports is either protective or a source of revenue.*
Obverse.	*No duty on imports fails both to protect native industries and to be a source of revenue.*
Converse.	*Among imposts that either protect native industries or are sources of revenue are duties on imports.*
Contrap.	*No impost that fails both to protect native industries and to be a source of revenue is a duty on imports.*
Inverse.	*Some imposts which are not duties on imports neither protect native industries nor increase the revenue.*

The obverted forms of the last three eductions can be easily supplied.

Eductions from a particular disjunctive.

(ii.) The symbolic form of the obverse of the particular disjunctive *Some S's are either P or Q* is *Some S's are not both $\bar{P}$ and $\bar{Q}$.* Thus, the obverse of 'Some arguments are either inconclusive or elliptical' is 'Some arguments are not both conclusive and fully stated.' The forms of the converses are the same as those from the universal disjunctive.

BOOK IV.

SYLLOGISMS.

CHAPTER I.

GENERAL NATURE OF SYLLOGISM.

107. Definition of Syllogism.

Syllogism—an inference from two propositions, containing a common element and one being universal, of a third proposition.

A Syllogism is an inference in which, from two propositions, which contain a common element, and one, at least, of which is universal, a new proposition is derived, which is not merely the sum of the two first, and whose truth follows from theirs as a necessary consequence.

The word Syllogism (Grk. *συλλογισμός*) seems to have originally signified 'Computation,' and to have been borrowed by Aristotle from Mathematics. It may, however, be considered as retaining its strict etymological meaning—'a collecting together'—and as implying that the elements of a syllogism are thought together. The word thus emphasizes the fact that a syllogistic inference is one indivisible act of thought.

Every syllogism is an inference from the general.

As one of the propositions given as data must be universal, every syllogism is an inference from the general; in many cases it is an argument from the general to the particular or individual. Syllogism is the one means by which a general principle can be applied to specific instances; and, in no

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case, can the derived proposition be more general than those from which it is drawn.

The force of a syllogism depends upon the necessity of the inference.

The whole force of a syllogism depends upon the necessity with which the inferred proposition follows from those given as data, and this necessity must be evident from the mere form of the argument.

The *matter* of a syllogism is given in its terms, which vary according to the subject to which the argument refers. Its *form* consists in that relation of the terms by which they are united in two propositions necessitating a certain conclusion. Syllogistic inference is, thus, purely formal, and can, consequently, be entirely represented by symbols (*cf.* § 10). We are concerned in a syllogism, not with the truth or falsity of either of the individual propositions which compose it but, simply with the dependence of one of them upon the other two, so that, if we grant the latter, we, of necessity, accept the former. The derived proposition, therefore, propounds no truth which was not contained in the data. But this is no objection to the syllogism as a process of inference ; it is, indeed, a necessity if that process is to be wholly regulated—as we shall show in the next chapter that it is—by the Laws of Thought (*see* § 109).

If the given data are objectively true, the proposition inferred from them must also be true ; but, if the given data are objectively false, it may accidentally happen that the derived proposition is true in fact. This is, however, a mere coincidence ; its truth is known from other sources, and is not established by the syllogism. For example, from the data

Lions are herbivorous
Cows are lions

we derive the proposition *Cows are herbivorous*, which is true, but whose truth cannot be held to be a consequence of the given data, which are both false.

The propositions from which the inference is made must have a common element.

It is essential that the propositions which form the data should have a common element, as, otherwise, they would have no bond of connexion with each other, and, consequently, no third proposition could be drawn from their

conjunction. But this common element does not appear in the derived judgment, which is an assertion connecting the remaining elements of the syllogism.

Elements of a Syllogism—the propositions and terms which compose it.

The *Elements of a Syllogism* are the propositions and terms which compose it. 'Terms' is here used widely to cover, not only the true terms of categorical propositions, but also the propositions which form the antecedents and consequents of hypothetical propositions (*cf.* § 76). The three propositions which compose a syllogism are called its *Proximate Matter*, and the terms (in the wide sense just noted) which are united in those propositions are styled its *Remote Matter.*

Conclusion—the derived proposition.

Premises—the propositions from which the inference is made.

The derived proposition is the *Conclusion* of the syllogism, and the two propositions from which it is derived are the *Premises.* These names are applicable when the syllogism is stated in the ordinary and strictly logical form, in which the premises (*propositiones præmissæ*) precede the conclusion—as when we say 'Everything which tends to reduce the supply of any article tends to raise its price; Protective Duties tend to reduce the supply of those articles on which they are imposed; therefore, Protective Duties tend to raise the price of those articles on which they are imposed.' But, when the conclusion is put forward first, as a thesis to be proved, it was called by the old logicians the *Question*, and the propositions which establish it, and which are then introduced by 'because,' or some other causal conjunction, were termed the *Reason.* In this form, the syllogism given above would read—'Protective Duties tend to raise the price of those articles on which they are imposed, because they tend to reduce the supply of those articles; and everything which tends to reduce the supply of an article tends to raise its price.' These latter terms are, however, but little used by modern writers.

Middle Term—the element common to the two premises—*M*.

The element common to the two premises is called the *Middle Term*, and is usually symbolized by *M*; whilst the other two terms are styled the *Extremes.*

Major Term—the predicate of the conclusion—*P*. *Minor Term*—the subject of the conclusion—*S*

Distinguishing between the extremes, that which is the predicate of the conclusion is called the *Major Term*, and is commonly expressed by the symbol *P*; and that which is the subject of the conclusion is named the *Minor Term*, and is generally

Major Premise—that containing P and M. *Minor Premise*—that containing S and M.

represented by S. The premise in which the major and middle terms occur is known as the *Major Premise;* that in which the minor and middle terms are found is called the *Minor Premise.* The order in which the premises are stated is, of course, of no consequence so far as the validity of the argument is concerned; but, as it is customary to state the major premise first, that order must be regarded as the legitimate logical form of a syllogism.

The terms Minor, Middle, and Major, refer primarily to the extent of the terms in a syllogism consisting of three A propositions.

The use of the words Minor, Middle, and Major, to denote the terms of a syllogism arose from the consideration of that form of syllogism in which the conclusion is a universal affirmative proposition, and both whose premises are also universal affirmatives. This syllogism may be symbolized by

$$\begin{array}{c} M\ a\ P \\ \underline{S\ a\ M} \\ \therefore S\ a\ P \end{array}$$

Here, as the extent of the predicate of an affirmative proposition must be, at least, as great as, and is generally greater than, that of the subject, it is plain that P must be at least as wide as, and is probably wider than, M in extent, and similarly with M and S. Hence, the extent of M is, in most cases, intermediate between that of S and that of P, and, in other cases, is coincident with that of, at least, one of those terms.

This relation of Extent does not hold in all syllogisms, and is not essential.

This relation of extent does not hold in all syllogisms and is not essential to the validity of syllogistic argument. For instance

$$\begin{array}{c} M\ a\ P \\ \underline{M\ a\ S} \\ \therefore S\ i\ P \end{array}$$

is a perfectly valid argument, though S is here greater than, or at least as great as, M in extent. Similarly, when one of the premises is negative, this relation of extent is not assured. For example, in

$$\begin{array}{c} M\ e\ P \\ \underline{S\ a\ M} \\ \therefore S\ e\ P \end{array}$$

the inference is perfectly just whether P be greater than,

equal to, or less than, M in extent; we cannot tell which is the case, nor is it material, as the total exclusion of P, which does not depend on its extent relatively to that of M, is secured. The names Minor, Middle, and Major, are not, therefore, appropriate in all cases, if they are regarded as referring to the extension of the terms; but they are universally accepted and recognized, and are as convenient as any others which could be invented. In another sense, moreover, the expression 'Middle Term' is quite appropriate, for that term in every syllogism *mediates* the conclusion, and is the middle bond of union connecting the premises.

The names Minor, Middle, Major, are primarily applicable to syllogisms composed of categorical propositions.

This terminology of Terms and Premises is primarily applicable to syllogisms which are entirely composed of categorical propositions, but it may be transferred, in a large measure, to those which consist, wholly or in part, of hypothetical or disjunctive propositions. This will be discussed more fully later on (*see* § 112).

In naming Terms and Premises by reference to the conclusion we assume only the empty form $S—P$ as the conclusion.

It has been urged that this naming of the terms and premises is a ὕστερον πρότερον, and, therefore, fallacious; for, it is said, the conclusion is assumed in order to name the premises, and, therefore, that is first assumed which should only follow from the premises. But this objection is not valid; for the reference is not to any definite proposition ($S\ a\ P$, $S\ e\ P$, etc.) as conclusion, but simply to the empty and universal form of proposition $S—P$. This can be assumed, and the naming of the terms and propositions based on it, without begging the question as to what the conclusion really is in any syllogism, or whether, indeed, any conclusion at all can be drawn from any given premises. Such assumption is necessary to preserve the distinctions of Figure and Mood (*see* Ch. III), on which so large a part of syllogistic doctrine depends.

Other names have been given to the elements of the syllogism.

The middle term was called, by old writers on Logic, the *Argument*, as it is what is assumed in order to argue. But that name is now used to denote the whole syllogism, or the process of inference, which those writers named *Argumentation*. The major premise was frequently termed the *Principle;* the reference being to that most perfect form of syllogism in which the major premise states a general principle, and the minor—hence called the *Reason*—brings some special case under it. The major was also styled simply *The Proposition*, and the minor the *Assumption*, whilst

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the conclusion was frequently termed the *Deduction* or *Collection.* Mediate, as opposed to Immediate Inference, is frequently called *Discursive*, and the process of reasoning is, similarly, termed *Discourse.* Discursive Reasoning is, therefore, that in which an element is used in the process of inference which does not appear in the conclusion. The name implies that, as we pass on to the conclusion, we drop the premises from sight, and retain the statement of the final fact as the one thing we are then concerned with.

108. Kinds of Syllogisms.

Syllogisms are of different kinds, according to the relations of the premises.

As there are different kinds of propositions—Categorical, Hypothetical, and Disjunctive (*see* § 67)—all of which can be used in syllogistic arguments, it follows that syllogisms can be of different kinds—or *relations* as it is technically called (*cf.* § 48).

In a *Pure Syllogism* all the constituent propositions are of the same kind, and may be Categorical, Hypothetical, or Disjunctive.

When both the premises in a syllogism are of the same character as regards the relation of the terms—categorical, hypothetical, or disjunctive—the syllogism is said to be *Pure*, and the conclusion is, in every case, of the same relation as the premises. Thus, two categorical premises yield a categorical conclusion, two hypothetical premises necessitate a hypothetical conclusion, and from two disjunctive premises there follows a disjunctive conclusion. There are, therefore, three kinds of pure syllogisms—the *Categorical*, the *Hypothetical* and the *Disjunctive.*

In a *Mixed Syllogism* the premises are of different relations. The major may be hypothetical or disjunctive, and the minor categorical;

When the premises are propositions of different relations the syllogism is called *Mixed.* In the first place, the major premise may be either hypothetical or disjunctive, and the minor categorical. A syllogism in which this order was reversed would be impossible, as the minor premise must state, in a definite manner, the special case which is to be brought under the more general statement of the major premise. This gives two kinds of Mixed Syllogisms—the *Hypothetical*, and the *Disjunctive.* These Hypothetical Syllogisms are sometimes called Hypothetico-Categorical, but it is more usual to name a mixed syllogism in accordance with the relation of the major premise. To avoid confusion, we shall always call syllogisms in which all the propositions are hypothetical or disjunctive propositions

Pure Hypothetical and *Pure Disjunctive Syllogisms;* whilst those with categorical minor premises and conclusions we shall style *Mixed Hypothetical* and *Mixed Disjunctive Syllogisms*, according to the character of the major premise. In the second place, the major premise may be hypothetical and the minor disjunctive. This gives that peculiar form of mixed syllogism called the *Dilemma*, in which, according to the number of terms in the major premise, the conclusion is either categorical or disjunctive.

or, the major may be hypothetical and the minor disjunctive—this is called the *Dilemma*.

We thus get the following table of kinds of syllogisms :—

Table of kinds of syllogisms.

Syllogisms - -
- 1. Pure - -
  - (*a.*) Categorical.
  - (*b.*) Hypothetical.
  - (*c.*) Disjunctive.
- 2. Mixed - -
  - (*a.*) Hypothetical.
  - (*b.*) Disjunctive.
  - (*c.*) Dilemmas.

The distinction between Pure Hypothetical and Pure Disjunctive Syllogisms on the one hand, and Categorical Syllogisms on the other is not of as great importance as is the distinction between hypothetical, disjunctive, and categorical propositions; for, in all cases the force of the syllogism depends on the necessity with which the conclusion follows from the premises, and the same rules will be found to apply to all kinds of Pure Syllogism. But the Mixed Syllogisms require somewhat different treatment.

The same rules apply to all Pure Syllogisms, but Mixed Syllogisms require different treatment.

We shall, in the next three chapters, confine our attention to Pure Syllogisms, working out the details fully with categorical syllogisms, and then showing how they can be applied to pure hypothetical and pure disjunctive syllogisms. We shall then, in Chapter V, discuss Mixed Syllogisms.

CHAPTER II.

CANONS OF PURE SYLLOGISMS.

109. Basis of Pure Syllogistic Reasoning.

Syllogistic Reasoning rests on the Laws of Thought—Affirmative Categorical Syllogisms on the Principle of Identity; Negative Categorical Syllogisms on that of Contradiction; Pure Hypothetical Syllogisms on the same principles, together with that of Sufficient Reason.

Syllogistic, like all other purely formal reasoning, rests ultimately upon the Laws of Thought. The Principle of Identity (*see* § 17) is the basis of every affirmative categorical syllogism, and that of Contradiction (*see* § 18) of every negative categorical syllogism. For pure hypothetical syllogisms an additional reference is required to the Principle of Sufficient Reason (*see* § 20).

As both the premises of every syllogism contain the same middle term (*see* § 107), each affirmative categorical premise must state that an element of identity exists between that term and one of the extremes, and each negative categorical premise must assert a separation between the middle term and one of the extremes. If, then, both premises are affirmative categoricals, the extremes are connected with each other mediately in so far as each is identical with the middle term, and identity to the same extent is established between them. Or, as Mansel puts it (*Prolegomena Logica*, p. 206), "what is given as identical with the whole "or a part of any concept must be identical with the whole "or a part of that which is identical with the same concept." We here use 'Identity' in the sense—explained in § 17—of identity amidst diversity, so as to make the principle cover such statements as *S is P*. Thus, symbolically, if *S is M,* and *M is P,* then *S is P.* Of course, if restrictions of quantity are introduced into the premises, they limit the identity, and the same limitation must appear in the

conclusion. If, out of two categorical premises, one is negative, then, as one extreme is excluded from *M*, it is excluded from everything which is identical with *M*, and, therefore, from the other extreme; for the other premise must be affirmative, and a term cannot at the same time agree with *M* and with a term which is incompatible with *M*. Thus, symbolically, if *S is M*, and *M is not P*, then *S is not P*. These principles apply equally to pure syllogisms whose premises are hypothetical propositions. But here the proposition which forms the 'middle term' of such a syllogism gives the reason why the proposition which forms the 'minor term' is the antecedent, whose affirmation is the ground for the assertion of the proposition which forms the 'major term,' and is, therefore, the consequent of the conclusion. Thus, symbolically, from *If* **A**, *then* **B**; and *if* **B**, *then* **C**, it follows that *If* **A**, *then* **C**; the 'Sufficient Reason' being found in the relation of both these extremes to **B**.

110. Axioms of Categorical Syllogisms.

(i.) **Axioms applicable to all forms of Categorical Syllogism.**

Logicians have usually developed the Laws of Thought into Axioms of Syllogism, but none of these are ultimate.

Instead of appealing directly to the simple statements of the Laws of Thought, logicians have been accustomed to give various axioms—which are more or less expansions of those statements—as the bases of syllogistic reasoning from categorical propositions. We cannot regard such axioms, however, as really ultimate; they are only *axiomata media*—or 'middle axioms'—which, so far as they are not mere expressions of the simple principles of thought, must be derived from those principles.

Whately's Axioms.

* (*a*) **Whately** (*Elements of Logic*, 5th Ed., pp. 83-4) gives the following two axioms:—

"1. If two terms agree with one and the same third, "they agree with each other.

"2. If one term agrees and another disagrees with one "and the same third, these two disagree with "each other."

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Hamilton's Axiom.

* (*b*) **Sir W. Hamilton** (*Lect. on Log.*, vol. ii., p. 357) propounded one such axiom, which he called "the Supreme Canon of Categorical Syllogisms," in these words :

"In so far as two Notions (notions proper or individuals) "either both agree, or, one agreeing, the other "does not agree, with a common third Notion, in "so far these Notions do or do not agree with "each other."

Thomson's Axiom.

* (*c*) The late **Archbishop Thomson** (*Laws of Thought*, p. 163) gives a statement of the axiom, which he calls the 'General Canon of Mediate Inference,' and which differs from Hamilton's in little but form. He says :

"The agreement or disagreement of one conception with "another is ascertained by a third conception, "inasmuch as this, wholly or by the same part, "agrees with both, or with only one of the "conceptions to be compared."

None of these are underivable, but are based on the Principles of Identity and Contradiction.

They should be called Axioms rather than Canons.

* A verbal objection may be made to Thomson's statement. For, if, as the words "is ascertained" seem to imply, two conceptions can only be compared mediately through a third conception, then all comparison is impossible ; for neither conception can be compared with this third conception. It would, therefore, be better to read "may be" for "is." But, putting this on one side as a fault of expression rather than of meaning, it is evident that the statements both of Hamilton and of Thomson are—allowing for the Conceptualist language in which they are expressed and the Nominalist phraseology of Whately—simply summaries of Whately's two axioms. Their accuracy is undoubted, but it is not correct to speak of any such statement as 'the supreme canon,' if by 'supreme' is meant ultimate or underivable ; for each is merely a more developed statement of the Principles of Identity and Contradiction. The word 'Canon' is, moreover, a not very appropriate name for such statements. A Canon is a rule, and Hamilton's and Thomson's statements are not rules, but axioms, or general principles, from which rules may be deduced.

(ii.) **Axioms applicable to only one form of categorical syllogism.**

(*a*) The *Dictum de omni et nullo.* The scholastic logicians regarded as the perfect type of categorical syllogism that in which the middle term is the subject of the major premise and the predicate of the minor premise—that is, in which the empty schema is

Logicians commonly regarded one type of syllogism as fundamental;

$$\begin{array}{l} M\text{——}P \\ S\text{——}M \\ \hline \therefore S\text{——}P. \end{array}$$

viz.—

$$\begin{array}{l} M\text{——}P \\ S\text{——}M \\ \hline \therefore S\text{——}P \end{array}$$

All other forms of syllogism can be reduced to this by applying the various modes of eduction (*see* §§ 102, 103) to the premises (*see* §§ 126-30). The validity of such other forms can, therefore, be tested, by first reducing them to this standard form, and then enquiring whether or not they conform to the general axiom which applies directly to this form only. These logicians, therefore, gave one axiom as the fundamental principle of syllogistic reasoning. This is the time-honoured *Dictum de omni et nullo*, which is, perhaps, most satisfactorily expressed by saying:

and tested all others by reducing them to this form.

The *Dictum* is the Axiom which applies to this form.

Whatever is distributively predicated, whether affirmatively or negatively, of any class may be predicated in like manner of anything which can be asserted to belong to that class.

Statement of the *Dictum*.

This axiom is, however, no more fundamental than are those more generally applicable principles which we have already examined. Like them, it is simply an expanded statement of the Principles of Identity and Contradiction; for, to predicate anything of a term used distributively is to make the same predication of each of the constituents of the denotation of that term.

It is not fundamental, but is an expanded statement of the Principles of Identity and Contradiction.

The Latin form in which the *Dictum* was commonly given by the older logicians was *Quicquid de omni valet, valet etiam de quibusdam et de singulis. Quicquid de nullo valet, nec de quibusdam valet, nec de singulis.* The common rendering of this into English was 'Whatever can be affirmed, or denied, of a class may be 'affirmed, or denied, of everything included in the class.' But this is not satisfactory. The original proposition is not made of a class

The 'class' statement of the Dictum is objectionable;

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for the reference is not to a term used collectively but to one used distributively, and the logical class is indefinite and fixed by connotation.

as a class—*i.e.*, with a class name used collectively—but of the individuals which compose the class—*i.e.*, with a class name used distributively [*cf.* § 27 (ii.)]. The *Dictum* itself makes this clear, for it says *de omni*, not *de cuncto*. This rendering of the axiom was founded upon the class-inclusion view of the import of propositions (*see* § 85), and is apt to suggest that the 'class' whose name forms the middle term of the syllogism, is a definitely determined and constituted body of individuals; instead of which, it must be remembered, Logic regards a class as comprising an indefinite number of individuals possessing in common certain specified attributes [*cf.* § 28 (iv.)].

The *Nota notæ* is an axiom nearly equivalent to the *Dictum*, but stated in a connotative form.

Mill accepted it as the practical form of his axioms.

(*b*) The *Nota notæ*. The *Dictum* reads the subject in each premise—*i.e.*, the middle and minor terms—in denotation, though the major term retains its natural connotative force as a predicate. Those logicians who regarded the connotation of each term as its most important element (*cf.* § 88) framed an axiom corresponding to the *Dictum*, but expressing this connotative view. This axiom is *Nota notæ est nota rei ipsius. Repugnans notæ, repugnat rei ipsi.* Mill adopts this mode of statement as the form of his axioms best adapted "as an aid for our practical exigencies. . . . "In this altered phraseology, both these axioms may be "brought under one general expression; namely, that what- "ever has any mark, has that which it is a mark of. Or, "when the minor premise as well as the major is universal, "we may state it thus: Whatever is a mark of any mark, is "a mark of that which this last is a mark of" (*Logic*, Bk. II, ch. ii, § 4).

In applying this to negative syllogisms it is necessary to remember that an attribute may be 'a mark of' the *absence* of another attribute. It will be noticed that both statements start with the minor term: the former says—*S* has the mark *M*, which is a mark of *P*, therefore, *S* has *P*: whilst the latter puts it—*S* is a mark of *M*, which is a mark of *P*, therefore *S* is a mark of *P*. The former statement keeps closer to the *nota notæ*, the latter is a legitimate generalization of that axiom on the lines on which it is founded.

111. General Rules or Canons of Categorical Syllogisms.

(i.) **Derivation of Rules from the 'Dictum.'** The *Dictum de omni et nullo*, as has been said [*see* § 110 (ii.) (*a*)], is directly applicable to syllogisms in whose premises the middle term is the subject of the major, and the predicate of the minor, premise. To all other forms of syllogism it applies indirectly through this form. The *Dictum* may, therefore, be taken as the *axioma medium* of all syllogistic inference (*cf.* § 109, 110); and, consequently, all rules which govern such inferences must be deducible from it. An examination of the *Dictum* will give these in a specific form, corresponding to its own direct reference to one form only of syllogism; but, by a slight generalization they can be made directly applicable to all forms of syllogism. Such an examination shows that:—

As the *Dictum* is the axiom of all syllogistic inference, all rules which govern such inference are deducible from it.

1. The *Dictum* speaks of three, and of only three, terms. There is the 'Whatever is predicated'—which is the major term; the 'class' of which it is predicated—the middle term; and the 'anything asserted to belong to that class'—the minor term. This gives the rule that *a syllogism must have three, and only three, terms.*

The *Dictum* provides:— 1. That there be three, and only three, terms.

2. Similarly, there are three, but only three, propositions contemplated by the *Dictum*. There is that in which the original predication is made of the 'class'—the major premise; that which declares something 'to belong to that class'—the minor premise; and that in which the original predication is made of that included something—the conclusion. Hence, the rule that *a syllogism must consist of three, and only three, propositions.*

2. That there be three, and only three, propositions.

3. The Dictum says the original predication is made of some 'class.' Now this 'class' is, as has just been said (*see* 1), the middle term, which is directly regarded by the *Dictum* as the subject of the major premise. Thus, the *Dictum* tells us that in this form of syllogism the middle term must be distributed (*cf.* § 72) in the major premise. Generalizing this, we get the rule that *the middle term must be distributed in one, at least, of the premises.*

3. That the middle term be distributed.

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4. That no term be distributed in the conclusion which is not distributed in the premises.

4. The *Dictum* says the original predication may be made of 'anything' which can be asserted to belong to the class; therefore, that predication must not be made of a term *more definite than* this 'anything.' Hence, if the 'anything' is undistributed in the premise it must be undistributed in the conclusion. Similarly, the *same* predication which is made of the 'class' in the major premise can be made of this 'anything' in the conclusion; we are, therefore, not justified in making a *more definite* predication, and hence, if this predication is made by means of an undistributed term in the predicate of the major premise, it must be made by a similarly undistributed term in the predicate of the conclusion. Generalizing this, we get the rule that *no term may be distributed in the conclusion which is not distributed in one of the premises.*

5. That one, at least, of the premises be affirmative.

5. According to the *Dictum* the minor premise, in the form of syllogism it directly refers to, must be affirmative, for it must declare that something *can be* included in the 'class' (*i.e.*, in the middle term). This, when generalized, gives the rule that *one, at least, of the premises must be affirmative.*

6. That a negative premise necessitates a negative conclusion, and *vice versâ.*

6. The *Dictum* recognizes the possibility of the original predication—that is, the major premise in such a syllogism as it directly applies to—being either affirmative or negative, and declares that the predication in the conclusion must be made 'in like manner.' As, according to 5, the minor premise in such a syllogism is always affirmative, it follows that when both premises are affirmative the conclusion is affirmative, and when the major premise is negative, then, and only then, the conclusion is negative as well. By generalizing this, we get the rule that *a negative premise necessitates a negative conclusion, and there cannot be a negative conclusion without a negative premise.*

Each rule applies directly to every form of syllogism.

(ii.) Examination of the Rules of the Syllogism. We, thus, get the traditional six general rules, or canons, of the syllogism. Each of these is directly applicable to every form of syllogism, and no syllogism is a valid inference in which any one of these rules is violated. An examination

of them shows that the first two relate to the nature of a syllogism, the second two to quantity—or distribution of terms, and the last two to quality. They may, therefore, be thus summarized :— Statement of Rules of Syllogism.

A. Relating to Nature of Syllogism :

I. *A syllogism must contain three, and only three, terms.*

II. *A syllogism must consist of three, and only three, propositions.*

B. Relating to Quantity :

III. *The middle term must be distributed in one, at least, of the premises.*

IV. *No term may be distributed in the conclusion which is not distributed in a premise.*

C. Relating to Quality :

V. *One, at least, of the premises must be affirmative.*

VI. *A negative premise necessitates a negative conclusion, and to prove a negative conclusion requires a negative premise.*

We will now examine each of these rules in detail.

Rules I and II decide what is a syllogism.

The fallacy of Four Terms is often due to ambiguity in one of the terms, usually in the middle term.

Rules I and II.—These are not rules of syllogistic inference, but rules for deciding whether or not we have a syllogism at all. Rule I forbids all ambiguity in the use of the terms employed in the syllogism ; for, if any term is used ambiguously, it is really two terms (*see* § 26), and so the argument really contains four, instead of three, terms, and is not a true syllogism at all, though it may, at first sight, appear to be one. If there is ambiguity it is most likely to occur in the case of the middle term, and hence, Rule III is frequently stated with the additional words 'and must not be ambiguous.' But this is unnecessary, for Rule I provides against that error, and also against a similar fault in connexion with either the major or the minor term, which, if not so common, is equally fatal, when it does occur, to the validity of the inference.

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A good example of an ambiguous middle is given by De Morgan (*Formal Logic*, pp. 241-2)—

"*All criminal actions ought to be punished by law:*
"*Prosecutions for theft are criminal actions;*
"*∴ Prosecutions for theft ought to be punished by law.*

"Here the middle term is doubly ambiguous, both *criminal* "and *action* having different senses in the two premises." If the middle term is not exactly the same in both premises it is evident that there is no connecting link between the major and minor terms. There must be a common element, and this must be identical in the two premises. Mere resemblance, however close, is not enough; for then *S* and *P* might resemble *M* in different ways, and so no connexion be established between them.

But, as *M* is connected with *P* in the one premise, and with *S* in the other, it occurs in different contexts, and has different specifications. We thus have that diversity which is a necessary concomitant of identity (*cf.* § 17). Both this diversity and this identity are necessary to inference. If *M* were not identical in both premises, it could not form a connecting bond between *S* and *P*; and if it had not different aspects in the two premises, it could not be connected with the ideas *S* and *P*, which are different from each other.

Again, if *S* or *P* is used in a different sense in the conclusion from that which it bears in the premise in which it occurs, the inference is invalid; for the premises justify only the predication of that *same P* which was connected in the major premise with *M*, of that *same S* which was related in the minor premise to the same *M*.

Rules I and II are involved in the definition of a syllogism.

Of Rule II but little need be said. If there are three terms, two of which are to occur in each proposition, and the same two in no two propositions, it is evident there must be three, and only three, propositions. The very definition of a syllogism secures this rule directly, and Rule I indirectly; for two premises with a common term contain evidently three,

and only three terms, and the conclusion relates the two terms which are not common to the two premises.

If *M* is undistributed no bond of connexion between *S* and *P* is secured,

Rule III.—The violation of this rule is called the *Fallacy of the Undistributed Middle.* It is essential that the middle term should be distributed in one, at least, of the premises, as only thus can there be any assurance that there exists that element of identity which is necessary to constitute a bond of connexion between the extremes. Unless it is certain that the extremes are related to one and the same part of the middle term, there can be no inference as to the relation in which those extremes stand to each other. Now, if only an indefinite reference is made to the middle term in each premise, either the same, or an entirely different, part of its extent may be, in fact, involved in each case. For example, because All Englishmen are Europeans, and All Frenchmen are Europeans, it does not follow that All (or any) Frenchmen are Englishmen. In fact, every possible relation between *S* and *P* is consistent with the two propositions *All P is M* and *All S* is *M*, where *M* is an undistributed middle term. This is seen at a glance by a reference to Euler's diagrams (*see* § 91), which give all the possible objective relations of two classes. Each of those five figures may be entirely enclosed in a larger circle representing *M*, and in each case *P a M* and *S a M* will hold true. Thus, it is evident that from two such propositions no inference whatever can be drawn as to the relation of *S* and *P*. Similar ambiguity will be found to follow from every other case in which, in a pair of propositions, *M* is not once distributed. As, then, we have no security when *M* is undistributed that there is any bond of connexion whatever between *S* and *P*, we can draw no inference concerning the relation of those two terms. For, by the Law of Parsimony (*cf.* § 79), formal inference must depend upon that bare minimum of assertion which the premises must be held to make unconditionally; and, therefore, as it is possible that the same part of *M* is not referred to in both premises, we must not assume that it is so referred to in any particular case. Only, then, by securing the whole

hence, there can be no inference as to their relation to each other.

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M need not be distributed in more than one premise.

of the middle term in one premise can we be certain that there is an identical element in both premises. And, though the middle term may be distributed in both premises, yet a single distribution is sufficient to secure this. For, if the whole extent of *M* is related to one of the extremes, no matter what part of *M* is related to the other extreme, it must be identical with some at least of the *M* referred to in the former case. The real mediation is, of course, through this common part of *M*, whatever its extent may be; what that is Formal Logic does not enquire, it deals only with the definite 'all' and the indefinite 'some,' and rests on the assurance that the former must, of necessity, include the latter.

If we have an *Illicit Process* of either term our conclusion is more definite than the premises warrant.

Rule IV.—The violation of this rule is called Illicit Process. If the minor term is distributed in the conclusion and not in the minor premise, we have the *Fallacy of Illicit Process of the Minor Term;* if the same unwarranted treatment is accorded to the major term, it gives rise to the *Fallacy of Illicit Process of the Major Term.* As the conclusion must follow necessarily from the premises, we can never be justified in making a predication about a definite *All S* when the minor premise only refers to *Some S.* If merely an indefinite part of *S* is related to *M*, that relation can give us no right to trace, through *M*, a connexion between *P* and *all S.* Because all criminals are deserving of punishment, and *some* Englishmen are criminals, it does not follow that *all* Englishmen are deserving of punishment. The conclusion must be no more definite than the premises warrant. And the same holds of the major term. We are justified in relating the whole of *P* to *S* only when the whole of *P* has been previously related to *M* in the major premise. It will be noticed that we can only use *P* universally in the conclusion when that conclusion is negative, for *P* is always its predicate, and the predicate of an affirmative proposition is always undistributed (*see* § 72). In this case, therefore, one of the premises must be negative (Rule VI). If that premise be the minor, then *P* must be the subject of the major premise, which must be an **A** proposition; but if the negative premise be the major,

Illicit Major is only possible with a negative conclusion.

P may always be its predicate, though it can be its subject only when it is universal (*cf.* § 72). In every other case, if *P* is distributed in the conclusion, we have Illicit Process of the Major. For example, if we argue from 'All fishes are oviparous,' and 'No birds are fishes,' to 'No birds are oviparous,' our inference is invalid. In this case, as in most such cases, the conclusion is also false in fact. But even were it true in fact it would not be a valid inference from the premises; its truth would not be a result of their truth, but would be an accidental coincidence and known only through some other source. For instance, 'All fishes are cold-blooded, No whales are fishes, therefore No whales are cold-blooded' is exactly as invalid an inference as the one just considered, though the proposition given as its 'conclusion' is objectively true. For there is nothing in the premises to deny to whales the attribute 'cold-blooded,' as will be seen by substituting the word 'snakes' for 'whales,' when the 'conclusion' becomes false in fact. Thus, no conclusion is justified in which any term is distributed which is undistributed in the premise in which it occurs. The violation of this rule may be compared with the simple conversion of an **A** proposition, a process which has been already shown to be illicit [*see* § 102 (ii.) (*a*)].

Two negative premises yield no conclusion, as no connexion is asserted between *M* and either *S* or *P*.

Rule V.—From two negative premises no conclusion can be drawn; for, from mere negation of relation no statement of relation can be deduced. It is only when one of the extremes is connected with the middle term, that we can, through that connexion, infer its agreement with, or separation from, the other extreme. For, if both *S* and *P* are declared to be separated from *M*, there is, clearly, no bond of union to connect them with each other. They may, or may not, be related in fact; but whatever relation they hold, it is impossible to infer it from the negation of relation with the common element which is contained in the premises. Compare, for instance, the pairs of negative propositions: 'No cows are carnivorous—No sheep are carnivorous'; 'No men are immortal—No negroes are immortal.' In the first case the minor term is, in fact, wholly excluded from, and, in the

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second case, wholly included in, the major term; but neither the exclusion nor the inclusion can be deduced from the premises, which simply separate both the classes represented by the major and minor terms from the common attribute expressed by the middle term; and which, being identical in form, must, if they give any conclusion, always give one of the same form.

Two negative premises are consistent with every one of the possible relations between S and P.

It will be profitable to examine this rule more in detail, as its accuracy has been questioned. There are three possible combinations of negative premises which we will consider separately :—

This holds if the premises are both **E**;

1. Both premises may be *negative universal*—*i.e.*, **E**—propositions. In this case, both S and P are wholly separated from M. They may, at the same time be (1) wholly separated from each other, (2) partly coincident and partly separated, or (3) one may be wholly included in the other. Distinguishing in this last case between inclusion when the extent of the terms coincide, inclusion without coincidence of S by P, and of P by S, we have here all the five possible objective relations between S and P (*cf.* § 85). This will be made evident by a reference to Euler's diagrams (*see* § 91), which express these relations; for, to each of the five figures which represent the relations of S and P we may add a circle M, lying entirely outside both the circles which represent S and P, and thus signifying the entire exclusion of both S and P from M. We see, therefore, that this exclusion is consistent with every conceivable relation of S and P to each other.

or, one **E** and one **O**;

2. One premise may be *universal*, and the other *particular*, *negative*—*i.e.*, **E** and **O**. In this case M is entirely excluded from one of the extremes, and partially, at least, from the other. But the particular never excludes the possibility of the universal being true in fact [*cf.* § 71 (ii.)]. Hence, the whole indefiniteness which obtains when both premises are **E** propositions remains when one is **E** and the other is **O**, and there is added the possibility of other relations, not between S and P but, between those terms and M. These may be shown on Euler's diagrams (*see* § 91) by drawing a circle to represent M, which shall intersect one of the circles representing S and P, and lie entirely outside the other. This requires seven additional diagrams besides the five which are necessary when both premises are **E**. For each of the Figures IV and V gives two diagrams, according as the S or the P circle is intersected by the new circle representing M. The indefiniteness

of the conclusion is not increased—that is impossible, as there are only five conceivable relations in fact between S and P—but the indefiniteness of the relations represented by the premises which should lead to a conclusion is seen to be more than doubled.

or both **O**;

3. Both premises may be *particular negative—i.e.*, **O**—propositions. In this case, each extreme is partially, at least, separated from M. Here again, the particular does not exclude the possibility that the universal is true in fact. All the former indefiniteness, then, still remains, and it is still further increased so far as the relations of S and P to M are concerned. For the circle representing M may now be drawn to intersect both the S and the P circle. This gives four more diagrams, in addition to the twelve we already have; for this double intersection will give a fresh arrangement of circles in every case except in that of Fig. I, where to intersect either S or P is, necessarily, to intersect them both, as they are coincident; this case was, then, represented in the diagrams necessary to represent **E** and **O** premises.

no inference can, therefore, be drawn from such premises.

In every case, then, we see that every possible relation of S and P is consistent with two negative premises; such premises can, therefore, necessitate no conclusion; that is, no inference can be made from them.

Jevons denies this;

* This has been denied by Jevons. He says: "The old "rule informed us that from two negative premises no "conclusion could be drawn, but it is a fact that the rule in "this bare form does not hold universally true; and I am "not aware that any precise explanation has been given of "the conditions under which it is or is not imperative. "Consider the following example:

" *Whatever is not metallic is not capable of powerful*
" *magnetic influence,*
" *Carbon is not metallic.*
" ∴ *Carbon is not capable of powerful magnetic influence.*

" Here we have two distinctly negative premises, and yet "they yield a perfectly valid negative conclusion. The "syllogistic rule is actually falsified in its bare and general "statement" (*Principles of Science*, 2nd Ed., p. 63).

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Expressed symbolically this argument is

$$\begin{array}{l} \textit{No non-M is P} \\ \underline{\textit{S is not M}} \\ \therefore\ \textit{S is not P}; \end{array}$$

but the minor premise in his example is really affirmative.

where we have, apparently, four terms, S, P, M, and *non-M*. But, if we examine the argument more closely, we shall see that what the minor premise really asserts is that S is included in those things of which P is denied, *i.e.*, the *non-M's*. Hence, the middle term is really *non-M*, and it is this which is predicated *affirmatively* of S in the minor premise. We reduce the above argument, therefore, to a simple syllogism by obverting the minor premise [*cf.* § 102 (i.)], when we get

$$\begin{array}{l} \bar{M}\ e\ P \\ \underline{S\ a\ \bar{M}} \\ \therefore S\ e\ P \end{array}$$

which is a perfectly valid syllogism, and in which Rule V is observed, as the minor premise is affirmative.

Mr. Bradley objects to expressing the minor premise in the affirmative form,

Mr. Bradley will not allow this to be any true answer. Referring to the example just quoted from Jevons, he says: "This argument "no doubt has *quaternio terminorum* and is vicious technically, "but the fact remains that from two denials you somehow *have* "proved a further denial. 'A is not B, what is not B is not C, "'therefore A is not C;' the premises are surely negative to start "with, and it appears pedantic either to urge on one side that 'A "'is *non-B*' is simply positive, or on the other that B and *non-B* "afford no junction. If from negative premises I can get my con-"clusion, it seems idle to object that I have first transformed one "premise; for that objection does not show that the premises are "not negative, and it does not show that I have failed to get my "conclusion" (*Principles of Logic*, p. 254). Certainly, the conclusion is obtained, and, assuredly, it is a valid inference; but it is neither 'pedantic' nor 'idle' to urge that one premise has already been transformed when they are both stated as negative propositions, and must be reduced back to its natural affirmative form before the inference can be made. For, as Ueberweg remarks, "this reduc-

"tion is not an artificial mean, contrived in order to violently "reconcile an actual exception to a rule falsely considered to be "universally valid. We only arrive naturally at the conclusion "when we think the minor premise in the form: *S* falls under the "notion of those beings which are not *M*" (*Logic*, Eng. trans., p. 384). It must be remembered that we can, by obversion, always reduce an affirmative proposition to a negative, and *vice versâ*. The decision as to which form is appropriate to any particular case must be decided by considering whether the judgment to be expressed emphasizes the negative or the affirmative element present in every negative proposition (*cf.* § 70). In this case, the minor emphasizes the affirmative element.

but Ueberweg shows it is the only natural reading.

Ueberweg further points out that this case had not been overlooked by the older logicians, who explained it in this very way, and he thinks it "not improbable that the doctrine of qualitative "Æquipollence between two judgments [*i.e.*, Obversion] owes its "origin to the explanation of the syllogistic case" (*ibid.*). The authors of the *Port Royal Logic* had also considered and solved the apparent difficulty exactly as we have just done. The 'precise 'explanation' which Jevons desiderated had, then, been frequently given. In fact, he had given it himself. In his *Elementary Lessons in Logic* (which was published before the second edition of the *Principles of Science*) he says: "It must not however be "supposed that the mere occurrence of a negative particle (*not* or "*no*) in a proposition renders it negative in the manner con- "templated by this rule. Thus the argument

This case has been explained by the older logicians,

and by Jevons himself.

> "'*What is not compound is an element,*
> "*Gold is not compound;*
> "∴ *Gold is an element,*'

"contains negatives in both premises, but is nevertheless valid, "because the negative in both cases affects the middle term, which "is really the negative term *not-compound*" (p. 134). The rule then holds without exception that in every syllogism there must be at least one affirmative premise.

If the extremes are asserted to stand in opposed relations with *M* they cannot agree with each other;

Rule VI.—If one premise is negative, the other must, by Rule V, be affirmative. Hence, the two extremes are related to the middle term in opposite ways. Now, if two terms agree with each other, they must, necessarily, stand in the same relation to any third term. If, then, the relations

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of S and P respectively to M are not the same, but contradictory, relations, S and P cannot agree with each other. We reach the same result in a slightly different way by considering that, in so far as anything agrees with M, it must be separated from everything from which M is separated. If, then, one premise declares the agreement of one extreme with M, and the other premise asserts the incompatibility with M of the other extreme, those extremes must be inferred to be incompatible with each other. Hence, a negative premise involves a negative conclusion. And the converse of this is also true. The non-agreement of S and P with each other must follow from the fact that one agrees with, and the other is separated, wholly or in part, from M. For, if they both agreed with M, they would agree with each other. Therefore, a negative conclusion can only be inferred when one of the premises is negative.

hence, a negative premise involves a negative conclusion, and a negative conclusion requires a negative premise.

Rules III–VI are the only ones which are rules of syllogistic inference.

* (iii.) **Simplification of the Rules of the Syllogism.** Our examination of Rules I and II showed that they are not rules of syllogistic inference at all, but rather preliminary statements of what a syllogism is. Thus, we have, really, only *four* rules of syllogistic *inference*—two relating to quantity and two to quality. By a direct application of these the validity of any argument given in the form of a syllogism can be tested. They are not, however, equally fundamental; for all, except the last part of Rule VI, are derivable, by a more or less indirect process, either from Rule III or from Rule IV—these last two being mutually inferrible.

These four are not all fundamental.

This has been admirably worked out by Dr. Keynes in his *Formal Logic*, and a large part of this subsection is taken from that work. We will now show in detail how this reduction in the number of fundamental rules is effected.

The first part of Rule VI can be deduced from Rule V.

(*a*) *The first part of Rule VI is a corollary from Rule V.*—The first part of Rule VI says that a negative premise necessitates a negative conclusion. Dr. Keynes (*Formal Logic*, 3rd Edit., p. 247) thus shows this to be a deduction from Rule V, which forbids two negative premises :—

"If two propositions **P** and **Q** together prove a third **R**, it is plain "that **P** and the denial of **R** prove the denial of **Q**. For **P** and **Q** "cannot be true together without **R**. Now, if possible, let **P** (a "negative) and **Q** (an affirmative) prove **R** (an affirmative). Then "**P** (a negative) and the denial of **R** (a negative) prove the denial of "**Q**. But two negatives prove nothing."

Rule V follows from Rule III.

(*b*) *Rule V is a corollary from Rule III.*—Rule V forbids two negative premises. This is shown by De Morgan (*Formal Logic*, p. 13) to be inferrible from the rule prohibiting undistributed middle. If the negative premises are both **E** propositions they may always be expressed in the form $P\ e\ M$, $S\ e\ M$; for if given in any other form they may be simply converted [*see* § 102 (ii.) (*b*)]. These by obversion [*see* § 102 (i.)] give the equivalent propositions

$$P\ a\ \bar{M}$$
$$S\ a\ \bar{M}$$

where, as the middle term, $\bar{M}$, is in each premise the predicate of an affirmative proposition, we have undistributed middle (*cf.* § 72). The same proof would hold when one of the premises is particular, so long as M is its predicate. But if one of the premises is particular and has M for its subject, then, as an **O** proposition cannot be converted [*see* § 102 (ii.) (*d*)], we cannot bring the argument into this form. But, by retaining the original statement of the particular premise, we can express the apparent syllogism in one of these two forms

$$P\ a\ \bar{M} \qquad\qquad M\ o\ P$$
$$M\ o\ S \qquad\qquad S\ a\ \bar{M}$$

in each of which there is no middle term. De Morgan does not prove this last case quite in this way. He contraposits the particular premise, which gives

$$P\ a\ \bar{M} \qquad\qquad \bar{P}\ i\ M$$
$$\bar{S}\ i\ M \qquad\qquad S\ a\ \bar{M}$$

but there is nothing gained by doing this, and the former method has the advantage of simplicity. If both premises are particular, the invalidity is seen by obverting both when M is predicate of both, by contrapositing both when M is subject of both, each of which processes will show undistributed middle; and by obverting the one which has M for its predicate when M is subject to one and predicate of the other, which will show four terms.

Dr. Keynes objects to the proof of invalidity by reduction to four

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terms. He says: "This is not satisfactory, since we may often "exhibit a valid syllogism in such a form that there appear to be "four terms; *e.g.*, *All M is P, All S is M*, may be converted into

"*All M is P*
"*No S is non-M*,

"in which there is no middle term" (*Formal Logic*, pp. 246-7). This is certainly the case, as we saw in our examination of Rule V in the last sub-section when we were criticizing Jevons' argument that two negative premises may yield a valid conclusion. But this objection does not hold against the reduction of two **E** propositions to undistributed middle, and as Mr. Keynes goes on to remark, "The "case in question may . . . be disposed of by saying that if we can "infer nothing from two universal negative premises, *à fortiori* we "cannot from two negative premises, one of which is particular" (*ibid.*). The only objection which it is possible to bring against the reduction to undistributed middle is that if *M* is written as the subject of the premises, whenever it is possible to do so, we get a real middle term which is, moreover, distributed. Thus, if the premises are both **E** they may be written *M e P, M e S*, which, by obversion, give

$$M\ a\ \overline{P}$$
$$M\ a\ \overline{S};$$

but it must be pointed out that, though we have here neither undistributed middle nor four terms yet, *M* does not mediate a connexion between *S* and *P*, which are the two extremes in the original form of our premises.

Rule IV may be inferred from Rule III;

(*c*) *Rule IV may be inferred from Rule III.*—That illicit process indirectly involves undistributed middle is thus shown by Mr. Keynes: "Let **P** and **Q** be the premises and **R** the conclusion of a "syllogism involving illicit major or minor, a term *X* which is un-"distributed in **P** being distributed in **R**. Then the contradictory "of **R** combined with **P** must prove the contradictory of **Q**. [For **P** "and **Q** cannot be true together without **R**.] But any term dis-"tributed in a proposition is undistributed in its contradictory. *X* "is therefore undistributed in the contradictory of **R**, and by hypo-"thesis it is undistributed in **P**. But *X* is the middle term of the "new syllogism, which is therefore guilty of the fallacy of undis-"tributed middle" (*Formal Logic*, 3rd Edit., pp. 247-8).

(*d*) *Rule III may be inferred from Rule IV.*—Mr. Keynes remarks that undistributed middle may be deduced from illicit process, as well as *vice versâ.* This may be thus shown: Let **P** and **Q** be the premises and **R** the conclusion of a syllogism involving undistributed middle, and let *X* be the undistributed middle term. Then **P** together with the contradictory of **R** must prove the contradictory of **Q**. For **P** and **Q** cannot be true together without **R**. But any term undistributed in a proposition is distributed in its contradictory [*cf.* §§ 97 (ii), 98]. Therefore *X* is distributed in the contradictory of **Q**. But, by hypothesis, *X* is undistributed in **P**, and as it does not appear in **R** it must be absent from its contradictory. Hence *X* is undistributed in the premises and distributed in the conclusion of the new syllogism, which is, therefore, guilty of the fallacy of illicit process.

and Rule III is inferrible from Rule IV.

* (*e*) **Results of the Simplification.**—There are, thus, seen to be only two ultimate rules, one relating to quantity and the other to quality. As, however, the two original rules relating to quantity can each be inferred from the other, we may adopt either of these as ultimate. This gives us two alternative pairs of independent rules. The first pair is :—

Hence, there are two alternative pairs of independent rules,

(1) The middle term must be distributed in one, at least, of the premises.

(2) To prove a negative conclusion requires a negative premise.

which are—
(1) Rule III
(2) Rule VI (second part),

The second, and alternative, pair is :—

(1) No term may be distributed in the conclusion which is not distributed in a premise.

(2) To prove a negative conclusion requires a negative premise.

or:—
(1) Rule IV
(2) Rule VI (second part).

This latter pair has the advantage of exact correspondence with the rules for Subalternation [*see* § 97 (i.)] and Conversion [*see* § 102 (ii.)] respectively, and of thus showing the fundamental identity of the mental process in immediate and in mediate inference.

This simplification is not important, as Rules III-VI are all required to *directly* test syllogistic arguments.

* This simplification is interesting but of no great practical importance; for we must still appeal to the last four rules given in sub-section (ii) as *direct* tests of the validity of any argument expressed in the form of a syllogism. An invalid syllogistic inference need not offend against either of the two independent rules to which those four have been reduced, except in a very indirect way, which is only made apparent by a more or less complex process of reasoning.

Three corollaries are deducible from the Rules.

(iv.) Corollaries from the Rules of the Syllogism.

Though the four rules of syllogistic inference [sub-§ (ii.), (Rules III-VI)] are not equally fundamental, yet all are so far independent that all are necessary for the immediate detection of invalidity in syllogistic inference; and for this purpose they are sufficient. There are, however, three corollaries from these rules which, though not absolutely requisite for the detection of syllogistic fallacy, are useful for that purpose. It was long customary to give the first two of these as independent rules of syllogism. They are :—

Statement of the Corollaries.

1. *From two particular premises nothing can be inferred.*
2. *If one premise is particular the conclusion must be particular.*
3. *From a particular major and a negative minor nothing can be inferred.*

We will now show in detail how each of these can be deduced from the rules already given.

Two particular premises cannot distribute enough terms to warrant any conclusion.

Cor. 1. The best proof that two particulars prove nothing is that given by De Morgan (*Formal Logic*, p. 14) :—"Since "both premises are particular in form, the middle term can "only enter one of them universally by being the predicate "of a negative proposition; consequently (Rule V) the other "premise must be affirmative, and, being particular, neither "of its terms is universal. Consequently both the terms as "to which the conclusion is to be drawn enter partially, and "the conclusion (Rule IV) can only be a particular *affirma-*

"*tive* proposition. But if one of the premises be negative, "the conclusion must be *negative* (Rule VI). This contra-"diction shows that the supposition of particular premises "producing a legitimate result is inadmissible."

* A somewhat different proof, based on an examination of each of the possible combinations of particular premises, may also be given :—In every valid syllogism the premises must contain one distributed term more than the conclusion. For if any term is distributed in the conclusion it must also be distributed in the premise in which it occurs, and, in addition to this, the middle term must be distributed once, at least, in the premises. From this it follows that no conclusion can be drawn from two particular premises. For, if they are both **O** propositions, by Rule V nothing can be inferred. If they are both **I** they contain no distributed term at all (*see* § 72) and, thus, break Rule III. If one is **I** and the other is **O** then the conclusion must be negative (Rule VI), and, consequently, it distributes the major term (*see* § 72). But **I** and **O** distribute only one term between them, and, therefore, cannot distribute both the major and the middle terms. Hence, nothing can be inferred, for to draw a conclusion would be to break either Rule III or Rule IV.

One universal and one particular premise can only distribute enough terms to warrant a particular conclusion.

Cor. 2. That a particular premise necessitates a particular conclusion may be thus proved :—If both premises are affirmative and one particular, they can, between them, distribute only one term, which must be the middle term (Rule III). Hence, both the extreme terms are undistributed in the premises, and, consequently, must be undistributed in the conclusion (Rule IV)—that is, the conclusion must be *particular* affirmative.

If, however, in such a syllogism, one premise is negative, it distributes its predicate, and the premises, therefore, contain between them two, but only two, distributed terms. One of these must be the middle term (Rule III). Hence, only one distributed term can enter the conclusion (Rule IV). But the conclusion must be negative (Rule VI), and it, therefore, distributes the major term, which must, consequently,

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be the second distributed term in the premises (Rule IV). The minor term is, therefore, undistributed in the premises and must be undistributed in the conclusion—that is, the conclusion must be *particular* negative.

As both premises cannot be negative (Rule V), these are the only possible cases.

This corollary can be proved from Cor. 1.

* A neat proof of this corollary is given by De Morgan (*Formal Logic*, p. 14), who thus deduces it from Corollary 1:—"If two propositions **P** and **Q** together prove "a third **R**, it is plain that **P** and the denial of **R** prove the "denial of **Q**. For **P** and **Q** cannot be true together without "**R**. Now, if possible, let **P** (a particular) and **Q** (a universal) "prove **R** (a universal). Then **P** (particular) and the denial "of **R** (particular) prove the denial of **Q**. But two particu-"lars can prove nothing."

A particular major and a negative minor cannot yield a conclusion because the major term is undistributed.

Cor. 3. That nothing can be inferred from a particular major and a negative minor may be thus proved:—As both premises cannot be negative the major is affirmative particular (I), and distributes neither of its terms. The major term, therefore, cannot be distributed in the conclusion (Rule IV). But, as one premise is negative, the conclusion must be negative. Therefore, the major term must be distributed in the conclusion. This contradiction shows that no valid inference can be made.

112. Application of the Rules to Pure Hypothetical and Pure Disjunctive Syllogisms.

All the rules of syllogism apply to pure hypothetical syllogisms.

(i.) **Pure Hypothetical Syllogisms.** Since hypothetical propositions, when we include Modal Particulars under the name, admit of the same distinctions of quality and quantity as categorical propositions (*see* § 78, *cf.* § 105), there can be forms of pure hypothetical syllogisms (*see* § 108) corresponding to every form of categorical syllogism. Hence, all the rules given in § 111 (ii.) apply to pure hypothetical syllogisms. The denotative, or conditional, forms bear a closer analogy to the ordinary quantified forms of the categorical syllogism than do the pure abstract hypothetical forms, and the application of the rules is more clearly seen when those

quantified forms are considered. The 'terms' here are, however, propositions—the consequent of the conclusion corresponding to the major term of a categorical syllogism, the antecedent of the conclusion to the minor term, and the element which appears only in the premises to the middle term. In considering the distribution of these 'terms' it must be remembered that, as 'always,' 'never,' 'sometimes,' 'sometimes not,' in conditional propositions correspond to 'all,' 'no,' 'some,' 'some not,' in categorical propositions, these words indicate the quantity of the antecedent. The quantity of the consequent must be determined by the same rule which decides the quantity of the predicate of a categorical proposition. That is to say, the consequent of a negative conditional proposition is distributed, and that of an affirmative conditional is undistributed (*cf.* § 72). For example, *If any S is M then always that S is P* does not distribute the proposition *that S is P;* for it neither states nor implies that the only possible condition of *S* being *P* is that it should be *M*—it is quite possible that *S* is *P* under many other conditions, as when it is *N* or *Q* or *X*. In short, the distribution of the 'terms' in a pure hypothetical syllogism must not be determined by a reference to those terms by themselves and out of connexion with their context, any more than in a categorical syllogism.

The 'terms' are propositions.

The quantity of the antecedent is shown by the words 'always,' etc.

The quantity of the consequent depends on the quality of the proposition.

(ii.) **Pure Disjunctive Syllogisms.** Since disjunctive propositions are all affirmative [*see* § 81 (i.)], the syllogistic rules (V and VI) relating to quality do not apply. The rule for securing the distribution of the middle term (III) can only be fulfilled when one of the alternatives in the minor premise is the negative of one of those in the major premise. This will be more fully considered in the next chapter [*see* § 125 (ii.)].

The rules of Quality do not apply to Pure Disjunctive Syllogisms.

CHAPTER III.

FIGURE AND MOOD.

113. Distinctions of Figure.

Figure is the disposition of M in the premises.

Figure is the form of a syllogism as determined by the position of the middle term in the two premises.

If account is taken of the premises alone, so that it is immaterial which of the extreme terms is the subject, and which the predicate, of the conclusion, only three figures are possible. For M must either be (1) subject in one premise and predicate in the other, (2) predicate in both, or (3) subject in both. If, however, it is determined which term shall be the subject, and which the predicate, of the conclusion, the distinction of major and minor is introduced into the premises. The first alternative now becomes twofold, according as M is subject in the major and predicate in the minor premise, or predicate in the major and subject in the minor.

There are four Figures—
I. $M—P$, $S—M$, $\therefore S—P$
II. $P—M$, $S—M$, $\therefore S—P$
III. $M—P$, $M—S$, $\therefore S—P$
IV. $P—M$, $M—S$, $\therefore S—P$

There are thus four possible Figures of syllogism :—

First Figure : M is subject in major, and predicate in minor, premise.

Second Figure : M is predicate in each premise.

Third Figure : M is subject in each premise.

Fourth Figure : M is predicate in major, and subject in minor, premise.

The empty forms of syllogisms arranged in Figures, and with the premises written in the usual order of major first (*cf.* § 107), are, therefore :—

Fig. I.	Fig. II.	Fig. III.	Fig. IV.
M——P	P——M	M——P	P——M
S——M	S——M	M——S	M——S
$\therefore S$——P	$\therefore S$——P	$\therefore S$——P	$\therefore S$——P

Of course, the distinction between the First and Fourth Figures in no way depends upon the order in which the premises are written, but upon the distinction between major and minor premise, which is due to the predetermined order of the terms in the conclusion (*cf.* § 107).

114. Axioms and Special Rules of the Four Figures.

Though all syllogistic reasoning is ultimately based upon the fundamental principles of thought (*see* § 109, *cf.* §§ 17-20), yet separate *axiomata media* may be given as the more immediate bases of inferences in each of the four figures. From these axioms special rules may be derived which directly secure the observance of the general rules of syllogism (*see* § 111) by arguments in the figure to which they apply. Each Figure has a special axiom.

(i.) **The First Figure.** Several axioms—which we have already considered in detail [*see* § 110 (ii.)]—have been given as the foundation of syllogistic inference in this figure, the most generally received of which is the *Dictum de omni et nullo*. As that axiom applies *directly* to all syllogisms in the First Figure, the following special rules of that figure may be immediately gathered from it :— The *Dictum de omni* is the axiom for Fig. I.

Special Rules of Fig. I.—
1. Major premise universal
2. Minor premise affirmative.

1. *The major premise must be universal.*
2. *The minor premise must be affirmative.*

The derivation of these rules was discussed in § 111 (i.) 3 and 5.

These special rules are merely applications to this particular form of syllogism of the General Rules discussed in § 111. Thus :—

Special Rule 2. If the minor premise were negative, the

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major must be affirmative (Gen. Rule V); P would, therefore, be undistributed in the major premise, of which it is the predicate (*see* § 72). But the conclusion must be negative (Gen. Rule VI), and P, as its predicate, must be distributed. But this is forbidden by Gen. Rule IV. Hence, such a syllogism is impossible; *i.e.*, the minor premise must be affirmative.

Special Rule 1. As the minor premise is affirmative it cannot distribute M, which is its predicate. M must, therefore, be distributed in the major premise (Gen. Rule III), of which it is the subject; *i.e.*, the major premise must be universal.

The *Dictum de diverso* is the axiom for Fig. II.

(ii.) **The Second Figure.** The axiom on which syllogistic reasonings in this figure are based is called the *Dictum de diverso.* It is most accurately stated by Mansel (*Aldrich, Art. Log. Rud.*, 3rd Ed., p. 84) in the words: "If a certain "attribute can be predicated (affirmatively or negatively) of "every member of a class, any subject, of which it cannot "be so predicated, does not belong to the class."

Special Rules of Fig. II.—
1. Major premise universal.
2. One premise negative.

From this axiom the following special rules of the Second Figure can be immediately derived:—

1. *The major premise must be universal.*
2. *One of the premises must be negative.*

The first rule is involved in the words "predicated . . . "of *every* member of a class" and the second in the words "any subject of which it *cannot* be so predicated." From this second rule it follows, by General Rule VI, that the conclusion must be negative. This is also evident from the words "does *not* belong to the class" with which the axiom ends.

These rules are merely applications of the General Rules of syllogism (*see* § 111). Thus:—

Special Rule 2. M must be distributed once, at least, in the premises (Gen. Rule III), and, as M is predicate of both premises, this can only be secured when one of them is negative (*see* § 72).

Special Rule 1. As one premise is negative the conclusion must be negative (Gen. Rule VI), and distribute its predicate, the major term *P* (*see* § 72). *P* must, therefore, be distributed in the major premise (Gen. Rule IV), of which it is the subject; *i.e.*, the major premise must be universal.

The *Dictum de exemplo* is the axiom of Fig. III.

(iii.) **The Third Figure.** The axiom which forms the basis of syllogistic reasoning in the Third Figure is called the *Dictum de exemplo*. It may be thus stated: "If any-"thing which is stated to belong to a certain class is affirmed "to possess, or to be devoid of, certain attributes, then those "attributes may be predicated in like manner of some "members of that class."

Special Rules of Fig. III.— 1. Minor premise affirmative. 2. Conclusion particular.

An examination of this axiom will immediately make clear that the special rules of the Third Figure are:—

1. *The minor premise must be affirmative.*
2. *The conclusion must be particular.*

The first is evident from the words "is stated to belong to "a certain class" being used of the subject of the conclusion, and the second from the predication in the conclusion being restricted to "*some* members of that class."

These rules are merely applications to syllogisms in the Third Figure of the General Rule given in § 111. Thus:—

Special Rule 1. If the minor premise were negative, the major premise would necessarily be affirmative (Gen. Rule V). Then, as *P* is the predicate of the major premise, it is undistributed (*see* § 72). But a negative premise necessitates a negative conclusion (Gen. Rule VI), and this would distribute *P*. But this is impossible (Gen. Rule IV). Hence the minor premise cannot be negative.

Special Rule 2. As the minor premise is affirmative, and *S* is its predicate, *S* is undistributed (*see* § 72). Therefore, *S* can only be the subject of a particular conclusion (Gen. Rule IV). Thus, the second special rule is seen to be implied in the first.

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The *Dictum de reciproco* is the axiom of Fig. IV.

(iv.) **The Fourth Figure.** The axiom of the Fourth Figure was called by Lambert the *Dictum de reciproco*, but no one statement of it has been generally agreed upon even by those logicians who accept that Figure, which many do not. The following may be suggested: 'Whatever has a 'predicate affirmed, or universally denied, of it, may itself 'be predicated, particularly and with like quality, of any 'thing which is affirmed of that predicate; and whatever 'has a predicate universally affirmed of it may itself be 'universally denied of any thing which is universally denied 'of that predicate.'

Special Rules of Fig. IV.— 1. If major affirmative, minor universal. 2. If minor affirmative, conclusion particular. 3. If one premise negative, major universal.

The following special rules of the Fourth Figure may be deduced from this axiom :—

1. *If the major premise is affirmative, the minor must be universal.*
2. *If the minor premise is affirmative, the conclusion must be particular.*
3. *If either premise is negative, the major must be universal.*

The first rule is explicitly stated in the last clause of the axiom, where the minor premise is negative. When the minor premise is affirmative it is implicitiy involved in the axiom, for we can only be sure that anything has really been affirmed about the predicate of an affirmative proposition (which is undistributed) when the affirmation is made of the same term distributed, so as to secure every part of its denotation.

The second rule is explicitly stated in the first clause of the axiom, and the third rule is involved in the words "universally denied" in the first clause, and "universally "affirmed" in the second.

These special rules are only applications of the General Rules (*see* § 111) to the Fourth Figure. Thus :—

Special Rule 1. If the major is affirmative, *M*, which is its predicate, is undistributed (*see* § 72). But *M* must be distributed in one of the premises (Gen. Rule III). Therefore, *M* must be distributed in the minor premise, of which it is the subject ; *i.e.*, the minor premise must be universal.

Special Rule 2. If the minor premise is affirmative, *S*, which is its predicate, is undistributed (*see* § 72). It must, therefore, be undistributed in the conclusion, of which it is the subject (Gen. Rule IV); *i.e.*, the conclusion must be particular.

Special Rule 3. If either premise is negative, the conclusion must be negative (Gen. Rule VI), and distribute *P* (*see* § 72). *P* must, therefore, be distributed in the major premise, of which it is the subject (Gen. Rule IV); *i.e.*, the major premise must be universal.

It will be noticed that these rules are all hypothetical. This is because in the Fourth Figure either premise may be negative, or both may be affirmative; so that in this Figure, we cannot give a general rule as to the quality of either premise.

From these rules we deduce the following corollary :— Corollary—

Cor. Neither of the premises can be a particular negative proposition. Neither premise can be O.

This follows from Rules 1 and 3. For a negative premise requires the major to be universal (Rule 3). The major, therefore, cannot be **O**. And if the major is affirmative, the minor must be universal (Rule 1), and, consequently, cannot be **O**. Hence, neither premise can be particular negative.

(v.) **Classification of Special Rules.** The special rules of each Figure are intended to guard against any infraction of the General Rules III and IV, *i.e.* they are rules of *quantity*. No special rule is given when the application of one of these general rules is obvious and immediate. The following table shows which special rule in each Figure provides against the fallacy of breaking either of those General Rules : The special rules are rules of quantity.

Fallacy guarded against	Fig. I	Fig. II	Fig. III	Fig. IV
Undistributed Middle .	1	2		1
Illicit Major	2	1	1	3
Illicit Minor			2	2

General Rule IV is immediately applicable to the minor term of Figures I and II; or we may say more specifically: If the minor premise in Figures I and II is particular, the conclusion must be particular.

Corollary 1 to the General Rules of Syllogism [*see* § 111 (iv.)] secures us against an Undistributed Middle in Figure III.

* 115. Characteristics of each Figure.

Syllogisms in every figure are conclusive.

The scholastic logicians, following Aristotle, regarded the first figure as the only perfect and cogent form of syllogistic inference, and asserted that the validity of syllogisms expressed in any of the other figures could only be made evident by reducing them to that figure [*cf.* §§ 110 (ii.) (*a*); 126]. But, as all syllogistic argument is really based on the fundamental principles of thought (*see* § 109, *cf.* §§ 17-20), such reasoning is perfectly conclusive in every one of the four figures; in any one, denial of the conclusion involves a denial of one of those fundamental principles. Moreover, there are various kinds of arguments, directed to the establishment of certain classes of conclusions, which fall into either the second or the third figure more naturally than into the first. Each figure, indeed, has its peculiar characteristics, and, with the exception of the fourth, its appropriate sphere. These we will now briefly examine.

Figures II and III are appropriate to certain kinds of arguments.

Fig. I best shows the character of syllogistic inference.

(i.) **The First Figure.** In the first figure alone the distinctive character of syllogistic inference—the subsumption of a special case under a general rule—is shown by the very form of the argument. Moreover, conclusions of each of the four forms of categorical proposition—**A, E, I, O**—can be proved in this figure; and **A** can be the conclusion in no other figure (*see* § 116). It is, thus, the figure in which deductions from general scientific principles are most frequently expressed; for deductive science chiefly aims at establishing universal affirmative propositions. Further, in this figure neither of the extreme terms suffers an inversion of position; for the minor term is subject, and the major term predicate, both in premise and in conclusion. Thus,

A, E, I, O, can all be proved in Fig. I; **A** can be proved in no other figure.

S and *P* occupy same positions in premises as in conclusion.

that element of each term—whether denotation or connotation—which is predominant in the premises remains predominant in the conclusion. Book IV. Ch. III.

On these accounts, and because the *Dictum de omni et nullo* applies directly to syllogisms in the first figure only, it was regarded by Aristotle as the only perfect figure, an opinion in which he was followed not only by his scholastic disciples but by such modern logicians of eminence as Sir W. Hamilton. Its superiority over the other figures, however, may be granted without rejecting them as worthless.

Fig. II can prove negatives only.

(ii.) **The Second Figure.** In this figure negative conclusions only can be proved [*see* § 114 (ii.)]; it is, consequently, most employed in arguments intended to disprove some assertion. It has been called the *Exclusive Figure*, because, by means of a succession of syllogisms in it we can exclude, one by one, every possible predicate of a subject but one. Thus:—

By a series of syllogisms in Fig. II we can exclude all possible predicates but one—*Abscissio infiniti.*

S either is, or is not, A;
But, Every A is X,
and S is not X,
∴ S is not A.
If S is not A it either is, or is not, B;
But, Every B is Y,
and S is not Y,
∴ S is not B.

And so on, till we are left with only one possible conclusion —*S is P.* Such a process is called *abscissio infiniti*, because it is a repeated excision of what the subject is not. The following, which can be easily thrown into the above form, may be given as an illustration:—This act either was, or was not, done deliberately; but the doer is not a thoughtless man who would act without deliberation. If it was done deliberately it either was, or was not, done from a sense of duty; but the doer is not a man who would disregard his conscience. If it was done from a sense of duty, these painful consequences either were, or were not, foreseen; but these consequences always follow such an act, and so could

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not have been unforeseen. Hence, these painful consequences were foreseen, and we must conclude that the man voluntarily faced personal discomfort from a sense of duty.

P undergoes inversion of position.

The second figure is not so natural as the first, in that one of the extremes suffers inversion, the major term being subject in the major premise and predicate in the conclusion. This involves a change from the denotative to the connotative reading of that term (*cf.* §§ 68, 84).

Fig. III can prove particulars only.

(iii.) **The Third Figure.** In this figure particular conclusions only can be proved; it is, therefore, specially adapted to the establishment of exceptions to a general rule. Arguments in which the middle term is singular, or definite in quantity, in both premises fall naturally into the third figure, as such terms are the true subjects of the propositions in which they occur (*cf.* § 68). In this figure, as in the second, there is one inversion of position, the minor term being predicate in its premise, and subject in the conclusion. This, of course, involves a change from the connotative to the denotative aspect of that term.

S undergoes inversion of position.

Fig. IV is of little importance.

(iv.) **The Fourth Figure.** But few syllogisms find a natural expression in this figure, as in it there is a complete inversion of the order of thought. The minor term is predicate in its premise and subject in the conclusion, whilst the major term is subject in its premise and predicate in the conclusion. Each of the extreme terms, therefore, appears in a different aspect in the conclusion from that which it bears in its premise. The chief value of the fourth figure, indeed, is theoretical; as it is a possible arrangement of terms, its recognition as such is necessary to complete the formal doctrine of figure (*cf.* § 113).

Both *S* and *P* undergo inversion of position.

Summary.

(v.) **Summary.** We have now shown that each figure —with the exception of the fourth—has its appropriate sphere, though the first is the most natural, as it retains one order of thought throughout. The special uses of each are thus expressed by Lambert: "The first figure is "suited to the discovery or proof of the properties of a "thing; the second to the discovery or proof of the dis-

"tinctions between things; the third to the discovery or "proof of instances and exceptions; the fourth to the "discovery or exclusion of the different species of a genus." On this last we may observe that the relation of species and genus would be much more satisfactorily established by a syllogism in the first figure, in which the name of the species is the minor, and that of the genus the major, term than by one in the fourth figure, in which the major term denotes the species and the minor term the genus.

116. Determination of Valid Moods.

Mood depends on the quality and quantity of premises and conclusion.

Mood is the form of a syllogism as determined by the quality and quantity of the three constituent propositions, *e.g.*, **A A A, E A E, A O O,** are different moods of syllogism. As a mood may be valid in one figure and not in another, the full description of a syllogism requires the statement both of its mood and of its figure. We must now enquire how many such fully specified syllogistic forms are valid; *i.e.*, we must determine the number of valid moods of syllogism, using the word 'mood' in a narrower sense to denote this more specific description. Such determination may be made either directly, by enquiring what premises are capable of yielding each of the four possible forms of conclusion—**A, E, I, O**; or indirectly, by examining all possible combinations of premises and excluding those which offend against any of the syllogistic rules. The latter mode of procedure is the more commonly adopted in text-books on Logic, but it is both less philosophical and less scientific than the former. We will, therefore, examine the direct methods only.

In a narrower sense 'Mood' specifies both mood and figure.

The number of valid moods can be determined directly:

(i.) **Direct Determination.** We may directly determine the number of valid syllogistic forms—or 'moods' in the narrower sense—by appealing immediately to the fundamental Principles of Thought which form the ultimate basis of syllogistic reasoning (*see* § 109; *cf.* §§ 17 and 18); to the General Rules of Syllogism (*see* § 111); or to the Special Rules of each Figure (*see* § 114). We will examine the two former of these in turn.

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(a) By reference to the Laws of Thought.

A can only be proved in the mood **A A A** in Fig. I.

(*a*) *By Reference to the Fundamental Principles of Thought.* The conclusion to a categorical syllogism must be of one of the forms **A**, **E**, **I**, or **O**.

(1) *To prove* **A**. If P is to be affirmed of every S through the medium of M, it is evident, by the Principle of Identity, (*see* § 17), that P must be affirmed of every M, and that the connotation of M must be affirmed of every S; *i.e.*, that every S must be M, and every M must be P. Thus, the only premises which yield an **A** conclusion are $M\ a\ P$, $S\ a\ M$; and the syllogism is

$$\begin{array}{c} M\ a\ P \\ \underline{S\ a\ M} \\ \therefore S\ a\ P \end{array}$$

which is the mood **A A A** in Figure 1.

E can be proved in: **E A E** in Fig. I. **E A E**, **A E E** in Fig. II. **A E E** in Fig. IV.

(2) *To prove* **E**. If P is to be denied of every S through the medium of M, then, by the Principles of Identity and Contradiction (*see* §§ 17, 18), M must be affirmed of the whole extent of the denotation of one of the extremes, and entirely excluded from the denotation of the other.

Now, M is entirely separated from P when the major premise is either $M\ e\ P$ or $P\ e\ M$. Combining each of these with $S\ a\ M$, in which M is affirmed of every S, we get

$$\begin{array}{ccc} (1) & \qquad & (2) \\ M\ e\ P & & P\ e\ M \\ \underline{S\ a\ M} & & \underline{S\ a\ M} \\ \therefore S\ e\ P & & \therefore S\ e\ P \end{array}$$

Each of these is of the form **E A E**, the first in Figure I, the second in Figure II.

Similarly, M is entirely separated from S when the minor premise is either $S\ e\ M$ or $M\ e\ S$. Combining each of these with $P\ a\ M$, in which M is affirmed of every P, we get

$$\begin{array}{ccc} (3) & \qquad & (4) \\ P\ a\ M & & P\ a\ M \\ \underline{S\ e\ M} & & \underline{M\ e\ S} \\ \therefore S\ e\ P & & \therefore S\ e\ P \end{array}$$

Each of these is of the form **A E E**, the first in Figure II, and the second in Figure IV.

There are, thus, four moods in which **E** can be proved—one in Figure I, two in Figure II, and one in Figure IV.

I can be proved in: A I I in Fig. I. A I I, I A I, A A I in Fig. III. I A I, A A I in Fig. IV

(3) *To prove* **I.** If P is to be affirmed of an indefinite part of the denotation of S through the medium of M, then by the Principle of Identity (*see* § 17), an indefinite portion of the denotation of each of the extremes must agree with one and the same portion of the denotation of M. This can only be assured when one, at least, of the extremes is affirmed of *every* M, and, at the same time, agreement to a more or less indefinite extent is predicated between M and the other extreme.

If P is affirmed of every M, and there is wholly indefinite agreement between S and M, we have

(1)	(2)
$M\ a\ P$	$M\ a\ P$
$S\ i\ M$	$M\ i\ S$
$\therefore S\ i\ P$	$\therefore S\ i\ P$

These are both of the form **A I I**, the first in Figure I, and the second in Figure III.

If S is affirmed of every M, and there is agreement, wholly or partially indefinite, between the denotation of M and that of P, we have

(3)	(4)	(5)	(6)
$M\ i\ P$	$M\ a\ P$	$P\ i\ M$	$P\ a\ M$
$M\ a\ S$	$M\ a\ S$	$M\ a\ S$	$M\ a\ S$
$\therefore S\ i\ P$	$\therefore S\ i\ P$	$\therefore S\ i\ P$	$\therefore S\ i\ P$

Of these (3) and (5) are in the mood **I A I**, and (4) and (6) in **A A I**; (3) and (4) are in Figure III, and (5) and (6) in Figure IV. There are thus seen to be six moods in which **I** can be proved—one in Figure I, three in Figure III, and two in Figure IV.

(4) *To prove* **O.** If P is to be denied of an indefinite

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O can be proved in:
EIO in Fig. I.
EIO, AOO in Fig. II.
EAO, EIO, OAO in Fig. III.
EAO, EIO in Fig. IV.

portion of the denotation of *S* through the medium of *M*, then, by the Principles of Identity and Contradiction (*see* §§ 17, 18), either

(α) *P* must be denied of certain *M's* which are affirmed to be *S*, or

(β) *M* must be both affirmed of every *P* and denied of some *S's*.

(α) In the first case either *M* must be entirely separated from every *P*, and agree, in whole or to an indefinite extent, with some *S's*; or *P* must be denied of an indefinite number of *M's* whilst every *M* is affirmed to be *S*. The first condition is fulfilled when the major premise is either $M\ e\ P$ or $P\ e\ M$, and the minor $S\ i\ M$, $M\ a\ S$, or $M\ i\ S$. Combining each of these minors with each of the majors we get

(1)	(2)	(3)	(4)	(5)	(6)
$M\ e\ P$	$P\ e\ M$	$M\ e\ P$	$M\ e\ P$	$P\ e\ M$	$P\ e\ M$
$S\ i\ M$	$S\ i\ M$	$M\ a\ S$	$M\ i\ S$	$M\ a\ S$	$M\ i\ S$
$S\ o\ P$	$\therefore S\ o\ P$	$\therefore S\ o\ P$	$\therefore S\ o\ P$	$\therefore S\ o\ P$	$\therefore S\ o\ P$

Of these (3) and (5) are in the mood **E A O**, and all the others in the mood **E I O**; the first is in Figure I, the second in Figure II, the third and fourth in Figure III, and the fifth and sixth in Figure IV. The second condition of the first case is fulfilled by the syllogism

$$\begin{array}{c} (7) \\ M\ o\ P \\ M\ a\ S \\ \hline \therefore S\ o\ P \end{array}$$

which is in the mood **O A O** in Figure III.

(β) The second case gives the syllogism

$$\begin{array}{c} (8) \\ P\ a\ M \\ S\ o\ M \\ \hline \therefore S\ o\ P \end{array}$$

which is in the mood **A O O** in Figure II.

There are, thus, eight moods in which **O** can be proved;

one in Figure I, two in Figure II, three in Figure III, and two in Figure IV.

Summary.

Collecting our results it appears that :—

A can be proved in only one mood, and only in Figure I.

E can be proved in four moods, and in every Figure except the Third.

I can be proved in six moods, and in every Figure except the Second.

O can be proved in eight moods, and in every Figure.

Thus **O** is seen to be proved in the greatest number of moods, and **A** in the smallest. But these propositions are contradictories, and the establishment of the one disproves the other. Hence, it is often said that **A** is the most difficult proposition to establish and the easiest to disprove. At the same time, it must be remembered that "*universal* "*affirmative* conclusions have the highest *scientific value*, be-"cause they advance our knowledge in a positive manner "and admit of reliable application to the individual. The "*universal negatives* come next ; they guarantee only a nega-"tive but a distinctly definite view. Then come the *par-*"*ticular affirmatives*, which promise a positive advance, but "leave us helpless in the application to individual cases. "Lastly, the *particular negative* conclusions are of the lowest "value. Particular propositions, however, are by no means "without scientific meaning. Their special service is to ward "off false generalizations. The universal negative or affirma-"tive judgment, falsely held to be true, is proved not true "by the particular affirmative or negative conclusion, which "is its contradictory opposite" (Ueberweg, *Logic*, Eng. trans., pp. 436-7).

(*b*) By reference to General Rules of Syllogism.

(*b*) *By Reference to the General Rules of Syllogism* (*see* § 111).

(1) *To prove* **A**. Both premises must be affirmative (Rule VI) ; and, consequently, distribute only their subjects

(*see* § 72). S, being distributed in the conclusion, must be distributed in—*i.e.*, be the subject of—the minor premise (Rule IV). This leaves M to be distributed in the major premise (Rule III), of which it is, therefore, the subject. Thus, we get the syllogism

$$\begin{array}{c} M\ a\ P \\ S\ a\ M \\ \hline \therefore S\ a\ P \end{array}$$

which is of the form **A A A** in Figure I.

(2) *To prove* **E**. One of the premises must be negative (Rule VI) and one must be affirmative (Rule V). Both S and P are distributed in the conclusion (*see* § 72), and must, consequently, be distributed in the minor and major premises respectively (Rule IV). M must also be distributed in one of the premises (Rule III). But the premises can between them distribute three terms only when both are universal; one is, therefore, **E**, and the other **A**. In the **E** premise, both M and one of the extremes are distributed; the other extreme must, therefore, be distributed in—*i.e.*, be the subject of—the **A** premise. In the **E** premise M may be either the subject or the predicate, as both are distributed. Hence, we get four possible syllogisms with an **E** conclusion

$$\begin{array}{cccc} (1) & (2) & (3) & (4) \\ M\ e\ P & P\ e\ M & P\ a\ M & P\ a\ M \\ S\ a\ M & S\ a\ M & S\ e\ M & M\ e\ S \\ \hline \therefore S\ e\ P & \therefore S\ e\ P & \therefore S\ e\ P & \therefore S\ e\ P \end{array}$$

Of these (1) and (2) are in the mood **E A E**, and (3) and (4) in **A E E**. The first is in Figure I, the second and third in Figure II, and the fourth in Figure IV.

(3) *To prove* **I**. Both premises must be affirmative (Rule VI). As neither S nor P is distributed in the conclusion, M is the only term whose distribution is necessary in the premises. It is immaterial whether P is or is not distributed in the major premise as this cannot affect the conclusion. But

the distribution of S in the minor premise would permit the conclusion to be **A**, and, therefore, in premises which can yield *only* **I**, S must be undistributed. This leaves us any combination of affirmative premises in which M is, and S is not, the subject of an **A** proposition. We thus get

$$
\begin{array}{cccccc}
(1) & (2) & (3) & (4) & (5) & (6) \\
M\,a\,P & M\,a\,P & M\,i\,P & M\,a\,P & P\,i\,M & P\,a\,M \\
S\,i\,M & M\,i\,S & M\,a\,S & M\,a\,S & M\,a\,S & M\,a\,S \\
\hline
\therefore S\,i\,P & \therefore S\,i\,P & \therefore S\,i\,P & \therefore S\,i\,P & \therefore S\,i\,P & \therefore S\,i\,P
\end{array}
$$

Of these (1) and (2) are in the mood **A I I**, (3) and (5) in **I A I**, and (4) and (6) in **A A I.** The first is in Figure I, the second, third, and fourth in Figure III, the fifth and sixth in Figure IV.

(4) *To prove* **O.** One premise must be negative (Rule VI). P is distributed in the conclusion (*see* § 72), and must, consequently, be distributed in the major premise (Rule IV). M must be distributed in the premises (Rule III). If the major premise is **E**, both P and M are distributed in it, and either may be its subject; no term need be distributed in the minor premise, but M may be, *i.e.*, the minor premise is either **I** or $M\ a\ S$. If the major premise is **O**, P must be its predicate, and M must be distributed in the affirmative minor premise, which will be $M\ a\ S$. If the major premise is **A**, P must be its subject, and M alone should be distributed in the negative minor premise, which will be $S\ o\ M$. Hence we get

$$
\begin{array}{cccc}
(1) & (2) & (3) & (4) \\
M\,e\,P & P\,e\,M & M\,e\,P & M\,e\,P \\
S\,i\,M & S\,i\,M & M\,a\,S & M\,i\,S \\
\hline
\therefore S\,o\,P & \therefore S\,o\,P & \therefore S\,o\,P & \therefore S\,o\,P
\end{array}
$$

$$
\begin{array}{cccc}
(5) & (6) & (7) & (8) \\
P\,e\,M & P\,e\,M & M\,o\,P & P\,a\,M \\
M\,a\,S & M\,i\,S & M\,a\,S & S\,o\,M \\
\therefore S\,o\,P & \therefore S\,o\,P & \therefore S\,o\,P & \therefore S\,o\,P
\end{array}
$$

Of these (3) and (5) are in the mood **E A O**, (7) in **O A O**,

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(8) in **A O O**, and all the others in **E I O**. The first is in Figure I, and second and eighth in Figure II, the third, fourth, and seventh in Figure III, and the fifth and sixth in Figure IV.

These results are, of course, identical with those we obtained by the more philosophical method of appealing directly to first principles.

The names in the mnemonic lines specify the moods by indicating the quality and quantity of the three propositions by the letters *a*, *e*, *i*, *o*.

(ii.) **The Mnemonic Lines.** Each method of determination has led us to the result that there are nineteen valid moods, in the narrower sense of the term—four in Figure I, four in Figure II, six in Figure III, and five in Figure IV. It is customary to designate these moods by the names which compose the following mnemonic lines; each of these names containing three vowels, and thus specifying a mood by indicating the quality and quantity of the constituent propositions by the usual symbols—**A, E, I, O**; thus *Cesare* denotes the mood **E A E** in Figure II :—

> *Barbără*, *Cēlārent*, *Dărĭi*, *Fĕrĭō*que prioris :
> *Cēsărĕ*, *Cāmēstres*, *Festīnŏ*, *Bărōcŏ*, secundæ :
> Tertia, *Dāraptī*, *Dĭsămis*, *Dātĭsī*, *Fĕlapton*,
> *Bōcardō*, *Fĕrĭsŏn*, habet : Quarta insuper addit
> *Brāmantip*, *Cămĕnes*, *Dĭmăris*, *Fēsāpŏ*, *Frĕsĭson*.

These mnemonics are given here for the convenience of referring to the moods by their ordinary names, but the full explanation of their import must be deferred till we treat of Reduction, in the next chapter.

* 117. Fundamental and Strengthened Syllogisms.

Fundamental Syllogism—no term unnecessarily distributed.

Of the nineteen valid moods of syllogism, fifteen may be called *Fundamental*, as in them neither premise is stronger than is necessary to produce the conclusion; *i.e.*, neither of the extreme terms is distributed in the premises without being distributed in the conclusion, and the middle term is distributed only once. But there are two moods in Figure III—*Darapti* and *Felapton*—and one in Figure IV—*Fesapo*, in which the middle term is distributed in both premises,

and one mood in Figure IV—*Bramantip*—in which the major term is distributed in the major premise, but not in the conclusion. These are called *Strengthened Syllogisms*, as in *Darapti* and *Felapton* either premise, in *Fesapo* the minor, and in *Bramantip* the major, premise may be weakened to its subaltern particular proposition without affecting the conclusion, and when this is done the syllogism is in one of the fundamental moods.

Strengthened Syllogism—a term distributed in the premises more than is required.

118. Subaltern Moods or Weakened Syllogisms.

Weakened Syllogism or *Subaltern Mood*—particular, instead of universal, conclusion.

When from given premises a conclusion is deduced which is weaker than the premises warrant, the syllogism is said to be *Weakened*, or to be in a *Subaltern Mood*. There can, thus, be a subaltern mood corresponding to every mood with a universal conclusion. There are five universal moods (*see* § 116), viz., *Barbara* and *Celarent* in Figure I, *Cesare* and *Camestres* in Figure II, and *Camenes* in Figure IV; and the corresponding subalterns may be named *Barbari*, *Celaront*, *Cesaro*, *Camestros*, and *Camenos*. The particular conclusions drawn in these moods are, no doubt, justified by the premises, but such weakened conclusions are misleading, as they suggest that the universal cannot be deduced. As, however, the predication in the minor premise has been made of all the denotation of the minor term, nothing can be predicated in the conclusion of one part of that denotation which cannot be similarly predicated of any other part, and, consequently, of the whole. Such syllogisms are, therefore, not admitted into the list of independent legitimate syllogisms, for their conclusions can be obtained by subalternation [*see* § 97 (i.)] from the conclusion of the corresponding fundamental syllogism. They are, indeed, practically useless, as only a part of what really results from the premises is taken.

Such syllogisms are superfluous.

* Each of these subaltern moods is a strengthened syllogism, except *Camenos* (**A E O** in Figure IV); for in each of the other four moods the minor premise may be weakened without affecting the conclusion.

* Including subaltern moods, then, there are two strengthened syllogisms in each figure:—

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In Figure I - - (A A I), (E A O);
In Figure II - (E A O), (A E O);
In Figure III - A A I, E A O;
In Figure IV - A A I, E A O;

but those in brackets, being subalterns, are superfluous. Of course, the other strengthened syllogisms, although in them a particular conclusion is inferred from two universal premises, are not subaltern moods; for the particular conclusion is the utmost the premises will allow, their superfluous information referring, not to the minor but, either to the middle or to the major term.

119. Valid Moods of the First Figure.

Fig. I shows by its form the fundamental nature of syllogistic inference

In the First Figure the fundamental nature of syllogistic reasoning — the application of a general rule to a special instance—is most clearly seen. The major premise gives the general principle, whilst the minor premise states the special case to which that general principle is to be applied Reasonings of the greatest scientific value are, therefore, most naturally expressed in this figure. There are, as we have seen (*see* § 116) four valid moods in this figure, each of which has one of the four forms of categorical proposition for its conclusion. But the difference between a mood with a particular conclusion and that with the universal conclusion of the same quality is merely in the degree of definiteness with which the general principle can be applied to the objects denoted by the minor term. The forms of argument which are ultimately distinct are, therefore, two; one proving the presence, and the other the absence, of an attribute in the special case under consideration.

A A A and E A E are the ultimately distinct moods.

Moods of Fig. I:

We will now consider each of the four moods in some detail.

1. *Barbara*—
M a P
S a M
∴ *S a P*

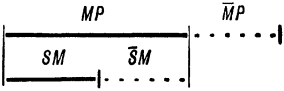
(i.) **Barbara.** This is the most important of all the forms of syllogistic inference, and the one most frequently employed — though often elliptically — not only in all branches of science but in common life; for to establish a universal connexion between subject and attribute is the

constant effort of thought, and object of research. Its schema is

$$\begin{array}{r} M\ a\ P \\ S\ a\ M \\ \hline \therefore S\ a\ P \end{array}$$

The necessity of the **A** conclusion is plainly seen when the mood is represented by the diagrams suggested in § 94—

Diagram of *Barbara.*



The line — whether wholly or only partially unbroken— which represents the total extent of *M* being drawn of the same length in each premise, it is evident that the conclusion must be *Every S is P.*

Examples of *Barbara*—

A short examination of examples in some of the chief domains of thought will establish the great importance of this mood.

From Mathematics.

All direct *mathematical* demonstrations of affirmative theorems are given exclusively in such syllogisms, though they are frequently abbreviated by the omission of an explicit statement of the major premise. For example, the whole argument of the First Proposition in the First Book of Euclid consists of three syllogisms in *Barbara* — two proving the equality of each of the newly constructed lines with that given, and each assuming as its major premise that 'all lines drawn from the centre of a circle to the circumference are equal to each other'; and one establishing the equality of the two newly constructed lines with each other, the implicit major premise being that 'things which are equal to the same thing are equal to one another.' And so throughout. "This syllogistic concatenation is the spinal "cord of mathematical demonstration. The mathematician "shortens *the form of expression*, but the syllogistic *form of* "*thought* cannot be removed without destroying the force of "the demonstration itself" (Ueberweg, *Logic*, Eng. trans., pp. 404-5).

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From Physics.

In *Physics* again the syllogistic form of thought is the only one by which particular phenomena can be explained; and here, again, *Barbara* is the most important mood. From the general law of the radiation of heat—that, unless some medium intervenes, a warm body radiates part of its heat through the atmosphere to a colder body surrounding it—we infer that, as the surface of the earth on a clear night is a warm body under those conditions, it will thus become cooled. Similarly, "the explanation of the formation of dew rests on "the syllogism: Every cooling object whose temperature is "below that of the so-called point of dew, attracts to itself "out of the atmosphere a part of the watery vapour con-"tained in it, and causes it to precipitate itself on it; the "superficies of the earth, and especially of plants, are colder "in clear nights than the atmosphere, in consequence of the "radiation of the heat to the space around; and, therefore, "when the cooling exceeds a certain limit, they attract a "portion of the watery vapour contained in the atmosphere, "and make it precipitate itself on them" (Ueberweg, *ibid.*, p. 406).

From Grammar.

In *Grammar*, again, we see the same syllogistic process. For example, we have in English Grammar the general rule that names ending in *-y* not immediately preceded by a vowel form the plural by changing the *-y* into *-i* and adding *-es;* now *lady* is such a noun; therefore, the plural of *lady* is *ladies.* In French, a verb expressing doubt is followed by the subjunctive mood; *douter* is such a verb, therefore, *douter* is followed by the subjunctive mood.

From Law.

All application of *Law* is equally syllogistic. The whole aim of legal procedure is to determine whether or not a particular case does, or does not, fall under a certain general rule, and, if it does, what are the resultant consequences. Thus, in a criminal trial, the law which has been violated furnishes the major premise, the examination of the acts of the accused person supplies the minor premise, whilst the verdict of 'Guilty' or 'Not Guilty' gives the conclusion from those premises; a conclusion to which the sentence of the judge gives practical effect.

From Medicine.

In *Medicine* the reasonings are equally syllogistic. The whole of diagnosis is an attempt to subsume a particular ailment under some general class of disease, of which the appropriate treatment is more or less known. The diagnosis is itself syllogistic; for it is an inference that the case in question is a certain kind of disease, because it exhibits certain symptoms which are the marks of that disease. Thus, the diagnosis gives the appropriate minor premise, 'This is a case of such a disease'; the treatment adopted by the physician is the practical expression of the conclusion that such and such a remedy will be efficacious. For example, 'Lupus is cured by Dr. Koch's lymph; this is a case of lupus; therefore, this case is to be treated by Dr. Koch's lymph.'

From Economics.

Reasonings in *Economics* are of a like kind. Thus, we have the general principle—which is itself deduced syllogistically from the conception of the relations of Supply and Demand—that, other things being equal, everything which tends to limit the supply of a commodity tends to raise its price; but protective duties levied on imports tend to restrict the supply of the commodities on which they are imposed; hence, it follows that such duties have a tendency to raise the prices of those commodities.

From Ethics.

In *Ethics*, too, our judgments that such and such conduct is worthy of praise are the results of a syllogistic process by which we subsume the conduct in question under a general rule. For instance, when we praise a particular hero for his patriotism we do so because we apply to his special case the general rule that all patriotism is praiseworthy.

From History.

The explanation of *historical* phenomena is another case of syllogistic inference. Thus, Schiller explains the length and violence of the Thirty Years' War by bringing it under the general principle that all religious wars are marked by the greatest pertinacity and bitterness, because every man takes one side or the other from personal feelings and not, as in ordinary wars between nations, simply on account of the place of his birth. Similarly, the fall of the Roman Empire is understood when it is regarded as an instance of the

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general law that, as nations adopt luxurious and vicious habits, they lose their pristine vigour, become effeminate, and fall an easy prey to more hardy barbarians. In like manner, both experience and reason teach us that nations which are ground down by oppression will at length burst out into revolution; by this general law we may explain the French Revolution of the Eighteenth Century, and may even foretell, with more or less assurance, the probable fate of one or two modern European states.

2. *Celarent*— $\begin{array}{l} M\,e\,P \\ \underline{S\,a\,M} \\ \therefore S\,e\,P \end{array}$

(ii.) **Celarent.** This is the typical mood in which it is proved that a certain subject does not possess certain attributes. As it is neither of so much importance, nor of so much interest, to prove what a thing is not as to show what it is, this mood is not so universally used as *Barbara.* Its schema is

$$\begin{array}{c} M\,e\,P \\ \underline{S\,a\,M} \\ \therefore S\,e\,P \end{array}$$

The necessity of the conclusion is again evident from the diagram

Diagram of *Celarent.*

which shows the entire exclusion of S from P.

Examples of *Celarent.*

As examples we may give:—'What is involuntary is not to be overcome by punishment; stupidity is involuntary; hence, stupidity cannot be overcome by punishment.' 'Duties on imports levied solely for the purposes of revenue are not protective; all English import duties are of this class; therefore, no English import duty is protective.'

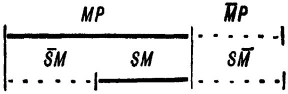
3. *Darii*— $\begin{array}{l} M\,a\,P \\ \underline{S\,i\,M} \\ \therefore S\,i\,P \end{array}$

(iii.) **Darii.** This mood is merely an indefinite form of the process of reasoning employed in *Barbara.* Its schema is

$$\begin{array}{c} M\,a\,P \\ \underline{S\,i\,M} \\ \therefore S\,i\,P \end{array}$$

and it is represented by the diagram

Diagram of *Darii*.



where $S\bar{M}$ is written under $\bar{M}P$ so as not to appear to exclude the contingency that any possibly existing $S\bar{M}$ may be P. Of course, in these diagrams, the order in which the lines denoting the various possible classes are written is of no importance.

Examples of *Darii*.

As examples of *Darii* we may give: 'Every act which is done from a strict sense of duty is formally right; some acts which mankind generally condemn are done from such a motive; therefore, some acts which are generally condemned are formally right.' 'All just governments aim at securing the welfare of their people; some autocracies have been just; therefore, some autocracies have aimed at securing the welfare of the governed.'

* The value of this mood, as of all others in which particular propositions are proved, is limited, but not destroyed, by the indefiniteness of the conclusion. For all that is indefinite is whether or not any *S's* exist of which the predication contained in the conclusion *cannot* be made. The value of the knowledge that Some *S's* are P (or, in moods with particular negative conclusions, that Some *S's* are not P) is not to be denied because the premises leave us without information concerning other possibly existing *S's*. The desire for knowledge, no doubt, must hold this limited information to be insufficient; but it is not insufficient in the sense that our conclusion has excluded the possibility that the predication can be made of every S. So long as the purely indefinite character of 'some' is borne in mind, there can be no fallacy in such an inference; for a particular conclusion does not imply the sub-contrary proposition; and the assertion of the minor premise assumes that some *S's* are known to be cases to which the general principle which is the basis of the inference may be applied.

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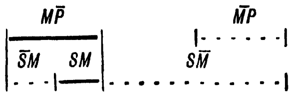
4. *Ferio*— $M\,e\,P$, $S\,i\,M$, $\therefore S\,o\,P$

(iv.) **Ferio.** This mood occupies the same relation to *Celarent* as *Darii* does to *Barbara.* The remarks made at the end of the last sub-section are, therefore, applicable here. Its schema is

$$\frac{\begin{array}{c}M\,e\,P\\ S\,i\,M\end{array}}{\therefore S\,o\,P}$$

and its diagram

Diagram of *Ferio.*



which shows the definite exclusion of Some *S's* from *P*, but leaves the existence and relation of any other *S's* problematic; so that, if any such exist, they may, or may not, be *P*.

Examples of *Ferio.*

As examples of *Ferio* may be given :— 'No act done from a right motive is deserving of punishment; some acts whose consequences are disastrous are done from a right motive; therefore, some acts whose consequences are disastrous are not deserving of punishment.' 'No protective duty is imposed for purposes of revenue; some of the French import duties are protective; therefore, some of those duties are not imposed for purposes of revenue.'

120. Valid Moods of the Second Figure.

Moods of Fig. II:

In this figure, **E** and **O** are the only possible conclusions, and each of these can be proved in two moods, in one of which the major, and in the other the minor, premise is negative. There are, therefore, four valid moods to be considered.

1. *Cesare*— $P\,e\,M$, $S\,a\,M$, $\therefore S\,e\,P$

(i.) **Cesare.** The schema of this mood is

$$\frac{\begin{array}{c}P\,e\,M\\ S\,a\,M\end{array}}{\therefore S\,e\,P}$$

and it is represented by the diagram

Diagram of Cesare.

Examples of Cesare.

The following examples may be given from Aristotle's *Nicomachean Ethics* (II, 4):—'The emotions do not make men either praiseworthy or blameworthy; the virtues and vices do this; therefore, the virtues and vices are not emotions.' 'The affections are not acts of choice; the virtues are acts of choice; therefore, the virtues are not affections.' 'Opinion is not limited in its range of objects; moral choice is so limited; therefore, moral choice is not opinion' (*ibid.*, III, 4).

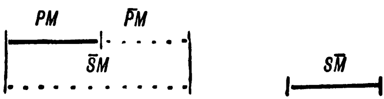
2. Camestres— $P\,a\,M$, $S\,e\,M$, $\therefore S\,e\,P$

(ii.) **Camestres.** The schema of this mood is

$$\begin{array}{l} P\,a\,M \\ S\,e\,M \\ \hline \therefore S\,e\,P \end{array}$$

and its diagram

Diagram of Camestres.



Examples of Camestres.

We may again illustrate from Aristotle's *Ethics.* 'The faculties, or capacities for feeling emotions, are natural gifts; virtues are not natural gifts; therefore, virtues are not faculties' (II, 4). Or we may say: 'All acts which are fit subjects for moral judgment are deliberate; no impulsive act is deliberate; therefore, no impulsive act is a fit subject for moral judgment.' The way for the discovery of the existence, place, and size of the planet Neptune was prepared by reasoning, which was really in this mood. The astronomer Leverrier argued that the sum total of the worlds belonging to our solar system must determine the orbit of Uranus.

and, as the known worlds did not fully do this, that, therefore, all the worlds of our solar system were not known.

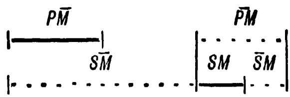
3. *Festino*— PeM SiM $\therefore SoP$

(iii.) **Festino.** This mood is the indefinite form corresponding to *Cesare.* Its schema is

$$\begin{array}{c} P\,e\,M \\ S\,i\,M \\ \hline \therefore S\,o\,P \end{array}$$

and its diagram

Diagram of *Festino.*



which shows that Some S's do not possess P, but leaves quite indefinite the relation between P and any other S's which may possibly exist.

Examples of *Festino.*

Examples of *Festino* are :—'No truthful man prevaricates ; some statesmen prevaricate ; therefore, some statesmen are not truthful.' 'No wise men are superstitious ; some educated men are superstitious ; therefore, some educated men are not wise.'

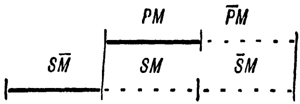
4. *Baroco*— PaM SoM $\therefore SoP$

(iv.) **Baroco.** This mood bears the same relation to *Camestres* that *Festino* does to *Cesare.* Its schema is

$$\begin{array}{c} P\,a\,M \\ S\,o\,M \\ \hline \therefore S\,o\,P \end{array}$$

and its diagram

Diagram of *Caroco.*



Of course, there is no suggestion that SM—if any such class exists—corresponds with PM; the boundaries of the classes represented by dotted lines are absolutely indefinite.

Examples of *Baroco*.

As examples of *Baroco* we may give—'Whatever is true is self-consistent; some of Hamilton's logical theories are not self-consistent; therefore, some of Hamilton's logical theories are not true.' 'All truly moral acts are done from a right motive; some acts which benefit others are not done from such a motive; therefore, some acts which benefit others are not truly moral.' 'All moral choice is fixed on the possible; some wishes relate to the impossible; therefore, some wishes are not of the nature of moral choice' (Aristotle, *Ethics*, III, 4).

121. Valid Moods of the Third Figure.

Moods of Fig. III:

In this figure, only **I** and **O** propositions can be proved, but each can be the conclusion of three moods; for both premises may be universal, or either may be particular. There are, therefore, six valid moods to be considered.

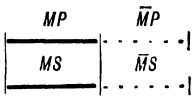
1. *Darapti*— $M\,a\,P$, $M\,a\,S$, $\therefore S\,i\,P$

(i.) **Darapti.** The schema of this mood is

$$\begin{array}{c} M\,a\,P \\ \underline{M\,a\,S} \\ \therefore S\,i\,P \end{array}$$

and its diagram

Diagram of *Darapti*.



where it is evident that the part of S which is M must correspond with the part of P which is M, but the extent, and, indeed, the existence, of any other S and P are left problematic.

Examples of *Darapti*.

An example of *Darapti* is found in Aristotle's *Ethics* (III, 7), where he argues that as 'to do or to forbear doing what is creditable or the contrary is in our own power, and these respectively constitute the being good or bad, therefore, the being good or vicious characters is in our own power.' Another example is: 'All whales are mammals; all whales are water-animals; therefore, some water-animals are mammals.'

The fact that in this mood the middle term is distributed in each premise makes it a peculiarly appropriate form in which to express those syllogisms which have two singular propositions as premises. Professor Bain (*Deductive Logic*, p. 159) denies that these are genuine syllogistic inferences at all. He takes the example

Socrates fought at Delium,
Socrates was the master of Plato,

$\therefore$ *The master of Plato fought at Delium;*

and says that "the proposition 'Socrates was the master of Plato, "and fought at Delium,' compounded out of the two premises, is "obviously nothing more than a grammatical abbreviation." The step to the conclusion "contents itself with reproducing a part of "the meaning, or saying less than had been previously said . . . "Now, we never consider that we have made a real inference, a "step in advance, when we repeat *less* than we are entitled to say, "or drop from a complex statement some portion not desired at the "moment." But the same argument would apply to every syllogism, and with especial ease to all those in the Third Figure. For, in every syllogism, the premises can be combined into a single statement, and the conclusion always says "less than had been previously said." Indeed, the fact that "we repeat less than we are entitled to say," which Professor Bain regards as fatal to the claim of such an argument to be considered a true syllogism, is the characteristic of all discursive thought, which is so called for the very reason that it *does* leave out of sight the data of which it has made full use, and concerns itself only with the predication which can be made about the special case it is considering.

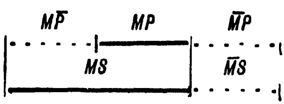
2. *Disamis*— MiP / MaS / $\therefore SiP$

(ii.) **Disamis.** The arguments in this mood are very similar to those in *Darapti*. The schema of the mood is

$$\begin{array}{c} M\,i\,P \\ \underline{M\,a\,S} \\ \therefore S\,i\,P \end{array}$$

and its diagram is

Diagarm of *Disamis.*



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As exampies we may give : 'Some pronouns in English are inflected; all such pronouns are words of English origin; therefore, some words of English origin are inflected.' 'Some gratifications of appetite are injurious to health ; all such gratifications are pleasant at the moment; therefore, some things which give pleasure at the moment are injurious to health.'

Examples of *Disamis.*

3. *Datisi*— $M\ a\ P$ / $M\ i\ S$ / $\therefore S\ i\ P$

(iii.) **Datisi.** This mood, again, is very like the last two. In fact, as an I proposition can be simply converted [*see* § 102 (ii.) (*c*)], it is a matter of very small moment whether an argument is expressed in *Disamis* or in *Datisi.* The schema of the mood is

$$\begin{array}{c} M\ a\ P \\ M\ i\ S \\ \hline \therefore\ S\ i\ P \end{array}$$

and it is represented by the diagram

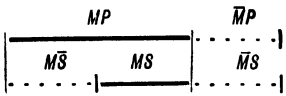

Diagram of *Datisi.*

Examples are : 'All wars cause much suffering ; some wars are justifiable ; therefore, some justifiable courses of conduct cause much suffering.' 'All diseases entail suffering ; some diseases are preventible ; therefore, some preventible causes of suffering exist.'

Examples of *Datisi.*

4. *Felapton*— $M\ e\ P$ / $M\ a\ S$ / $\therefore S\ o\ P$

(iv.) **Felapton.** This mood is the negative corresponding to the affirmative *Darapti.* Its schema is

$$\begin{array}{c} M\ e\ P \\ M\ a\ S \\ \hline \therefore\ S\ o\ P \end{array}$$

and the diagram which represents it is

Diagram of *Felapton.*

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Example of *Felapton.*

An example is: 'No brave man fears death; all brave men fear dishonour; therefore, some who fear dishonour do not fear death.'

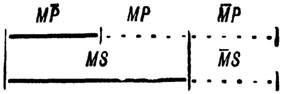
5. *Bocardo*— $M\,o\,P$, $M\,a\,S$, $\therefore S\,o\,P$

(v.) **Bocardo.** This mood gives the same conclusion as its strengthened form, *Felapton;* it is the negative corresponding to the affirmative *Disamis.* Its schema is

$$\begin{array}{c} M\,o\,P \\ M\,a\,S \\ \hline \therefore S\,o\,P \end{array}$$

and its diagram

Diagram of *Bocardo.*



which makes it plain that the S which coincides with $M\bar{P}$ is not P.

Example of *Bocardo.*

A good example is given by Ueberweg (*Logic*, Eng. trans., pp. 425-6): "Some persons accused of witchcraft have not "believed themselves to be free from the guilt laid to "their charge; all those accused of witchcraft were accused "of a merely feigned crime: hence some who were accused "of a merely feigned crime have not believed themselves "free from the guilt laid to their charge."

6. *Ferison*— $M\,e\,P$, $M\,i\,S$, $\therefore S\,o\,P$

(vi.) **Ferison.** This is *Felapton* with a weakened minor premise, and corresponds to the affirmative *Datisi.* Its schema is

$$\begin{array}{c} M\,e\,P \\ M\,i\,S \\ \hline \therefore S\,o\,P \end{array}$$

and its diagram

Diagram of *Ferison.*

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Examples of *Ferison.*

As examples we may give: 'No truly moral act is done without deliberation; some such acts are followed by painful consequences; therefore, some acts whose consequences are painful are not done without deliberation.' 'No aggressive war is justifiable; some aggressive wars are successful; therefore, some successful wars are not justifiable.'

122. Valid Moods of the Fourth Figure.

Moods of Fig. IV:

Comparatively few arguments fall naturally into the Fourth Figure. The arguments with **A**, **E**, and **I** conclusions respectively which can be expressed in it generally fall into Figure I. But if we wish to fix attention on the term which in the First Figure would be the predicate of the conclusion, we throw the argument into the Fourth Figure, where that term becomes the subject of the conclusion. The two moods in Figure IV with an **O** conclusion can generally be expressed, at least as naturally, in the Third Figure.

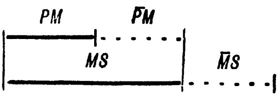
1. *Bramantip*— PaM, MaS, $\therefore SiP$

(i.) Bramantip. The schema of this mood is

$$\begin{array}{c} P\,a\,M \\ M\,a\,S \\ \hline \therefore S\,i\,P \end{array}$$

Diagram of *Bramantip.*

and it is represented by the diagram



As an argument which falls more naturally into this mood than into *Barbara*, with the extreme terms transposed in the conclusion, may be given: 'All the important operations of nature are common; things which are common escape our attention; therefore, some things which escape our attention are important operations of nature.' Similarly, from the premises 'All moderate physical exercise is beneficial to health; everything beneficial to health is inculcated by the Moral Code,' we shall, if our attention is concentrated on moral pre-

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cepts, most naturally conclude that 'Amongst the commands of the Moral Code is one which insists on moderate physical exercise.'

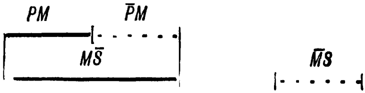
2. Camenes— $\frac{P\,a\,M \quad M\,e\,S}{\therefore S\,e\,P}$

(ii.) **Camenes.** This mood holds the same relation to *Celarent* as *Bramantip* does to *Barbara.* Its schema is

$$\begin{array}{c} P\,a\,M \\ M\,e\,S \\ \hline \therefore S\,e\,P \end{array}$$

and it is represented by the diagram

Diagram of *Camenes.*



In this diagram *S* is entirely represented by a dotted line, which implies that its existence is doubtful. The only conclusion, therefore, which is formally justified by the premises is of the conditional form—*If any S exists, it is not P.* As examples may be given: 'All squares are parallelograms; no parallelogram is a trapezoid; therefore, no trapezoid is a square.' 'All truly brave men prefer death to dishonour; no one who prefers death to dishonour is capable of a mean action; therefore, no one who is capable of a mean action is truly brave.' In both these examples we know independently that *S* exists.

Examples of *Camenes.*

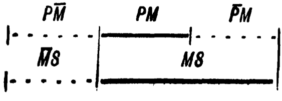
3. Dimaris— $\frac{P\,i\,M \quad M\,a\,S}{\therefore S\,i\,P}$

(iii.) **Dimaris.** This corresponds with *Darii,* as the two preceding moods do with *Barbara* and *Celarent* respectively. Its schema is

$$\begin{array}{c} P\,i\,M \\ M\,a\,S \\ \hline \therefore S\,i\,P \end{array}$$

and its diagram

Diagram of *Dimaris.*



Example of *Dimaris.*

An example is 'Some parallelograms are squares; all squares are regular figures; therefore, some regular figures are parallelograms.'

4. *Fesapo*— $P\,e\,M$, $M\,a\,S$, $\therefore S\,o\,P$

(iv.) **Fesapo.** The schema of this mood is

$$\begin{array}{l} P\,e\,M \\ \underline{M\,a\,S} \\ \therefore S\,o\,P \end{array}$$

and it is represented by the diagram

Diagram of *Fesapo.*

As the minor premise assures us of the existence of M in that sphere of existence to which the syllogism refers, we may simply convert the major premise to the categorical proposition $M\,e\,P$ [*cf.* §§ 89, 102 (ii) (*b*)], and the syllogism is then in *Felapton* in Figure III.

Examples of *Fesapo.*

As examples may be given: 'No trades-unionist is employed in this factory; all who are employed here earn good wages; therefore, some who earn good wages are not trades-unionists.' 'No inference which falls under Aristotle's definition of inferences in the First Figure is either of the form *Fesapo* or of the form *Fresison;* every inference of these forms is in the Fourth Figure; therefore, some inferences in the Fourth Figure do not fall under Aristotle's definition of inferences of the First Figure.'

5. *Fresison*— $P\,e\,M$, $M\,i\,S$, $\therefore S\,o\,P$

(v.) **Fresison.** This mood gives the same conclusion as its strengthened form, *Fesapo*. Its schema is

$$\begin{array}{l} P\,e\,M \\ \underline{M\,i\,S} \\ \therefore S\,o\,P \end{array}$$

and it is represented by the diagram

Diagram of *Fresison.*

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Example of *Fresison.*

As an example may be given: 'No noble man does mean actions; some who do mean actions succeed in life; therefore, some who succeed in life are not noble.'

123. Syllogisms and Implications of Existence.

The ordinary treatment of syllogism tacitly claims existence for every term.

The great majority of writers on Logic do not examine how far the legitimacy of the various syllogistic moods recognized as valid is dependent upon the implications of existence contained in the premises. As De Morgan says (*Formal Logic*, p. 111): "Existence "as objects, or existence as ideas, is tacitly claimed for the terms of "every syllogism." But this assumption should not be taken on trust; for the inference in any syllogism is formally invalid, if the conclusion contains an implication of existence which is not present in the premises.

If all propositions imply existence of *S*, and affirmatives of *P*, every mood is valid except *Camenes*, and its subaltern *Camenos*, in Fig. IV;

Our enquiry in § 89 led to the adoption of the view that all propositions imply the existence of their subjects in the appropriate sphere; that in affirmative propositions this involves the existence of the predicate in the same sphere; but that in negative propositions the predicate does not necessarily exist in the same sphere as the subject, though it does in some sphere. We must, therefore, see what effect this view will have upon the legitimacy of the inferences in the moods of syllogism which are generally accepted as valid because they break none of the syllogistic rules.

As one premise in every syllogism is affirmative, the existence of the middle term is always guaranteed. The extremes are, therefore, the only terms we have to consider in this connexion. Now, the inference contained in any syllogism is valid, if the conclusion does not imply the existence of any term whose existence is not guaranteed by the premises. If both premises are affirmative, the existence of every term is assured; consequently, all such syllogisms are legitimate. If the conclusion is negative, it implies the existence of *S*, but not that of *P*. Hence, this conclusion can legitimately follow from the premises only when they imply the existence of *S*. This implication is present in every case except when *S* is the predicate of a negative minor premise. *S* occupies this position in *Camenes* alone of the nineteen recognized moods. It follows that the conclusion of this mood is illegitimate as a formal inference when stated categorically. The same objection necessarily holds against the subaltern mood *Camenos*, which is the weakened form of *Camenes* (*cf.* § 118). This problematic character

of the conclusion of *Camenes* is apparent from the diagram for that mood [*see* § 122 (ii)].

124. The Representation of Syllogisms by Diagrams.

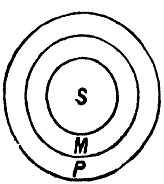
The meaning of diagrams should be obvious.

The main purpose of applying diagrams to the representation of syllogisms is to make immediately obvious to the eye, by means of geometrical figures, the relation established between the extreme terms by the premises, and, thus, to render easier the apprehension of the conclusion. The scheme of diagrams adopted in this work has been thus employed in the consideration of the valid moods of each figure (*see* §§ 119-22). It remains for us to examine how far the other schemes of diagrams described in §§ 91-3 fulfil the same purpose.

Euler's Circles are most commonly employed,

but they can only represent syllogisms by complex combinations of diagrams.

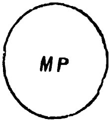
* (i.) **Euler's Diagrams.** The diagrams most commonly adopted by logicians are the circles of Euler. The fundamental objections to the application of these diagrams to the fourfold scheme of propositions have been already stated (*see* § 91). As every proposition—except **E**—requires a plurality of diagrams for its complete representation, it is evident that the combination of the two premises of a syllogism can only be fully set forth by a series of diagrams, which must, by its very complexity, go far to prevent that immediate obviousness which is an essential feature of any diagrams which are to be an aid, and not a hindrance, in apprehending the result of an argument. Simplicity has often been attained by representing each proposition by only one of the diagrams which express it; but this is erroneous and misleading. To show what is asserted by the premises, every case must be set forth, as is done by Ueberweg and Mr. Keynes. For instance, to represent *Barbara* by



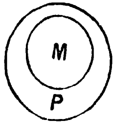
Representation of *Barbara* by Euler's circles.

—as is done by Jevons (*Primer of Logic*, p. 54) and by Mr. Stock (*Deductive Logic*, p. 200)—is to suggest that 'some' means 'some but not all,' and to ignore its absolute indefiniteness. Each premise of *Barbara* requires two diagrams to express it; thus—

M a P

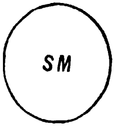


(i)

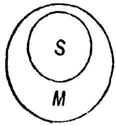


(ii)

S a M



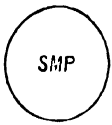
(*a*)



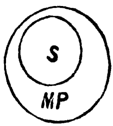
(*b*)

To represent the conclusion, we must combine each of the diagrams which express the major premise with each of those setting forth the minor premise. This gives a combination of four diagrams, and unless they are all considered, we cannot be sure that the result given by those we have examined will not be inconsistent with that yielded by those we have omitted. Thus—

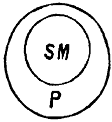
(i) and (*a*) give



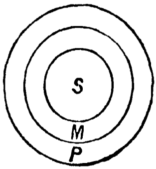
(i) and (*b*) give



(ii) and (*a*) give



(ii) and (*b*) give



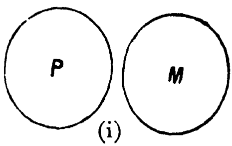
If we omit the consideration of *M*, the last three diagrams reduce to one so far as *S* and *P* are concerned, and we are left with the two diagrams which express *S a P*.

Similarly, if we combine **E** and **A** propositions as premises, we require two diagrams to represent the syllogism, for **A** can only be fully expressed by using the two diagrams given above, and **E** by diagram V. on p. 217. There are, therefore, two combinations, and these, moreover, will be lettered and interpreted differently according as the **A** proposition is the major or the minor premise.

Representation of *Festino* by Euler's circles.

If we take a syllogism involving a particular premise, the representation becomes still more complex. To take *Festino*, for instance, the major premise requires only one diagram, but for the minor four are needed—

P e M



(i)

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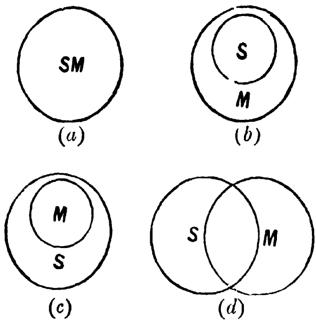
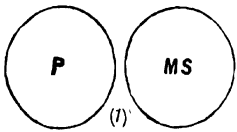
$S\ i\ M$

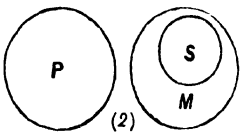


The combination of major and minor in every possible way yields no less than eight diagrams—

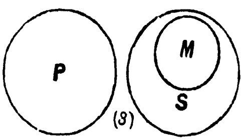
(i) and (*a*) give

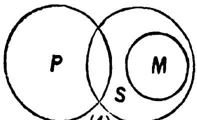


(i) and (*b*) give

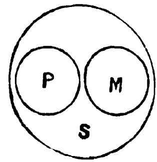


(i) and (*c*) give



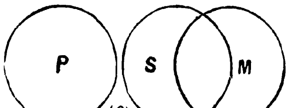


(4)

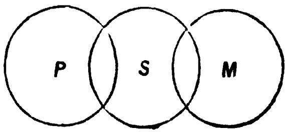


(5)

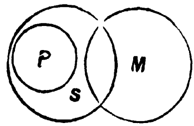
(i) and (*d*) give



(6)



(7)



(8)

From this group of figures, we have, by disregarding M. to find the relation of S and P. On examination we find that (1), (2), (3), (6) express the relation of entire mutual exclusion between S and P; that (4) and (7) represent the partial coincidence and partial exclusion of those terms, and (5) and (8) give the case in which P is entirely included in, but does not form the whole of, S. We reach, then, the three diagrams which express the proposition $S\ o\ P$. Of these diagrams Mr. Stock (*Deductive Logic*, p. 202) gives (2), (4), (6), (7), and omits the others. This writer has represented every valid mood by Euler's diagrams (*ibid.*, pp. 200-210), but in no case has he given all the Figures necessary to a complete statement.

Probably the above examples are sufficient to convince the reader that, though it may be a useful exercise of ingenuity thus to represent the different moods of the syllogism yet, the result will scarcely make the reasoning more immediately self-evident. Indeed, the chief value of this system of diagrams is the negative one of showing what premises will

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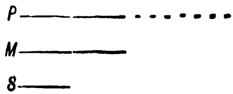
not yield a valid conclusion; when the diagrams are compatible with every possible relation between S and P—as in the case of two negative premises [*see* § 111 (ii)]—we know that no conclusion can be drawn.

Lambert's diagrams are less complex than Euler's.

(ii.) **Lambert's Diagrams.** Lambert's diagrams (*cf.* § 92) represent syllogisms with much less complexity than Euler's circles. To take the same moods as examples:—

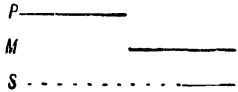
Barbara in Lambert's diagrams.

Barbara



Festino in Lambert's diagrams.

Festino



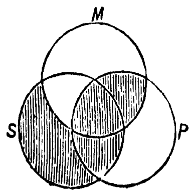
But, though simple, these diagrams are apt to be misleading—that for *Barbara*, for instance, *suggests* that S cannot be co-extensive with P, though it does not *imply* this, as the lengths of the lines do not indicate the relative extent of the classes they represent.

Dr. Venn's diagrams clearly present syllogisms with universal premises.

(iii.) **Dr. Venn's Diagrams.** Dr. Venn's system of diagrams, as has been already stated (*see* § 93), is well suited to represent universal, but not particular, propositions. Only the moods with two universal premises can, therefore, be conveniently represented in this way; but for such moods the diagrams are very neat and clear. As examples we will take *Cesare* and *Darapti*:—

Cesare in Dr. Venn's diagrams

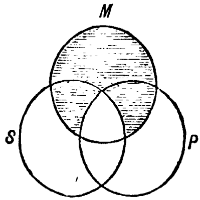
Cesare



The major premise—***P e M***—asserts the non-existence of the class ***P M*** (*cf.* § 89); we therefore shade it out in the diagram. Similarly, the minor premise—***S a M***—destroys ***S $\bar{M}$***, *i.e.*, all of ***S*** which is outside ***M***. We see at a glance that *No **S** is **P***.

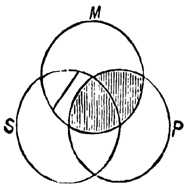
Darapti in Dr. Venn's diagrams.

Darapti



The major premise—***M a P***—destroys all ***M*** which is outside ***P***, and the minor premise—***M a S***—removes all of ***M*** which is outside ***S***. It is then immediately obvious that *Some **S** is **P***. The only way in which particular propositions can be distinguished from universals is by drawing a line through each compartment to be saved, and when this is done the conclusion is much less obvious. For example, *Festino* would be represented by

Festino in Dr. Venn's diagrams.



A further objection which lies against the use both of these diagrams and of those of Euler to represent the syllogism is that they give no indication of Figure. This objection is of more weight against Euler's system than against this of Dr. Venn; for the former was invented as an illustration of ordinary logic, but the latter is based on an interpretation of the import of propositions in which the distinction of subject and predicate no longer exists, and, when that distinction is removed, Figure necessarily disappears.

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125. Figure and Mood in Pure Hypothetical and Disjunctive Syllogisms.

Every mood of categorical syllogism has its corresponding form in pure hypothetical syllogisms,

(i.) **Pure Hypothetical Syllogisms.** As hypothetical propositions—including the modal particular forms—have the same distinctions of quality and quantity as categorical propositions (*see* § 78), it follows that they can be combined into syllogisms in exactly the same number of ways. There can, therefore, be forms of pure hypothetical syllogism corresponding to every figure and mood of categorical syllogism, and governed by the same rules [*cf.* § 112 (i.)]. But, as the universal hypothetical propositions are the only ones of much importance [*see* § 78 (ii.)], it follows that the important pure hypothetical syllogisms are those composed of such propositions; and of these, those which correspond in form to *Barbara* are the most useful, and the most frequently employed. Moreover, as the whole force of syllogistic inference consists in the necessity with which the conclusion follows from the premises (*cf.* § 107), and as this necessity is not affected by the hypothetical or categorical form in which those premises are expressed, it follows that such hypothetical premises can always be reduced to the categorical form without affecting the validity of the inference. This reduction is most conveniently made when the quantified —or conditional—forms of the hypothetical are employed, as they correspond most closely with the quantified form in which the propositions composing a categorical syllogism are usually written. Of course, when this is done, though the inference is equally necessary, the abstract and necessary character of the conclusion is more or less hidden.

but only those composed of universal propositions are important.

Hypothetical syllogisms can be expressed in categorical form.

Examples of pure hypothetical syllogisms—

It will be sufficient to give an example of a pure hypothetical syllogism in each figure expressing each of our propositions in the quantified denotative form.

1. Corresponding to *Barbara*.

Figure I. Corresponding to *Barbara* we have the form

If any S is X, that S is P,
If any S is M, that S is X,

∴ *If any S is M, that S is P.*

As a material example may be given: 'If any person is selfish, he is unhappy; if any child is spoilt, that child is selfish; therefore, if any child is spoilt, he is unhappy.'

2. Corresponding to Cesare.

Figure II. Corresponding to *Cesare* is the form

If any S is P, then never is it X,
If any S is M, then always it is X,

∴ *If any S is M, then never is it P.*

An example is: 'If any act is done from a sense of duty, it is never formally wrong; if any act is done from purely selfish motives, it is always formally wrong; therefore, if any act is done from purely selfish motives, it is not done from a sense of duty.'

3. Corresponding to Bocardo.

Figure III. Corresponding to *Bocardo* is the form

If an S is X, then sometimes it is not P,
If any S is X, then always it is M,

∴ *If an S is M, then sometimes it is not P.*

We may give as an example: 'If a war is just, it is sometimes not successful; if any war is just, it is always waged in defence of some right; therefore, if a war is waged in defence of some right, it is sometimes not successful.' Here it is evident nothing is lost by transferring the syllogism to the categorical form, and saying: 'Some just wars are not successful; all just wars are waged in defence of some right; therefore, some wars waged in defence of a right are not successful.' This has exactly the same force as the conditional form, for the latter does not imply that the want of success is a necessary consequence of the character of the war. But, in the examples with universal conclusions, it is evident there is such a dependence of consequent upon antecedent, which is lost if the syllogism be transferred to the categorical form.

4. Corresponding to Dimaris.

Figure IV. Corresponding to *Dimaris* is the form

If an S is P, it is sometimes X,
If any S is X, it is always M,

∴ *If an S is M, it is sometimes P.*

This may be illustrated by: 'If the currency of a country consists of inconvertible bank notes, it is sometimes depreciated; if the currency of any country is depreciated, it causes an artificial inflation of prices; therefore, if the currency of a country causes an artificial inflation of prices, it sometimes consists of inconvertible bank notes.' Here, again, it is evident that the antecedent does not state the necessary ground or reason for the consequent, and nothing is lost by reducing the whole argument to the categorical form.

Pure disjunctive syllogisms correspond to the affirmative moods of categorical syllogisms.

(ii.) **Pure Disjunctive Syllogisms.** The possibility of syllogisms consisting entirely of disjunctive propositions has not been usually considered by logicians. Indeed, it is only with certain limitations that such syllogisms are possible at all. They can, to begin with, only be syllogisms with an affirmative conclusion, as no disjunctive proposition can be negative [*see* § 81 (i.)]. Only the affirmative moods are, therefore, possible, and, of these, that corresponding to *Barbara* is the only one of any importance. Further, we only secure a middle term when one of the alternatives in the minor premise negatives one of those in the major premise. From

One of the alternatives in the minor must negative one of those in the major.

S is either P or Q
S is either P or R

no conclusion can be drawn, except that *S is either P or Q or R* which simply sums up the premises. But from

S is either P or Q
S is either $\bar{P}$ *or R*

we can draw the conclusion *S is either Q or R*. This will, perhaps, be more clearly seen if each premise is expressed as a hypothetical proposition. We can write the premises in the form

If S is $\bar{P}$ *it is Q*
If S is $\bar{R}$ *it is* $\bar{P}$

whence it follows that *If S is* $\overline{R}$ *it is Q*, which expresses the disjunctive *S is either Q or R*. Such syllogisms are, however, of infrequent occurrence. As the order of the alternatives is indifferent it will be seen that distinctions of figure have here no proper application.

CHAPTER IV.

REDUCTION OF SYLLOGISMS.

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126 Function of Reduction.

Reduction—changing the Figure or Mood of a syllogism.

Reduction is the process by which a given syllogistic argument is expressed in some other Figure or Mood.

Reduction to Fig. I is the most important.

Reduction has generally been confined to expressing in the First Figure arguments given in the other Figures, and though the process may be applied with equal ease to changing reasonings from any one figure to any other which contains the required conclusion, and even from one mood to another in the same figure [*cf.* § 128 (i) (*c*) (3)], yet these processes are of no great utility.

If the axiom for Fig. I is regarded as the principle of all syllogistic inference, Reduction is necessary.

There has been a good deal of dispute as to the place Reduction, in this narrower sense, should fill in syllogistic theory. The view taken on this point will depend upon the principle adopted as the basis of the syllogism. Aristotle and the scholastic logicians, who regarded the First as the only perfect figure, and the *dictum de omni et nullo* as the basis of all syllogistic inference, taught that reduction to Figure I is absolutely necessary to establish the validity of any syllogism not expressed in that figure [*see* § 110 (ii) (*a*)]. The same view is held by Kant and all other logicians who adopt a principle directly applicable to the First Figure as the basis of all syllogistic reasoning [*see* § 110 (ii)]. On the other hand, those who hold, with Lambert, that each figure rests on its own *dictum* (*see* § 114) regard reduction as both unnatural and unnecessary. The figure of a syllogism, they justly argue, is due to the nature of the pro-

Reduction is not necessary to prove validity, if each figure has its own axiom,

positions which form its premises, so that some arguments fall most naturally into figures other than the First, and to reduce them to that form is to substitute an awkward and unnatural expression for a simple and natural one. Moreover, the validity of such arguments is as immediately obvious as is that of the moods of the First Figure, and, consequently, Reduction is unnecessary. The view here adopted—that all syllogistic reasoning rests ultimately on the fundamental principles of thought, of which the *dicta* of the different figures are mere limited expressions (*see* §§ 109, 110, 114)—leads to the same conclusions. As these principles apply equally to syllogisms in all the figures, Reduction, as a proof of validity, is superfluous. But it does not follow that Reduction has no legitimate place in syllogistic theory. It is true that the reasoning does not become more cogent by being reduced to the First Figure, but its distinctive character is more immediately obvious in that figure than in any other [*see* § 115 (i)]. Reduction thus makes evident the essential unity of all forms of syllogistic inference, and systematizes the theory of syllogism by showing that all the various moods are, at bottom, expressions of but one principle.

nor if all syllogisms rest on the Laws of Thought;

but it shows the unity of the syllogistic process.

127. Explanation of the Mnemonic Lines.

The primary intention of the mnemonic lines given in § 116 (ii) is to indicate the processes by which syllogisms in figures other than the first can be reduced to that figure. This is most ingeniously done by means of the consonants employed. For convenience of reference we will here repeat the lines :—

The primary purpose of the mnemonic lines is to indicate the processes of reduction to Figure I.

Barbara, *Celarent*, *Darii*, *Ferio*que prioris :
Cesare, *Camestres*, *Festino*, *Baroco* [or *Faksoko*], secundæ :
Tertia, *Darapti*, *Disamis*, *Datisi*, *Felapton*,
Bocardo [or *Doksamosk*], *Ferison*, habet : Quarta insuper addit
Bramantip, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.

Book IV Ch. IV

The two additional names, given in square brackets, refer to the direct process of reduction, whilst *Baroco* and *Bocardo* indicate the indirect process adopted by the scholastic logicians.

Some writers replace the *c* in *Baroco* and *Bocardo* by *k*, but this letter is required for the two additional mnemonics for those moods, and cannot, therefore, be used in the older ones without confusion, as it would then denote two entirely different processes.

Explanation of the mnemonics:

The initial letters of the moods in the First Figure are the first four consonants. In the other figures :—

s—simple conversion.

s denotes simple conversion of the preceding proposition.

p—conversion *per accidens*.

p indicates that the preceding proposition is to be converted *per accidens*.

m—transpose premises.

m signifies metathesis, or transposition, of the premises.

k—obversion.

k denotes obversion of the preceding proposition.

ks—contraposition.

ks indicates obversion followed by conversion—*i.e.*, contraposition—of the preceding proposition.

sk—obverted conversion.

sk signifies that the simple converse of the preceding proposition is to be obverted.

c—indirect reduction.

c shows that the syllogism is to be reduced indirectly (*conversio syllogismi*, or change of the syllogism).

Each letter refers to the preceding proposition.

Transposition of premises necessitates conversion of new conclusion.

When one of these letters occurs in the middle of a word, one of the premises of the *original* syllogism is to undergo the process of eduction indicated. Now, when one of the changes indicated is the transposition of premises, the position of the extreme terms is reversed, and the major term of the original syllogism becomes the minor term of the new. The conclusion must, therefore, be converted to bring it to the original form. Thus every word in which *m* occurs ends in *s*, *p*, or *sk*, and these letters indicate that the conclusion of the *new* syllogism is to be converted. It will be noticed that no other significant letter ends a word. The only meaningless letters are thus seen to be *r*, *t*, *l*, *n*, and *b* and *d* when they are not initial. Several attempts so to change the forms of the

words as to omit meaningless letters, and to employ a distinctive letter for each mood have been made, but none of them is likely to replace the traditional forms.

128. Kinds of Reductions.

There are two kinds of Reduction—

It was indicated in the last secton that there are two kinds of Reduction—*Direct* and *Indirect*, the latter being usually restricted to the moods *Baroco* and *Bocardo*.

1. *Direct*—when same conclusion is deduced from premises changed by:

(i.) **Direct or Ostensive Reduction.** *Reduction is direct when the original conclusion is deduced from premises derived from those given.* The original premises are changed by conversion, transposition, or obversion.

(*a*) Conversion.

(*a*) *Conversion.*

(1) The moods *Cesare*, *Festino*, *Datisi*, *Ferison*, and *Fresison*, are reduced to the First Figure by simply converting one, or both, of the premises. For example, *Cesare* (Fig. II) becomes *Celarent* in Figure I :—

$$\begin{array}{lcl} P\,e\,M & \text{————} & M\,e\,P \\ S\,a\,M & & S\,a\,M \\ \hline \therefore S\,e\,P & & \therefore S\,e\,P \end{array}$$

and *Fresison* (Fig. IV) become *Ferio* (Fig. I) :—

$$\begin{array}{lcl} P\,e\,M & \text{————} & M\,e\,P \\ M\,i\,S & \text{————} & S\,i\,M \\ \hline \therefore S\,o\,P & & \therefore S\,o\,P \end{array}$$

A comparison of the diagrams of each of these moods (*see* §§ 119-22) will show that those in Figure III are absolutely identical with those in Fig. I, and that the others differ only in assuring the existence of P; for the existence of M is in no case doubtful in the syllogism, as it is implied in the minor premise.

(2) The moods *Darapti* and *Felapton* are reduced by converting the minor premise *per accidens*. Thus *Darapti* (Fig. III) becomes *Darii* (Fig. I) :—

$$\begin{array}{lcl} M\,a\,P & & M\,a\,P \\ M\,a\,S & \text{————} & S\,i\,M \\ \hline \therefore S\,i\,P & & \therefore S\,i\,P \end{array}$$

A comparison of diagrams again shows the equivalence of these moods as far as the relation of S and P is concerned, the presence of the possible class $\bar{S}M$ in the First Figure being quite immaterial.

(3) *Fesapo* (Fig. IV) is reduced to *Ferio* (Fig. I) by the simple conversion of its major, and the conversion *per accidens* of its minor premise :—

$$\begin{array}{rcr} P\,e\,M & \text{———————} & M\,e\,P \\ \underline{M\,a\,S} & \text{———————} & \underline{S\,i\,M} \\ \therefore S\,o\,P & & \therefore S\,o\,P \end{array}$$

The diagrams again illustrate the equivalence of the moods, the guarantee of the existence of P given in that for *Fesapo* not affecting the relation of S and P.

(b) Transposition.

(*b*) *Transposition of premises.* This, as has been seen, involves conversion of the new conclusion.

(1) The moods *Bramantip*, *Camenes*, and *Dimaris*, all in Figure IV, reduce to the First Figure by merely transposing the premises. Thus *Bramantip* becomes *Barbara* :—

$$\begin{array}{rcr} P\,a\,M & & M\,a\,S \\ \underline{M\,a\,S} & & \underline{P\,a\,M} \\ \therefore S\,i\,P & & \therefore P\,a\,S \\ & & \therefore \text{(by Conv.) } S\,i\,P \end{array}$$

A comparison of diagrams shows that they are identical if S and P are transposed in the premises — a transposition necessitated by the change in the order of the premises.

(2) *Camestres* and *Disamis* are reduced to the First Figure by transposing one premise with the simple converse of the other. Thus, *Disamis* (Fig. III) becomes *Darii* (Fig. I) :—

$$\begin{array}{rcr} M\,i\,P & & M\,a\,S \\ \underline{M\,a\,S} & & \underline{P\,i\,M} \\ \therefore S\,i\,P & & \therefore P\,i\,S \\ & & \therefore \text{(by conv) } S\,i\,P \end{array}$$

The diagrams again show the equivalence of the moods when S and P are transposed in the premises.

(c) Obversion.

(c) *Obversion.*

(1) The mnemonic *Faksoko* indicates that *Baroco* (Fig. II) may be reduced to *Ferio* (Fig. I) by contrapositing the major premise and obverting the minor. Thus :—

$$\begin{array}{lcl} P\,a\,M & \text{———————} & \bar{M}\,e\,P \\ \underline{S\,o\,M} & \text{———————} & \underline{S\,i\,\bar{M}} \\ \therefore S\,o\,P & & \therefore S\,o\,P \end{array}$$

A comparison of the diagrams shows that the $S\bar{M}$ in that for *Baroco* bears the same relation to the P as the SM does in that for *Ferio.*

(2) Similarly *Doksamosk* signifies that *Bocardo* (Fig. III) may be reduced to *Darii* (Fig. I) by contrapositing the major premise and making it the minor, and then obverting the simple converse of the new conclusion. Thus :—

$$\begin{array}{lcl} M\,o\,P & & M\,a\,S \\ \underline{M\,a\,S} & & \underline{\bar{P}\,i\,M} \\ \therefore S\,o\,P & & \therefore \bar{P}\,i\,S \\ & & \therefore \text{(by conv.) } S\,i\,\bar{P} \\ & & \therefore \text{(by obv.) } S\,o\,P \end{array}$$

A comparison of diagrams shows that $\bar{P}$ bears the same relation to S in that for *Bocardo*, as S does for P in that for *Darii.*

(3) By the use of obversion, any mood can be reduced to a mood of similar quantity, but opposite quality, in the same figure. For example, *Celarent* may be reduced to *Barbara* (Fig. I) by obverting the major premise :—

$$\begin{array}{lcl} M\,e\,P & \text{———————} & M\,a\,\bar{P} \\ \underline{S\,a\,M} & & \underline{S\,a\,M} \\ \therefore S\,e\,P & & \therefore S\,a\,\bar{P} \end{array}$$

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and *Disamis* to *Bocardo* (Fig. III) by obverting the major premise :—

$$\begin{array}{ccc} M\,i\,P & \text{———} & M\,o\,\bar{P} \\ \underline{M\,a\,S} & & \underline{M\,a\,S} \\ \therefore S\,i\,P & & \therefore S\,o\,\bar{P} \end{array}$$

but such reductions serve no useful purpose, as the difference between affirmation and negation must always remain fundamental (*cf.* § 70).

2. *Indirect*—proves a conclusion to be legitimate by showing that its contradictory is not.

(ii.) **Indirect Reduction.** *Reduction is indirect when a new syllogism is formed which establishes the validity of the original conclusion by showing the illegitimacy of its Contradictory.* This method is also called *Reductio ad impossibile*, but that name is not so appropriate as *Reductio per impossibile* or *Reductio ad absurdum.* It can be applied to any mood, though in practice it is usually confined to *Baroco* and *Bocardo*; and this application is the only one contemplated in the original mnemonics. The method is founded on the Principle of Contradiction (*see* § 18). When a conclusion is legitimately deduced from two given premises, it is formally true; when it is not so deduced from them, it is formally false. In judging of the validity of an inference, this formal truth, or self-consistency, is all we are concerned with. Now, if the conclusion is formally false, its contradictory must be formally true (*see* § 18). If this contradictory is combined with one of the original premises, a new syllogism is formed whose conclusion will either be identical with, or will contradict, the remaining original premise. If it contradicts it, it proves that the contradictory of the original conclusion was formally false, that is, that conclusion was formally true. Thus the validity of the original syllogism is established.

Method of Indirect Reduction.

Indirect Reduction of *Baroco.*

For example, *Baroco* is proved valid by a syllogism in *Barbara.* For if the conclusion, $S\ o\ P$, is formally false, then its contradictory, $S\ a\ P$, is formally true, *i.e.*, is an inference from the two premises $P\ a\ M$, $S\ o\ M$. Replacing the premise followed by *c* by this contradictory of the original con-

clusion, we get the following syllogism in *Barbara*, with P for its middle term :—

$$\begin{array}{lcl} P\ a\ M & \qquad & P\ a\ M \\ \underline{S\ o\ M} & & \underline{S\ a\ P} \\ \therefore S\ o\ P & & \therefore S\ a\ M \end{array}$$

Thus, if $S\ a\ P$ is formally true so is $S\ a\ M$. But $S\ a\ M$ contradicts $S\ o\ M$ which is one of the original premises, and is, therefore, formally false. Hence, $S\ a\ P$ is also formally false ; *i.e.*, the original conclusion, $S\ o\ P$, is formally true, and *Baroco* is a valid mood.

Indirect Reduction of *Bocardo*.

Similarly with *Bocardo*. If the conclusion, $S\ o\ P$, is formally false, its contradictory, $S\ a\ P$, is formally true. Replacing the premise followed by *c* by this proposition, we get a syllogism in *Barbara*, with S for its middle term :—

$$\begin{array}{lcl} M\ o\ P & \qquad & S\ a\ P \\ \underline{M\ a\ S} & & \underline{M\ a\ S} \\ \therefore S\ o\ P & & \therefore M\ a\ P \end{array}$$

But $M\ a\ P$ contradicts the original major premise $M\ o\ P$. Therefore, $M\ a\ P$ is formally false, and this entails the formal falsity of $S\ a\ P$. Therefore, the original conclusion, $S\ o\ P$, is formally true, and *Bocardo* is a valid mood.

This process, which was adopted by the scholastic logicians because of their dislike of negative terms, is, certainly, very cumbrous, and as both the moods to which it is commonly applied can be reduced much more simply by the direct method, it might well be banished from Logic. It should be noted that this indirect process is not reduction in the same sense as the direct method is ; in the latter, the new syllogism is the same argument as the old, in the former, it is an entirely different argument.

129. Reductions and Implications of Existence.

If every proposition implies the existence of S,

(1) On the view we have adopted (*see* § 89) that every proposition implies the existence of its subject, the simple conversion of E, and, consequently, the contraposition of A, which involves it, are invalid processes. Therefore, no reduction which involves

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but negatives do not imply that of P, the reduction of *Camenes* is invalid.

either of these processes is legitimate, unless the existence of the predicate of the E proposition which has to be converted is implied in the other premise. The simple conversion of E is involved in the reduction of the moods *Cesare, Camestres, Festino*, in Figure II, and of *Camenes*, *Fesapo*, and *Fresison* in Figure IV. In every case in which the E proposition to be converted is a premise, its predicate is M, whose existence is implied in the other premise. In *Camestres* the conclusion of the new syllogism has also to be converted, but its predicate is S, which is the subject of the original minor premise, and whose existence is, therefore, assured. In *Camenes*, however, S is the predicate of the original minor premise, which is negative; its existence, therefore, is not implied, and, consequently, the simple conversion of the new conclusion, $P\ e\ S$, is invalid. The reduction of *Camenes* is, therefore, an illegitimate process. The contraposition of A is only employed in the direct reduction of *Baroco* (*Faksoko*). Here, the obversion of the minor premise shows that the existence of $\bar{M}$ is implied in that premise, and so justifies the contraposition of the major, which involves the simple conversion of $P\ e\ \bar{M}$. The direct reduction of *Baroco* is, therefore, legitimate. The indirect reduction of *Baroco* and *Bocardo* is also valid, as, on this view, the doctrine of contradiction holds good. Our examination, then, confirms the conclusion we reached in § 123, that, on this theory of the existential import of proposition, *Camenes* alone of the recognized moods is invalid.

130. Reduction of Pure Hypothetical Syllogisms.

All pure hypothetical syllogisms can be reduced similarly to categoricals.

The validity of the reduction of any syllogism depends, upon the legitimacy of the processes of immediate inference involved. With hypothetical propositions, including the modal particulars, all these processes are valid (*see* § 105), and, therefore, pure hypothetical syllogisms can be reduced in exactly the same way as categorical syllogisms. For example, the pure hypothetical syllogism corresponding to

Examples—

Cesare (Fig. II) (*cf.* § 125) is reduced to the form in Figure I agreeing with *Celarent*, by simply converting the major premise, so that we get;—

(conv. of orig. major) *If any S is X, then never is it P,*
If any S is M, then always it is X,

∴ *If any S is M, then never is it P.*

Form agreeing with *Cesare.*

The form corresponding to *Bocardo* (Fig. III) (*cf.* § 125) is directly reduced to that agreeing with *Darii* by contrapositing the major premise and transposing the premises. The new conclusion has then to be converted, and the converse obverted. We thus get:—

Form agreeing with *Bocardo.*

(orig. minor) *If any S is X, then always it is M,*
(contrap. of orig. major) *If an S is* $\bar{P}$, *then sometimes it is X,*

If an S is $\bar{P}$, *then sometimes it is M;*
∴ (by conv.) *If an S is M, then sometimes it is* $\bar{P}$,
∴ (by obv.) *If an S is M, then sometimes it is not P.*

And the form corresponding to *Dimaris* (Fig. IV) (*cf.* § 125) is reduced to that agreeing with *Darii* by transposing the premises and converting the conclusion. Thus we get:—

Form agreeing with *Dimaris.*

(orig. minor) *If any S is X, it is always M,*
(orig. major) *If an S is P, it is sometimes X,*

∴ *If an S is P, it is sometimes M;*
∴ (by conv.) *If an S is M, it is sometimes P.*

CHAPTER V.

MIXED SYLLOGISMS.

131. Mixed Hypothetical Syllogisms.

The hypothetical premise is the major, the categorical is the minor.

When one of the premises of a syllogism is a hypothetical and the other a categorical, proposition, the former is called the major, as it furnishes the ground of the inference; whilst the latter is the minor, as it states a case in which the major is applicable. The inference conforms to the same principles whether the major premise is stated in the fundamental abstract connotative or in the derived concrete enumerative form, which we have called conditional (*see* § 76). But in the latter case the fundamental character of syllogistic inference—the application of a general principle to a special case—is perhaps more plainly seen than in the former. For, when the major premise is a conditional proposition, it lays down, in so many words, a general dependence of one phenomenon upon another, though it makes no assertion as to whether or not either of these phenomena occurs in any special instance. The categorical minor affirms, or denies, the occurrence of one of these phenomena in some special case, and thus enables us, by applying the general rule given in the major, to conclude as to the occurrence, or non-occurrence, of the other phenomenon in that same case. When, however, the major premise is stated in the abstract hypothetical form making explicit the ground for the connexion of content—*If S is M it is P*—then the application to reality is not made through some particular instance of *S*, but must be mediated by the

The character of syllogistic inference is more evident when the major is enumerative than when it is abstract in form.

ascertained nature of S itself; in other words the minor premise must be the generic judgment *S is M*, and the conclusion is the generic judgment *S is P*.

These inferences rest ultimately upon the Laws of Thought, with the Principle of Sufficient Reason as an *axioma medium*.

(i.) **Basis of Mixed Syllogistic Reasoning from a Hypothetical major premise.** As the inference in these syllogisms is as purely formal as when both the premises are categorical, it must ultimately rest on the fundamental principles of thought (*see* §§ 17-20). There is a very distinct reference to the Principle of Sufficient Reason (*see* § 20), which may indeed be regarded as the *axioma medium* of such syllogisms. This principle of thought and necessary postulate of knowledge compels us to grant the conclusion which follows from any data we have accepted. Applied to syllogisms with a hypothetical major premise this means that, if in the minor we assert the antecedent of the major to be true in fact, we must accept, as a conclusion, the truth of the consequent. But a stricter examination shows that this is an application of the Principle of Identity. On the other hand, if, in the minor, we deny the consequent of the major, we must, in the conclusion, reject the antecedent. For, by the Principle of Excluded Middle, the antecedent must be either true or false, and, if it were true, the consequent would be true; and by the Principle of Contradiction, neither the antecedent nor the consequent can be both true and false; therefore, the denial of the consequent necessitates that of the antecedent.

Inference follows from affirmation of A, or denial of O.

(ii.) **Determination of Valid Moods.** It is thus seen that the assertion of the truth of the antecedent of a hypothetical proposition justifies the assertion of the truth of the consequent, and the denial of the consequent necessitates the denial of the antecedent. But the same consequent may result from more than one antecedent; and, therefore, the denial of the given antecedent will not justify the denial of the consequent, nor will the assertion of the consequent warrant that of the given antecedent. For example, though if a man is shot through the heart he dies, yet men also die from other causes. The denial that he is shot through the

As O may follow from other antecedents besides A, no inference follows from denial of A, or affirmation of O.

heart will not, therefore, warrant the denial of his death; nor will the assertion of his death necessitate the statement that it was due to this particular cause. We may express symbolically the various antecedents which lead to the same consequent, using the most general formula of the hypothetical proposition, as in this respect it does not matter whether the consequent has the same subject as the antecedent or not (*cf.* § 76):—

If **A**, *then* **C**.
If **X**, *then* **C**.
If **Y**, *then* **C**.
If **Z**, *then* **C**.

Here, it is evident that if we deny **A**, we still leave open several possibilities of the occurrence of **C**, for either **X**, **Y**, or **Z**, may be true; and if we assert **C**, though we, thereby, assert *one* of its possible antecedents we cannot tell *which one;* nor have we, indeed, in either case, any security that all the possible antecedents of **C** are known to us. If, indeed, **A** is the only possible antecedent of **C**, its denial is a material justification for the rejection of **C**, and the affirmation of **C** is a material warranty for that of **A**. But these material conditions do not hold in all cases, and we are not, therefore, justified in assuming them in any; in formal inference we can deal only with that which holds universally.

To deny A is analogous to Illicit Major, and to affirm C corresponds to Undistributed Middle.

Now, as **C** may follow from several other antecedents besides **A**, it corresponds to an undistributed term. When, however, the denial of **C** is deduced from the denial of **A**, **C** is used universally in the conclusion. Again, when **C** is affirmed, it is affirmed in one case only out of several possible ones; to posit **A** as a result of such affirmation of **C** would be to disregard this. Thus, the fallacy of denying the antecedent is analogous to an illicit process of the major term, and that of affirming the consequent bears a similar resemblance to an undistributed middle. In each, the unwarranted assumption is made, that the major premise embraces every case in which the consequent can be true.

There are thus two, and only two, valid processes of syllogistic inference from a hypothetical major premise. They are covered by the canon :—

In *Modus Ponens*, by positing **A** we posit **C**; in *Modus Tollens*, by sublating **C** we sublate **A**; the former is a *Constructive*, the latter a *Destructive* Syllogism.

To posit the antecedent is to posit the consequent; to sublate the consequent is to sublate the antecedent.

In the former case the syllogism is said to be *Constructive*, or in the *Modus Ponens;* in the latter case, *Destructive*, or in the *Modus Tollens.*

When, in such a syllogism, the major premise is a negative hypothetical, it is more convenient, and equally natural, to regard the negation as belonging to the consequent [*see* § 78 (i) *ad fin.*]. The major may, then, take any one of four forms, as both the antecedent and the consequent may be either affirmative or negative. There can, therefore, be four forms both of the *Modus Ponens* and of the *Modus Tollens.* But it must be remembered that these names have no reference to the quality either of the minor premise or of the conclusion, but simply to whether the minor enables us, in the conclusion, to posit the consequent, or to deny the antecedent, of the major, whatever that antecedent or consequent may be. To each of these varieties of the two moods separate names are given by German logicians. These names, however, are based on the quality of the minor premise and the conclusion—*ponens* marking affirmative, and *tollens* negative, quality—and thus the same name may denote either a *Modus Ponens* or a *Modus Tollens.* Still using the one general formula to denote all forms of hypothetical propositions, these varieties of the two moods are thus expressed symbolically :—

Both **A** and **C** may be either affirmative or negative

(*A*) **Modus Ponens.**

Different forms of the *Modus Ponens.*

(1) *Modus ponendo ponens.*

If **A** *then* **C**,
A,
$\therefore$ **C**.

(2) *Modus ponendo tollens.*
If A, *then* not C,
A,
———
∴ Not C.

(3) *Modus tollendo ponens.*
If not A, *then* C
Not A,
———
∴ C.

(4) *Modus tollendo tollens*
If not A, *then* not C,
Not A,
———
∴ Not C.

Different forms of the *Modus Tollens.*

(*B*) **Modus Tollens.**

(1) *Modus tollendo tollens.*
If A, *then* C,
Not C,
———
∴ Not A.

(2) *Modus ponendo tollens.*
If A, *then* not C,
C,
———
∴ Not A.

(3) *Modus tollendo ponens.*
If not A, *then* C,
Not C,
———
∴ A.

(4) *Modus ponendo ponens.*
If not A, *then* not C,
C,
———
∴ A.

The *Modus Ponens* and the *Modus Tollens* are mutually convertible.

The identity of the names of the subordinate moods points out that the *Modus Ponens* and the *Modus Tollens* are, at bottom, identical. On comparing the majors of the moods with the same name it is seen that they are the

obverted contrapositives of each other, with the antecedent and consequent transposed. It follows that, if we obvert the contrapositive of the major of any form of the *Modus Ponens*, we shall get the corresponding form of the *Modus Tollens*; and that the latter can be similarly reduced to the former. For example, if we take the *modus ponendo ponens* of the *Modus Ponens*

If A, *then* C,
A,
∴ C,

and obvert the contrapositive of its major, we get

If not C, *then* not A,
A,
∴ C,

which is the *modus ponendo ponens* of the *Modus Tollens*. Similarly, if we take the *modus ponendo tollens* of the *Modus Tollens*,

If A, *then* not C,
C,
∴ Not A,

by obverting the contrapositive of its major we get

If C, *then* not A,
C,
∴ Not A,

which is the corresponding form of the *Modus Ponens*.

In *Modus Ponens* the conclusion must be the contradictory of A.

It must be borne in mind that as, in a hypothetical proposition when it is stated in the conditional or enumerative form, the subject of both the antecedent and the consequent are quantified, the minor may sublate the consequent of the major by affirming either its contradictory or its contrary; in each case, however, we are only justified in asserting the *contradictory* of the antecedent of the major as our conclusion. Thus, from the premises 'If all prophets spoke the truth, some would be believed; but none are believed' we

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are only justified in inferring that 'some prophets do not speak the truth,' not that 'no prophets do so.'

Examples of mixed hypothetical syllogisms.

(iii.) **Examples.** We will now give some material examples of the various forms of mixed hypothetical syllogisms :—

(*A*) **Modus Ponens.**

(1) *Modus ponendo ponens.* If any country increases in wealth, it increases in power; England is increasing in wealth; therefore, England is increasing in power.

(2) *Modus ponendo tollens.* If any import duty is imposed simply for revenue purposes, that duty is not protective; English import duties are imposed simply for purposes of revenue; therefore, English import duties are not protective.

(3) *Modus tollendo ponens.* If any swan is not white, it is black; Australian swans are not white; therefore, Australian swans are black.

(4) *Modus tollendo tollens.* If any war is not defensive, it is not just; the wars waged by Napoleon the Great were not defensive; therefore, those wars were not just.

(*B*) **Modus Tollens.**

(1) *Modus tollendo tollens.* If any country is civilized it has a population amongst whom education is general; the people of Russia are not generally educated; therefore, Russia is not a civilized country.

(2) *Modus ponendo tollens.* If any social institution is justifiable, it oppresses no class of the community; slavery does oppress a class; therefore, slavery is not a justifiable social institution.

(3) *Modus tollendo ponens.* If any railway is not required in the district through which it runs, it is a financial failure; the great English lines are not financial failures; therefore, they are required in the districts through which they run.

(4) *Modus ponendo ponens.* If any country has no capital invested abroad, its imports will not exceed its exports; England's imports do exceed her exports; therefore, England has capital invested abroad.

A few examples may be added of similar inferences when the hypothetical major has not been reduced to the fundamental form with the same subject to both antecedent and consequent.

(*A*) **Modus Ponens.**

(1) *Modus ponendo ponens.* If all men are fallible, all philosophers are fallible; but all men are fallible; therefore, all philosophers are fallible.

(2) *Modus ponendo tollens.* If all our acts are within our own control, no vice is involuntary; all our acts are within our own control; therefore, no vice is involuntary.

(3) *Modus tollendo ponens.* If vindictiveness is not a justifiable emotion, all punishment should be simply preventive; vindictiveness cannot be justified; therefore, all punishment should be simply preventive.

(4) *Modus tollendo tollens.* If seeking his own pleasure is not man's chief end, the Egoist is not truly moral; the seeking his own pleasure is not man's chief end; therefore, the Egoist is not truly moral.

(*B*) **Modus Tollens.**

(1) *Modus tollendo tollens.* If all prophets spoke the truth, some would be believed; but none are believed; therefore, some do not speak the truth.

(2) *Modus ponendo tollens.* If some of a man's deliberate acts are wholly determined by circumstances, he is not morally responsible for them; but a man is morally responsible for all his deliberate acts; therefore, no such acts are wholly determined by circumstances.

(3) *Modus tollendo ponens.* If no men were mad, lunatic asylums would be useless; but they are not useless; therefore, some men are mad.

(4) *Modus ponendo ponens.* If the earth did not rotate on its axis, there would be no alternation of day and night; there is such alternation; therefore, the earth does rotate on its axis.

Mixed hypothetical syllogisms can be expressed as categoricals; *Modus Ponens* in Fig. I; *Modus Tollens* in Fig. II.

(iv.) **Reduction to Categorical Form.** A hypothetical proposition cannot be satisfactorily reduced to the categorical form, as it includes an element of doubt as to the concrete existence of its elements which would disappear in such reduction (*see* § 77). But in a mixed hypothetical syllogism this element of doubt is removed by the categorical character of both the minor premise and the conclusion. The mediate inference of the syllogism will, therefore, be exhibited without material alteration, if we express the major in the form—*The case of* **A** *being true is the case of* **C** *being true.* The minor of the *Modus Ponens* may then be written—*This is the case of* **A** *being true*, and that of the *Modus Tollens* may take the form—*This is the case of* **C** *being false.* The *Modus Ponens* of these syllogisms is then seen to be in the First Figure, and the *Modus Tollens* in the Second. Such reduction is, however, awkward; and its only value is to give a fresh proof of the fundamental unity of the syllogistic process in whatever form it may be expressed.

132. Mixed Disjunctive Syllogisms.

A Mixed Disjunctive Syllogism, in the strict sense of the term, is one in which the inference is drawn from the disjunctive form of the major premise.

In a Mixed Disjunctive Syllogism the inference is drawn from the disjunction in the major premise.

(i.) **Basis of syllogistic inference from a disjunctive major premise.** If two alternatives are given in the major premise, the denial of one of them in the minor justifies the assertion of the other in the conclusion. Such an inference is purely formal, and is, therefore, based on the fundamental principles of thought (*see* §§ 17-19). Though the common formula for a disjunctive proposition is *S is either P or Q*, yet even here the alternation is, at bottom, between the two propositions *S is P* and *S is Q*. And an alternation may be equally well asserted between two propositions with different subjects, as *Either S is P or M* is *Q*. If, then, we denote the alternative propositions by **X** and **Y**, we shall have the simple formula for disjunctive propositions—*Either* **X** *or* **Y**, which is more comprehensive than the customary *S is either P or Q*. Now, if the major premise is the disjunctive proposition *Either* **X** *or* **Y**, we know that one, at least, of these alternatives must be true, *i.e.*, **not X** ensures **Y**. If the minor premise denies **X**, it must, by the principle of Excluded Middle (*see* § 19), affirm **not X**, and this, by the Principle of Identity (*see* § 17) justifies the affirmation of **Y**. But the alternatives may be both negative—*Either* **not X** *or* **not Y**, and this may be written *Not both* **X** *and* **Y**. Here again, if one of the alternatives is false, the other must be true ; *i.e.*, **X** ensures **not Y**. If, then, the minor posits **X**, it must, by the Principle of Contradiction (*see* § 18), deny **Y**, for, in this case, **X** and **Y** cannot be true together.

The inference rests on the Laws of Thought.

(ii.) **Forms of Mixed Disjunctive Syllogisms.** The denial of one alternative, then, justifies the affirmation of the other. And, if the number of alternatives is greater than two, the same rule holds — the denial of any number justifies the

To deny any of the alternatives is to affirm all the others.

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affirmation of the rest, categorically if only one is left, disjunctively if more than one remain. Thus :—

Either X *or* Y *or* Z,
Neither X *nor* Y,
∴ Z.

and :—

Either X *or* Y *or* Z,
Not X,
∴ *Either* Y *or* Z.

Every disjunction can be expressed as two alternatives.

But the number of alternatives may always be expressed as two by considering, for the moment, two or more of them as one ; and then both the minor premise and the conclusion retain the categorical form. This combination is most naturally effected when the alternative propositions have the same subject, so that the major premise can be written in the form *Every S is either P or Q or R*, which may be expressed as *Every S which is not P is either Q or R*. But such reduction of the number of alternatives is, of course, only apparent, and serves no good purpose.

The assertion of one alternative does not justify the denial of the other.

As a disjunctive proposition does not imply that the alternatives are mutually exclusive (*see* § 79), we cannot infer the denial of one of them from the assertion of the other. Those logicians who hold the opposite view, of course, assert that this can be done. But, even if the exclusive view were right, and *S is either P or Q* implied that *S* could not be *both P and Q*, yet when it is inferred that *S* is not *Q* because it is *P*, the inference is plainly made from the categorical proposition, *No P is Q*, which the disjunctive major premise is held to imply, instead of from that major premise itself. Such an argument, therefore, even if valid, would not be a disjunctive syllogism. We may, then, give as the canon of syllogistic inferences from a disjunctive proposition :—

Rule.

To sublate one member (or more) of any alternation is to posit the other member or members.

All mixed Disjunctive Syllogisms are in the *Modus tollendo ponens.*

This gives one mood only of mixed Disjunctive Syllogisms, commonly called the *Modus tollendo ponens* because it posits one alternative by sublating the other.

As, however, both the alternative members may be either affirmative or negative, this mood may take four forms, corresponding to the subordinate forms of the two more fundamental moods of mixed hypothetical syllogisms. Both minor premise and conclusion, therefore, may be either affirmative or negative categorical propositions. The forms are thus expressed symbolically, the first being the standard:— There are four forms of this mood, depending on the quality of the alternatives.

Statement of these forms.

(1) *Either* X *or* Y,
Not X,

∴ Y.

(2) *Either* X *or* not Y,
Not X,

∴ Not Y.

(3) *Either* not X *or* Y,
X,

∴ Y.

(4) *Either* not X *or* not Y,
X,

∴ Not Y.

(iii.) **Reduction of Mixed Disjunctive Syllogisms.** As every disjunctive proposition may be expressed in hypothetical form (*see* § 80), every disjunctive syllogism may be expressed as a mixed syllogism with a hypothetical major premise. When this is done, the above four forms are seen to be equivalent to (1) the *modus tollendo ponens*, (2) the *modus tollendo tollens*, (3) the *modus ponendo ponens*, and (4) the *modus ponendo tollens* of the *Modus Ponens* when the denial of the first alternative is taken as the antecedent of the hypothetical major premise, and to the same forms of the *Modus Tollens* when the denial of the second alternative is so taken. As every syllogism in the *Modus Ponens* is reducible to a categorical syllogism in the First Figure, and every syllogism in the *Modus Tollens* to a similar syllogism in the Second Figure [*see* § 131 (iv.)], it follows that every

Every Mixed Disjunctive Syllogism can be reduced to a mixed hypothetical syllogism;

and through these to a categorical syllogism in either Fig. I or Fig. II.

disjunctive syllogism can be expressed at will as a categorical syllogism in either of these figures. This again illustrates the essential unity of the syllogistic process, though the reduction has no other value.

Examples.

(iv.) **Examples.** As examples of the four possible forms of mixed disjunctive syllogisms we may give :—

(1) Every tax which provokes general dissatisfaction is either onerous in amount, or unjust in its incidence; the unpopular Poll Tax of Richard II was not onerous in amount; therefore, it was unjust in its incidence.

(2) Any country which maintains a protective tariff either intends to subordinate present to future advantage, or fails to see its own interests clearly; America, in maintaining her protective policy, has no intention of subordinating the interests of the present to those of the future; therefore, she fails to see her own interests clearly.

(3) Every revolution is either unjustifiable, or is provoked by oppression; the French Revolution of 1789 was justifiable; therefore, it was provoked by oppression.

(4) Any penalty which fails to diminish the crime of which it is the appointed punishment, is either of insufficient severity, or is sometimes not incurred by the criminal; the penalty for murder thus fails, and being death, is of sufficient severity; therefore, its infliction on the culprit is not certain.

We will add a few examples in which the alternatives in the major premise have not the same subject :—

(1) Either the ancient Athenians were highly civilized, or the highest artistic culture is possible amongst a people of inferior civilization; but this latter

alternative is impossible; therefore, the ancient Athenians were highly civilized.

(2) Either vice is voluntary, or man is not responsible for his actions; but man is so responsible; therefore, vice is voluntary.

(3) Either no man should be a slave, or some men are incapable of virtue; but no men are incapable of virtue; therefore, no man should be a slave.

(4) Either poverty is never due to misfortune, or desert sometimes goes unrewarded; but poverty is sometimes due to misfortune; therefore, desert does sometimes go unrewarded.

(v.) **Disjunctive Syllogisms in the wider sense.** Some logicians call every syllogism which contains a disjunctive premise a disjunctive syllogism. They thus obtain such syllogisms in every figure. For example:—

A syllogism with a disjunctive premise is not disjunctive unless the argument depends on the alternation.

FIG. I

M is either P or Q, etc.,
S is M,

∴ *S is either P or Q, etc.*

FIG. II

P is either M or N, etc.,
S is neither M or N, etc.

∴ *S is not P.*

FIG. III

M is either P or Q, etc.
M is S

∴ *Some S is either P or Q, etc.*

FIG. IV

P is M
M is either S or T, etc.

∴ *Something which is either S or T, etc., is P.*

But in such syllogisms as these the inference does not, in any sense, depend upon the disjunction. They are, indeed, merely categorical syllogisms with one or more complex terms; but this complexity has no bearing upon the process of inference, which is purely categorical. Such syllogisms should not, therefore, be called Disjunctive.

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133. Dilemmas.

Dilemma—a syllogism with a compound hypothetical major and a disjunctive minor.

A Dilemma is a syllogism with a compound hypothetical major premise and a disjunctive minor.

It gives a choice of alternatives.

In other words, the major contains a plurality either of antecedents or of consequents, which are either disjunctively affirmed, or disjunctively denied, in the minor. The peculiar feature of a dilemmatic argument is the choice of alternatives which it thus offers; and, when it is used in Rhetoric, the aim is to make these alternatives of such a kind that, whilst one must be accepted, all lead to results equally disagreeable to an opponent. Hence arose the saying 'to be on the horns of a dilemma.' Strictly speaking, a *Dilemma* contains only two alternatives; if three are offered we have a *Trilemma;* if four, a *Tetralemma;* and if more than four, a *Polylemma.* As these more complex forms are governed by the same principles as the dilemma, it will be sufficient to consider the latter.

(i.) Forms of the Dilemma.

A dilemma is either *Constructive* or *Destructive*,

(*a*) *Determination of Forms.* Like all mixed hypothetical syllogisms, a dilemma may be either *Constructive*—when the antecedents are affirmed; or *Destructive*—when the consequents are denied. In the former case, there must, of necessity, be two antecedents in the major premise, as otherwise the minor premise could not be disjunctive; but there may be either a single consequent—which the conclusion will affirm in the same form, which is usually the simple categorical; or two consequents—when the conclusion will always be disjunctive. In the former case the dilemma is *Simple;* in the latter case *Complex.* Similarly, the major premise of a destructive dilemma must contain two consequents, which may have either one or two antecedents, the dilemma being again *Simple* or *Complex* accordingly. We thus get four main forms of the dilemma, which may be expressed by the following formulæ, in which each letter represents a proposition:—

and either *Simple* or *Complex.*

Four main forms of Dilemma:

(1) *Simple Constructive.* (1) Simple Constructive.

(*a*) *If either* **A** *or* **B**, *then* **C**,
Either **A** *or* **B**,

∴ **C**.

(*b*) *If either* **A** *or* **B**, *then either* **C** *or* **D**,
Either **A** *or* **B**,

∴ *Either* **C** *or* **D**.

(2) *Simple Destructive.* (2) Simple Destructive.

(*a*) *If* **A**, *then both* **C** *and* **D**,
Either **not C** *or* **not D**,

∴ **Not A**.

(*b*) *If both* **A** *and* **B**, *then both* **C** *and* **D**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**.

(3) *Complex Constructive.* (3) Complex Constructive.

If **A**, *then* **C**, *and if* **B**, *then* **D**,
Either **A** *or* **B**,

∴ *Either* **C** *or* **D**.

(4) *Complex Destructive.* (4) Complex Destructive.

If **A**, *then* **C**, *and if* **B**, *then* **D**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**.

The second form of the Simple Constructive dilemma is simple because the alternative hypotheticals which form the major premise have only one consequent. The conclusion is disjunctive because this single consequent is disjunctive in form. Similarly, the second form of the simple destructive dilemma is not complex, although it has a disjunctive conclusion, for that conclusion is merely the simple denial of the one single antecedent of the major premise. It thus appears that these forms are not fundamental, but are only special cases of somewhat greater complexity of the simple forms (*cf.* Keynes, *Formal Logic*, 3rd Ed., pp. 317-8 *notes*).

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It will be noticed that the major premise of both forms of the simple destructive dilemma has its consequent copulative, and not disjunctive, in form. The reason is that when two consequents are alternatives their disjunctive denial will not justify the denial of the antecedent; for, if one of two alternatives is false, the other must be true (*cf.* § 79), and the truth of one consequent is all that the antecedent of such a proposition demands. It is necessary that *both* the consequents should be connected with the whole antecedent, in order that the denial of their conjunction may justify the rejection of the antecedent as a whole.

We will now illustrate each of the above forms.

Examples of Dilemmas.

(1) (*a*) *Simple Constructive.* The inhabitants of a besieged town might express their position in some such dilemma as this: 'If we hold out, we shall suffer loss by the bombardment destroying our property; if we surrender, we shall suffer loss through having to pay the enemy a heavy ransom; but we must adopt one or other of these two courses; therefore, whichever way we act, we are bound to suffer loss.'

(*b*) This form, which is more indefinite than the former, neither antecedent being limited to one consequent, is much less frequently employed. As an example of it we may give: 'If either England is over-populated or its industry is disorganized, many people must either emigrate or live in deep poverty; England at present suffers either from over-population or from disorganization of industry; therefore, many Englishmen must either emigrate or live in deep poverty.'

(2) (*a*) *Simple Destructive.* Euclid's proof of Proposition VII of the First Book may be exhibited as a dilemma of this kind: 'If two triangles on the same base, and on the same side of it, have their conterminous sides equal, then two angles are both equal and unequal to each other; but they are either not equal or not unequal; therefore, the existence of two such triangles is impossible.'

Whately (*Elements of Logic*, 5th Ed., pp. 117-8) gives the

following example of such an argument: "If we admit the "popular objections against Political Economy, we must "admit that it tends to an excessive increase of wealth: "and also, that it tends to impoverishment; but it cannot "do *both* of these; (*i.e.*, either not the one, or, not the other) "therefore we cannot admit the popular objections, &c."

(*b*) This form is very seldom used. As an example we may give: 'If compulsory education is unnecessary and no legal regulation of the conditions of the labour of children is justifiable, then all guardians of children both understand and try to perform their duty to those under their charge; but some guardians either do not understand their duty to their young wards or do not try to perform it; therefore, either compulsory education is necessary or some legal regulation of the conditions of children's labour is justifiable.'

(3) *Complex Constructive.* A good example of this form of dilemma is found in the oration of Demosthenes *On the Crown*, where he argues: 'If Æschines joined in the public rejoicings, he is inconsistent; if he did not, he is unpatriotic; but either he did or he did not; therefore, he is either inconsistent or unpatriotic.'

The following argument is in the same form: 'If the Czar of Russia is aware of the persecutions of the Jews in his country, he is a tyrant; if he is not aware of them, he neglects his duty; but either he is, or he is not, aware of them; therefore, either he is a tyrant or he neglects his duty.'

(4) *Complex Destructive.* This, again, is not a very common form. An example is: 'If the industry of England is well organized, there is work for every efficient labourer who seeks it, and if all labourers are industrious, all will seek work; but either some labourers cannot get work or they will not seek it; therefore, either the industry of England is not well organized or some labourers are idle.'

(*b*) *Mutual Convertibility of Forms.* Like the simpler mixed hypothetical syllogisms [*see* § 131 (ii.) *ad fin.*], the

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The Constructive and Destructive forms are mutually convertible by obverting the contrapositive of the major.

constructive and destructive dilemmas are, at bottom, identical; for any form of the one may be converted to the corresponding form of the other by obverting the contrapositive of the major premise. Thus, the complex destructive and complex constructive dilemmas are, fundamentally, the same, and each of the two forms of the simple destructive is mutually convertible with the corresponding form of the simple constructive dilemma. In illustration of this it will be sufficient to reduce each of the destructive to a constructive form.

Simple Destructive. (*a*) By obverting the contrapositive of the major premise and retaining the original minor, we get:—

If either **not C** *or* **not D**, *then* **not A**,
Either **not C** *or* **not D**,

∴ **Not A**;

which is the simple constructive form with negative, instead of affirmative, elements.

(*b*) Similarly, by obverting the contrapositive of the second form of the simple destructive we get the second form of the simple constructive, with negative elements:—

If either **not C** *or* **not D**, *then either* **not A** *or* **not B**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**.

Complex Destructive. The obverted contrapositive of the major premise being taken, we get:—

If **not C**, *then* **not A**, *and if* **not D**, *then* **not B**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**;

which is the complex constructive form with negative elements.

This convertibility may be illustrated by an example. A

man in bad health, and who has no income but his salary, may argue that his recovery is hopeless, either in the simple destructive dilemma: 'If I am to regain health, I must both give up work and live generously; but I cannot do both of these (*i.e.*, either I cannot do one, or I cannot do the other); therefore, I cannot regain health'; or in the simple constructive: 'If I either continue to work, or live meagrely, I cannot regain health; but I must either continue to work or live meagrely; therefore, I cannot regain health.'

Logicians differ as to the definition and forms of the dilemma.

(*c*) *Other Views.* Very great diversity exists amongst logicians as to what arguments are, and what are not, properly called dilemmas, and equally divergent definitions have been given of that form of reasoning. The forms we have given are the simplest, but they may be modified by having a hypothetical proposition for the minor premise, when, of course, the conclusion will also be hypothetical. Or again, the major may be written in the negative form when, of course, the conclusion will be negative in the constructive dilemmas, whilst in the destructive dilemmas the minor premise will be affirmative.

Whately, Mansel, and Jevons, reject the simple destructive form.

Several logicians, including Jevons, follow Whately and Mansel in recognizing only three forms of dilemma—the first form of the simple constructive, and the complex constructive and complex destructive. They so define the dilemma as to make it essential that the hypothetical major should have *more than one antecedent*. Whately (*Elements of Logic*, 5th Ed., pp. 117-8) rejects the simple destructive form, on the ground that the disjunctive denial of several consequents "comes to the same thing as *wholly* denying them; "since if they be not *all* true, the *one antecedent* must equally fall "to the ground; and the Syllogism will be equally simple." This is perfectly true, and, we may add, if it were not, such a form of reasoning would be absolutely invalid; for in every destructive hypothetical syllogism the consequent must be denied. Moreover, the same argument applies with equal force against the simple constructive form, for the disjunctive assertion of the antecedents comes to the same thing as *wholly* affirming them; since if one be true, the one consequent must equally follow. In fact, as these two forms are mutually convertible, they must stand or fall together; and the simple constructive is the most frequently employed, and the most generally acknowledged, form of dilemma. Hamilton, indeed,

But it stands on the same ground as the simple constructive.

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Hamilton includes only one form,

excludes it when (*Lect. on Logic*, vol. i., p. 350) he defines a dilemma as "a syllogism in which the sumption [*i.e.*, major premise] "is at once hypothetical and disjunctive, and the subsumption "[*i.e.*, minor premise] sublates the whole disjunction, as a conse-"quent, so that the antecedent is sublated in the conclusion." This gives the form

If **A**, *then either* **C** *or* **D**,
Neither **C** *nor* **D**,

∴ Not **A**,

which appears also to be the only one contemplated by Lotze (*see Logic*, Eng. trans., vol. i., p. 127, *cf. Outlines of Logic*, p. 70), and is recognised by Kant, Ueberweg, Thomson, Bain, and other logicians. To this Mansel (*Aldrich, Art. Log. Rud.*, 3rd Ed., p. 107) objects, on the ground that it is "merely a common dis-"junctive syllogism"; that is, its major may be expressed in the form

Either not **A**, *or* **C** *or* **D**.

This does not seem conclusive, as any hypothetical proposition can be similarly reduced to the disjunctive form; for example the major premise of the simple constructive dilemma may be expressed—*Either* **C**, *or neither* **A** *nor* **B**.

But, when we obvert the contrapositive of the major premise of the above form we get :—

If neither **C** *nor* **D**, *then* not **A**,
Neither **C** *nor* **D**,

∴ Not **A**.

which contains no true alternative of choice.

This cannot be a dilemma, for it contains no disjunction at all. It may be expressed :—

If both not **C** *and* not **D**, *then* not **A**,
Both not **C** *and* not **D**,

∴ Not **A** ;

which corresponds to the form with affirmative elements :—

If both **A** *and* **B**, *then* **C**,
Both **A** *and* **B**,

∴ **C** ;

in which the absence of any alternative element is still more plainly seen.

Ueberweg (*Logic*, Eng. trans., p. 455) regards as an essential feature of a dilemma that "whichever of the members of the dis- "junction may be true, the same conclusion results." This excludes all the complex forms, in which the conclusion is disjunctive, and includes the form we have just rejected as well as several forms in which the major premise is not hypothetical, and which he also enumerates under the head of "disjunctive inferences in the "wider sense" [*ibid.*, p. 456; *cf.* § 148 (v)]. But the element of doubt marked by the hypothetical character of the major premise is an indispensable characteristic of a dilemmatic argument, for without it there can be no real choice of alternatives.

Ueberweg excludes the complex forms, and includes some whose major is categorical.

Professor Fowler (*Deductive Logic*, pp. 114-8) recognizes both the complex dilemmas and the first form of each of the simple, and Mr. Stock (*Deductive Logic*, p. 271) adds to these the second form of the simple constructive, but does not give the corresponding form of the simple destructive.

Fowler and Stock recognize four forms.

Thomson (*Laws of Thought*, p. 203) defines a dilemma as "a "syllogism with a conditional [*i.e.*, hypothetical] premise, in which "either the antecedent or consequent is disjunctive." He gives three examples—the simple constructive, the form contemplated by Hamilton's definition, and the following modification of the latter:—

Thomson's definition is too wide.

If some A is B, either the m that are A, or the n that are A, are B,
But neither the m that are A, nor the n that are A, are B,

∴ A is not B,

where the letters symbolize terms. But this form must be rejected together with that of which it is a modification. Thomson, in fact, misses the essential point that the minor premise must be disjunctive, and his definition is much too wide, as it would cover many forms which certainly have no claim to be called dilemmas such as:—

If A, *then either* C *or* D,
A,

∴ *Either* C *or* D.

* (ii.) **Reduction of Dilemmas.** A dilemma is, formally, only a somewhat elaborate kind of mixed hypothetical syllogism, and is, consequently, governed by the same canon as such inferences, and may be reduced in the same way to the

Dilemmas, like other mixed hypothetical syllogisms, can be reduced to categorical forms.

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categorical form [*cf.* § 131 (ii), (iv)]. Thus, the Simple Constructive Dilemma may be expressed :—

The case of either **A** *or* **B** *being true is the case of* **C** *being true,*
This is the case of either **A** *or* **B** *being true,*
∴ *This is the case of* **C** *being true;*

and similarly with the other forms.

It may also be thrown into a series of pure hypothetical syllogisms.

The formal validity of a dilemma may also be exhibited by resolving it into a series of pure hypothetical syllogisms, by reducing the disjunctive minor to the hypothetical form (*see* § 80). For example, the Complex Constructive form of Dilemma may be reduced to two such syllogisms :—

(orig. major) *If* **B**, *then* **D**,
(from orig. minor) *If* **not A**, *then* **B**,
∴ *If* **not A**, *then* **D** ;
(from orig. major) *But, If* **not C**, *then* **not A**,
∴ *If* **not C**, *then* **D**,
i.e. Either **C** *or* **D**.

But such reductions are only interesting as affording a fresh proof that all syllogistic inference is of essentially the same character.

The disjunctive minor must exhaust all alternatives, or the dilemma is not cogent.

(iii.) **Rebutting a Dilemma.** The conclusiveness of a dilemma depends upon material, as well as formal, considerations. Not only must the connexion of antecedent and consequent be a real one, but the disjunction in the minor premise must exhaust every possible alternative. The difficulty of securing this is the reason dilemmatic arguments are so often fallacious.

A faulty dilemma may be rebutted by transposing the consequents, and changing their quality.

Very often a faulty dilemma can be *rebutted* or *retorted* by an equally cogent dilemma proving the opposite conclusion. In such a case, the consequents of the major change places, and their quality is changed. Thus

If **A**, *then* **C**, *and if* **B**, *then* **D**,
Either **A** *or* **B**,
∴ *Either* **C** *or* **D**,

may be rebutted by the dilemma

If **A**, *then* **not D**, *and if* **B**, *then* **not C**,
Ether **A** *or* **B**,

∴ *Either* **not C** *or* **not D**.

But the conclusion proved is not really incompatible with that of the original dilemma, for both can be satisfied by **C** and **not D** or by **D** and **not C** being true together. Only the complex constructive forms of the dilemma lend themselves to this treatment (though destructive dilemmas can be reduced to the constructive form and then rebutted), and, of course, only those in which some flaw exists in the original argument; a valid dilemma cannot be rebutted. There are several classical examples of dilemmas thus rebutted, the consideration of which will tend to make the subject clear.

Classical examples of rebutted dilemmas—

On public life.

An Athenian mother is said to have advised her son not to enter public life; 'for,' said she, 'if you act justly men will hate you, and if you act unjustly the gods will hate you; but you must act either justly or unjustly; therefore, public life must lead to your being hated.' This argument he rebutted by the equally cogent dilemma: 'If I act justly the gods will love me, and if I act unjustly men will love me; therefore, entering public life will make me beloved.' But, according to the given premises, a public man must always be both hated and loved; the given conclusions are not, therefore, incompatible.

The *Litigiosus.*

More famous is the *Litigiosus*. Protagoras agreed to train Euathlus as a lawyer, one-half the fee to be paid at once, and the other half when Euathlus won his first case. As Euathlus engaged in no suit, Protagoras sued him, and confronted him with this dilemma: 'Most foolish young man, if you lose this suit you must pay me by order of the court, and if you gain it you must pay me by our contract.' To which Euathlus retorted: 'Most sapient master, I shall not pay you; for if I lose this suit I am free from payment by our contract, and if I gain it, I am exonerated by the judgment of the court.' Of this difficulty several solutions have been offered. The most reasonable seems to be this: As Euathlus had until then won no case, the condition of the bargain was

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not fulfilled, and the judges should have decided in his favour. It was then open to Protagoras to bring a fresh suit, when the judgment must have gone against Euathlus.

The *Crocodilus*.

Somewhat similar is the *Crocodilus*. A crocodile had seized a child, but promised the mother that if she told him truly whether or not he was going to give it back, he would restore it. Fearing that if she said he was going to give it back, he would prove her wrong by devouring it, she answered, 'You will not give it back'; and argued: 'Now you must give it back—on the score of our agreement if my answer is true, and to prevent its becoming true if it is false.' But the crocodile answered: 'I cannot give it back, for if I did your answer would become false, and thus I should break our agreement; and even could your answer be correct I could not give it back, as that would make it false.' On this Lotze says: 'There is no way out of this "dilemma; as a matter of fact however both parties rest "their cases on unthinkable grounds; for the answer really "given can as little be true or untrue independently of the "actual result as could the answer she might have given, an "answer which only differs from this in being more "fortunate" (*Logic*, Eng. trans., vol. ii., p. 20). For, had she said 'You will give it back,' then its restoration would both have made her answer true and have fulfilled the agreement.

CHAPTER VI.

ABRIDGED AND CONJOINED SYLLOGISMS.

134. Enthymemes.

An Enthymeme is a syllogism abridged in expression by the omission of one of the constituent propositions.

Enthymeme— A syllogism in which one of the constituent propositions is omitted.

The most common form in which syllogistic arguments are met with is the enthymematic. The tendency of speech is always to state explicitly no more than is required for clearness; and as, in most cases, when two of the constituent propositions of a syllogism are given the third is sufficiently obvious, it would be an offence against brevity to express it in ordinary discourse. It is, therefore, but seldom that fully expressed syllogisms are met with outside treatises on Logic. Especially in epigrams and other witty sayings, the enthymematic form is common; and by means of it, charges can be insinuated which it would be impolitic to advance openly. As a good example of this we may refer to Shakespeare's famous version of Mark Antony's Oration over Cæsar's body. It is almost needless to say that when a speaker or writer draws an inference which he knows to be fallacious, he naturally adopts this shortened form of statement, as the fallacy is then less likely to be detected than it would be if the argument were set out at length. Thus, a false conclusion may be supported by a perfectly true premise, the implied premise being, of course, false.

Both premises are necessary in thought.

Although when one of the premises is omitted from a syllogism, the resulting enthymeme appears at first sight to draw the conclusion from only one premise, yet, it must be

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remembered that it does not really do so. The implied premise is equally necessary with that which is expressed as a ground for the conclusion. The abridged form of expression does not affect the form of the thought, and the inference is, therefore, fully mediate—not immediate, as it would be were the conclusion drawn from one premise alone. Indeed, as the distinction between an enthymeme and a fully expressed syllogism is, primarily, one of expression, it belongs to Rhetoric rather than to Logic.

It is more frequently the case that the omitted proposition is a premise than that it is the conclusion. Some logicians, indeed, have so defined enthymemes as to exclude the latter form altogether. But this limitation cannot be justified, as the omission of the conclusion is by no means uncommon.

Enthymemes are of three orders—

According to which proposition is omitted, enthymemes are, therefore, of three *orders*:—

1. Major omitted.

First Order—when the major premise is omitted.

2. Minor omitted.

Second Order—when the minor premise is omitted.

3. Conclusion omitted.

Third Order—when the conclusion is omitted.

For example, the argument of the fully stated syllogism: —'All democratic governments are liable to frequent changes in foreign policy; the English government is democratic; therefore, the English government is liable to frequent changes in foreign policy'—may be expressed by an enthymeme of each order:—

First Order. 'The English government is liable to frequent changes in foreign policy, because it is democratic.'

Second Order. 'The English government is liable to frequent changes in foreign policy, because all democratic governments are liable to this.'

Third Order. 'All democratic governments are liable to frequent changes in foreign policy, and the English government is democratic.'

When an enthymeme is of the first or second order, more frequently than not the conclusion is stated first, and the

premise given in its support is introduced by some such illative particle as 'because' or 'since.'

The figure of an enthymeme of the first or second order can be determined by the position of *S* or *P*.

When an enthymeme is of the third order, it is, of course, immediately obvious to which of the syllogistic figures it belongs. When it is of either the first or second order, this must be determined by the position in the given premise of either the minor or the major term. If the given premise contains the subject of the conclusion, it is, necessarily, the minor premise, and the enthymeme is of the first order; if it contains the predicate of the conclusion, the enthymeme is of the second order. Now, if both the given premise and the conclusion have the same subject, the enthymeme must be in either the First or the Second Figure; for in those figures only is *S* the subject of the minor premise. Similarly, if both the propositions have the same predicate, the figure is either the First or the Third, for in these *P* is predicate of the major premise. If the predicate of the conclusion is the subject of the given premise, the argument belongs either to Figure II or to Figure IV, in each of which *P* is the subject of the major premise. Finally, if the subject of the conclusion is the predicate of the given premise, the figure is either the Third or the Fourth, for in each of these *S* is the predicate of the minor premise.

An enthymeme need not be categorical.

In all cases of categorical enthymemes, one term is common to both the given propositions. Similarly, if the enthymeme is the abridged statement of a pure hypothetical, or of a pure disjunctive, syllogism, one of the propositions which, in those syllogisms, take the place of terms will be found in both the given propositions. For instance, 'If any child is spoilt, he is unhappy; because if any child is spoilt, he is sure to be selfish' is a pure hypothetical enthymeme of the first order. But if an enthymeme of the first order is an abridged statement of a mixed syllogism, the expressed propositions will contain no common element; as one of them will be the antecedent, and the other the consequent, of the hypothetical major. Thus, 'There is alternation of day and night, because the earth rotates on its axis' is such an enthymeme. But here we cannot limit the

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fully expressed syllogism to a single possible form, for the implied major premise may be 'If the earth rotates on its axis, there is alternation of day and night,' when the syllogism is a mixed hypothetical in the *modus ponens;* or 'Either the earth does not rotate on its axis, or there is alternation of day and night,' when the syllogism is disjunctive. But if the enthymeme is of either the second or the third order, there is no such choice.

135. Progressive and Regressive Chains of Reasoning.

Syllogisms may be connected with each other.

The matter with which thought deals forms in itself a connected whole, and the advance of knowledge continually makes this connexion more evident to us. We find that from those ultimate general principles which are so self-evident that they are called Axioms, we can deduce other principles of less generality, but which are yet themselves the immediate ground for others of still less scope; and this process may be carried on through several stages. Thus we get a train of syllogistic reasoning, in which the conclusion of each syllogism becomes a premise in that which follows; so that the last conclusion—which may be of a very specialized character—is shown to be ultimately, though indirectly, dependent upon some axiom of the widest generality. Such a process, which is very common in Mathematics, and is constantly employed by Euclid in his direct proofs, may be thus represented symbolically :—

(1)	$Y\ a\ P$	(major)	(1)	$S\ a\ Y$	(minor)
	$X\ a\ Y$	(minor)		$Y\ a\ X$	(major)
	$\therefore X\ a\ P$	(concl.)		$\therefore S\ a\ X$	(concl.)
(2)	$X\ a\ P$	(major)	(2)	$S\ a\ X$	(minor)
	$M\ a\ X$	(minor)		$X\ a\ M$	(major)
	$\therefore M\ a\ P$	(concl.)		$\therefore S\ a\ M$	(concl.)
(3)	$M\ a\ P$	(major)	(3)	$S\ a\ M$	(minor)
	$S\ a\ M$	(minor)		$M\ a\ P$	(major)
	$\therefore S\ a\ P$	(concl.)		$\therefore S\ a\ P$	(concl.)

In the former case, the conclusion of each syllogism forms the major premise of that which follows; in the latter, it becomes the minor premise, which has, in this case, been written first in each syllogism to make the connexion more prominent. Each two members of such a train of syllogisms are thus connected by a common proposition; and the syllogisms thus related are called respectively *Prosyllogism* and *Episyllogism* with respect to each other. These terms are, therefore, purely relative; and the same syllogism may be at once a prosyllogism and an episyllogism, with reference to different members of the chain of syllogisms in which it occurs; as is, in fact, the case with the second syllogism in each of the above examples. We may, therefore, give the following definitions:—

The conclusion of a *Prosyllogism* is a premise in an *Episyllogism*.

A Prosyllogism is a syllogism whose conclusion is a premise in the syllogism with which it is connected.

An Episyllogism is a syllogism one of whose premises is the conclusion of the syllogism with which it is connected.

When the reasoning starts with the prosyllogism, it is called *Progressive*, *Episyllogistic*, or *Synthetic*.

In the trains of reasoning we have just examined, the progress of thought has been from prosyllogism to episyllogism. Such a demonstration is called **Progressive, Episyllogistic,** or **Synthetic,** and it may either consist of categorical or of hypothetical propositions. A good example of the latter is given by Ueberweg (*Logic*, Eng. trans., p. 464): "If there "is a medium obstructing the motion of the planets, then "the path of the earth cannot be constant nor periodical, but "must always become less: If this be the case, then the "existence of organisms on the earth cannot have been (nor "can remain) eternal. Hence, if there is this medium, "organisms must have at one time come into existence, and "will wholly pass away. If organisms once existed for the "first time on the earth, they must have arisen out of in"organic matter. If this is the case, there has been an "original production (generatio æquivoca). Hence, if this "obstructive medium exists, there has been an original pro"duction."

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When the reasoning starts with the episyllogism, it is called *Regressive, Prosyllogistic,* or *Analytic.*

But, instead of starting from an axiom of the widest generality, in physical science it more frequently happens that the highest and most general principles are the last to be discovered. "Certain general propositions are first discovered (as, e.g. the laws of Kepler) under which the individual facts are syllogistically subsumed. The highest "principles are discovered later (e.g. the Newtonian law of "Gravitation) from which those general propositions are "necessary deductions" (Ueberweg, *ibid.*, p. 465). In such a course of reasoning, thought advances from the episyllogism to the prosyllogism, going backwards further and further towards first principles. A demonstration of this kind is, therefore, called **Regressive, Prosyllogistic,** or **Analytic.** It may be thus represented symbolically, the episyllogism being stated first, as in such a train it comes first in the order of thought:—

(1)	$S\ a\ P$	(concl.)	(1)	$S\ a\ P$	(concl.)
	$\because M\ a\ P$	(major)		$\because M\ a\ P$	(major)
	$S\ a\ M$	(minor)		$S\ a\ M$	(minor)
(2)	$M\ a\ P$	(concl.)	(2)	$M\ a\ P$	(concl.)
	$\because X\ a\ P$	(major)		$\because X\ a\ P$	(major)
	$M\ a\ X$	(minor)		$M\ a\ X$	(minor)
(3)	$X\ a\ P$	(concl.)	(3)	$S\ a\ M$	(concl.)
	$\because Y\ a\ P$	(major)		$\because Y\ a\ M$	(major)
	$X\ a\ Y$	(minor)		$S\ a\ Y$	(minor)

In the first example we have a continuous regressive chain, reaching back to the widest general statement $Y\ a\ P$, and the major only of each syllogism being established by a prosyllogism. In the second, each of the premises in the episyllogism is established by a prosyllogism, of which it forms the conclusion.

Polysyllogism —a chain of connected syllogisms.

Such a train of reasoning, whether progressive or regressive, is often called a *Polysyllogism.*

136. Sorites.

Sorites—a progressive chain of reasoning, with the conclusion of each prosyllogism omitted.

A Sorites is a progressive chain of reasoning whose expression is simplified by the omission of the conclusion of each of the prosyllogisms.

A Sorites is a series of enthymemes

The Sorites is, thus, a series of enthymemes, of which the first is of the third order, as both its premises are stated; and the last is of either the first or the second order, as one premise and the conclusion are given (*cf.* § 134). But each of the intermediate enthymemes is represented by one premise alone, as the other premise is the omitted conclusion of the preceding prosyllogism. In somewhat different words, therefore, it may be said that a sorites is a series of enthymemes, in each of which, except the first, one premise is implied by a prosyllogism, and the other is explicitly stated. From this it follows that a full analysis of a sorites resolves it into a number of separate syllogisms, less by one than the total number of premises.

In the Aristotelian Sorites, the omitted conclusions form the minor premises of the succeeding syllogisms; in the Goclenian Sorites they form the major.

(i.) **Kinds of Sorites.** It was seen in the last section that the conclusion of a prosyllogism may form either the minor or the major premise of the episyllogism. There are, consequently, two forms of sorites—the *Aristotelian*, in which the suppressed conclusions form the minor premises of the following episyllogisms; and the *Goclenian*, in which they form the major premises. The symbolic expression of each may be thus given :—

Aristotelian Sorites—Every S is X
Every X is Y
Every Y is Z
Every Z is P
∴ *Every S is P*

Goclenian Sorites—Every Z is P
Every Y is Z
Every X is Y
Every S is X
∴ *Every S is P*

It will be noticed that in the Aristotelian form the subject of the conclusion is stated first, and the predicate of the conclusion occurs in the last premise ; in the Goclenian form the subject of the conclusion is the subject of the last premise, and the predicate of the conclusion is found in the first premise. The latter form, therefore, corresponds the more closely to the customary order in which the premises of a syllogism are stated. Both forms contain the same premises, though their order is reversed. This led Hamilton to regard the Aristotelian form as expressing a reasoning in comprehension, and the Goclenian form as expressing one in extension ; since in the latter we start with the premise which contains the two terms of widest extension, and in the former we end with that premise.

A sorites may be analysed into syllogisms, one less in number than its premises.

If both forms are analysed into their constituent syllogisms, it will be seen that in the Aristotelian form the omitted conclusions—which we enclose in square brackets—form the minor, and in the Goclenian form, the major premises of the succeeding episyllogisms.

Analysis of Aristotelian Sorites.

(1) *Every X is Y* (major)
Every S is X (minor)
∴ [*Every S is Y*] (concl.)

(2) *Every Y is Z* (major)
[*Every S is Y*] (minor)
∴ [*Every S is Z*] (concl.)

(3) *Every Z is P* (major)
[*Every S is Z*] (minor)
∴ *Every S is P* (concl.)

Analysis of Goclenian Sorites.

(1) *Every Z is P* (major)
Every Y is Z (minor)
∴ [*Every Y is P*] (concl.)

(2) [*Every Y is P*] (major)
Every X is Y (minor)

∴ [*Every X is P*] (concl.)

(3) [*Every X is P*] (major)
Every S is X (minor)

∴ *Every S is P* (concl.)

Every sorites is a progressive chain of reasoning.

It is evident that the two forms agree in the fact that each omitted conclusion is a premise of the following syllogism. Now, this advance from previous to consequent inferences is the characteristic of progressive reasoning (*cf.* § 135); it is, therefore, an error to speak of the Goclenian Sorites, as some logicians have done, as a regressive form of reasoning.

A sorites may consist of hypothetical propositions; but only the last syllogism can be mixed;

Either form of sorites may be entirely composed of hypothetical propositions. In the Goclenian Sorites the last premise may be categorical, and then the concluding enthymeme is the abridged form of a mixed syllogism, in which the categorical minor premise either posits the antecedent, or sublates the consequent, of the implied conclusion of the preceding prosyllogism; *e.g.* :—

If **C**, *then* **D**,	*If* **C**, *then* **D**,
If **B**, *then* **C**,	*If* **B**, *then* **C**,
If **A**, *then* **B**,	*If* **A**, *then* **B**,
A,	**Not D**,
∴ **D**.	∴ **Not A**.

and that must be of the Goclenian form.

In the Aristotelian Sorites, however, the same result can only be obtained by adding to the sorites a categorical minor premise, and then regarding the implied conclusion of the preceding prosyllogism as the major, instead of the minor premise of the last episyllogism. In other words, a mixed syllogism at the end of a sorites must, in all cases, correspond to the Goclenian form; *e.g.* :—

If **A**, *then* **B**,	*If* **A**, *then* **B**,
If **B**, *then* **C**,	*If* **B**, *then* **C**,
If **C**, *then* **D**,	*If* **C**, *then* **D**,
A	**Not D**,
∴ **D**.	∴ **Not A**.

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Examples of sorites.

The following example of a categorical Aristotelian Sorites may be given from Aristotle (*Poet.* vi): 'Action is that in which happiness lies; what contains happiness is the end and aim; the end and aim is what is highest; therefore, action is what is highest.' As an instance of a similar sorites composed of hypothetical propositions we may give: 'If any man is avaricious, he is intent on increasing his wealth; if he is so intent, he is discontented; if he is discontented, he is unhappy; therefore, if any man is avaricious, that man is unhappy.' In the following the last syllogism is a mixed hypothetical: 'If the soul thinks, it is active; if it is active, it has strength; if it has strength, it is a substance; now the soul thinks; therefore, the soul is a substance.' In all these cases, if we reverse the order of premises, we get a sorites of the Goclenian form.

(ii.) Special Rules of the Sorites.

(*a*) *The Aristotelian Sorites.* In this form of sorites, the predicate of the last premise is, in the conclusion, affirmed or denied of the first subject, through one or more intermediate propositions. Each intermediate term must, therefore, be affirmatively predicable of the whole of the preceding one, or the chain of connexion is broken. This gives us the two following as

Rules of Aristotelian Sorites—
1. Only last premise negative.
2. Only first premise particular.

Special Rules of the Aristotelian Sorites :—

1. *Only one premise, and that the last, can be negative.*
2. *Only one premise, and that the first, can be particular.*

The necessity of these rules is evident when the sorites is analysed into its constituent syllogisms.

Rule 1. More than one premise cannot be negative; for, as a negative premise in any syllogism necessitates a negative conclusion (*see* § 111), if more than one premise in the sorites were negative, one of the constituent syllogisms would contain two negative premises.

If any premise in the sorites is negative, the conclusion must be negative ; therefore, the predicate of the conclusion must be distributed in the last premise, of which it is the predicate ; *i.e.*, the last premise must be negative.

Rule 2. As every premise except the last must be affirmative, it is evident that if any, except the first, were particular, it would involve the fallacy of undistributed middle.

(*b*) *The Goclenian Sorites.* In this form of sorites the predicate of the first premise is, in the conclusion, either affirmed, or denied, of the subject of the last, through one or more intermediate propositions. Each intermediate term must, therefore, be affirmed universally of the succeeding one, or the necessary connexion will not be secured. We, thus, get the two following as

Special Rules of the Goclenian Sorites :—

Rules of Goclenian Sorites— 1. Only first premise negative. 2. Only last premise particular.

1. *Only one premise, and that the first, can be negative.*
2. *Only one premise, and that the last, can be particular.*

A consideration of the constituent syllogisms again shows the necessity of these rules.

Rule 1. As in the Aristotelian sorites, a plurality of negative premises would result in one of the syllogisms containing two negative premises.

If any premise is negative, the conclusion must be negative ; therefore its predicate must be distributed in the first premise, of which it is the predicate ; *i.e.*, the first premise must be negative.

Rule 2. If any premise but the last were particular, the conclusion of the syllogism in which it occurred would also be particular, and, as that proposition would be the major premise of the succeeding syllogism, we should have the fallacy of undistributed middle.

The above rules assume, in each case, that the sorites is entirely in the First Figure ; *i.e.*, that each of the constituent syllogisms is in that figure. We must now enquire whether this is a necessary limitation of this form of argument.

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Hamilton held that a sorites could be in the Second and Third Figures, as well as in the First;

(iii.) **Figure of Sorites.** Hamilton held that a sorites is possible in the Second and Third Figures, as well as in the First. He says (*Lectures on Logic*, vol. ii, p. 403): "In Second and Third Figures, "there being no subordination of terms, the only Sorites competent "is that by repetition of the same middle. In First Figure, there is "a new middle term for every new progress of the Sorites; in "Second and Third, only one middle term for any number of "extremes." He thus indicates such forms as the following:—

Second Figure.	*Third Figure.*
No X is M	*Every M is X*
No Y is M	*Every M is Y*
No Z is M	*Every M is Z*
Every P is M	*Every M is P*
∴ *No X or Y or Z is P.*	∴ *Some X and Y and Z are P.*

but the reasonings he refers to are not sorites.

But neither of these is a chain argument, or sorites, at all. There is not one conclusion drawn from a succession of premises, all necessary to its establishment: but as many different conclusions as there are syllogisms, though they are summed up into one compound proposition (*cf.* § 75).

Dr. Keynes shows that sorites in Figs. II and III are possible.

Dr. Keynes, whilst agreeing in the rejection of Hamilton's forms, yet shows that a sorites is possible in both the Second and Third Figures (*see Formal Logic*, 3rd ed., pp. 330-2). In the Second Figure the form would be (the suppressed conclusions being enclosed in square brackets):—

Some A is not B,
Every C is B,
[∴ *Some A is not C*],
Every D is C,
[∴ *Some A is not D*],
Every E is D,
∴ *Some A is not E*

Of this Dr. Keynes says: "This is the only resolution of the "sorites possible unless the order of the premises is transposed, and "it will be seen that all the resulting syllogisms are in Figure II "and in the mood *Baroco*. The sorites may accordingly be said to "be in the same mood and figure. It is analogous to the Aristotelian "sorites, the subject of the conclusion appearing in the premise "stated first, and the suppressed premises being all *minors* in their "respective syllogisms" (*op. cit.*, p. 331).

As a Sorites in the Third Figure Dr. Keynes gives one of the following form (the omitted conclusions being in square brackets):—

Some D is not E,
Every D is C,
[∴ *Some C is not E*],
Every C is B,
[∴ *Some B is not E*],
Every B is A,
∴ *Some A is not E.*

"These syllogisms are all in Figure III and in the mood *Bocardo;* "and the sorites itself may be said to be in the same mood and "figure. It is analogous to the Goclenian sorites, the predicate of "the conclusion appearing in the premise stated first, and the sup-"pressed premises being *majors* in their respective syllogisms" (*ibid.*).

These examples establish Dr. Keynes' contention, and the criticisms passed upon it in the first edition of this book must be withdrawn. The examples on which that criticism was based, and which were given by Dr. Keynes in the earlier editions of his book, were susceptible, as he himself admits, of more than one analysis, and one of those analyses would resolve them into the first figure. But this is not possible with the examples given above.

The special rules of sorites given in (ii.), of course, do not apply to sorites in other figures. The occurrence of such sorites is so rare, and their importance so small, that it is not necessary to here work out such rules in detail. That task may be left to the ingenuity of the reader.

(iv.) History of Sorites. The name Sorites is not employed by Aristotle in the modern sense, though he alludes to such chains of arguments. It appears to have been first used in this way by the Stoics, from whom it was adopted by Cicero, though its common acceptance was much later. The Goclenian Sorites received its name from Goclenius, who first discussed it in his *Isagoge in Organum Aristotelis* (1598).

A chain of enthymemes was first called a sorites by the Stoics.

The Goclenian Sorites was first discussed by Goclenius.

Ancient writers used the name Sorites—σωρός, a heap—to denote a particular kind of fallacy, based on the difficulty of assigning an exact limit to a notion:—'Does one grain of corn make a heap?' 'No.' 'Do two?' 'No.' 'Do three?' 'No.' Thus the number may be successively increased by unity, till the person questioned

In ancient writers, Sorites was the name of a fallacy.

has either to contradict himself by affirming that *one* grain does make a heap of that which before its addition was not a heap, or to deny the name to a pile of corn of any assignable magnitude, no matter how enormous it might be. A similar sophism was that which the old logicians termed *Calvus*, and which began with the enquiry whether pulling *one* hair from a man's head made him bald. Similarly, it might be asked, 'When does a kitten become a cat?' Such fallacies really rest on a confusion between the collective and distributive use of terms (*cf.* § 171 (v.).

137. Epicheiremas.

Epicheirema—a regressive chain of reasoning with one premise of each prosyllogism omitted.

An Epicheirema is a regressive chain of reasoning abridged by the omission of one of the premises of each prosyllogism.

Each prosyllogism, therefore, appears in the epicheirema as an enthymeme, though the episyllogism is stated in full. Each prosyllogism furnishes a reason in support of one of the premises of the episyllogism, and the whole epicheirema may be described as a syllogism with a reason given in support of one or both of its premises. When one premise only is thus supported, the epicheirema is *Single;* when both are furnished with reasons, it is *Double;* and when those reasons themselves have other reasons attached to them, it is *Complex.* The progress of thought in an epicheirema is from the episyllogism to the prosyllogisms on which it depends; from the conclusion to the principles which support it.

Symbolic forms of epicheiremas.

Symbolic examples of the Double Epicheirema are :—

(1) *Every M is P, because it is X,*
Every S is M, because it is Y,

∴ *Every S is P.*

(2) *Every M is P, because every A is,*
Every S is M, because every B is,

∴ *Every S is P.*

In the first case the enthymemes expressing the reasons are both of the first order, the suppressed major premises

being *Every X is P*, and *Every Y is M*. In the second case both the enthymemes are of the second order, the implied minor premises being *Every M is A*, and *Every S is B*. Of course, both need not be of the same order. If we leave out one of the reasons in either of the above examples we have a single epicheirema. A complex Epicheirema would be :—

Every M is P, because it is X, and every X is Y,
Every S is M,
∴ *Every S is P.*

The full analysis of this would give the first example of a regressive chain of reasoning given towards the close of § 135. Additional examples can be framed by omitting other premises in the same complex reasoning. Of course, both premises may be similarly supported by a chain of reasonings, but the arguments then become very complex.

Examples of epicheiremas

We will now illustrate what has been said by material examples of the two forms of double epicheirema given above.

(1) 'All unnecessary duties on imports are impolitic, as they impede the trade of the country; the American protective duties are unnecessary, as they support industries which are quite able to stand alone; therefore, the American protective tariff is impolitic.'

(2) 'All Malays are cruel, because all savages are; all the aboriginal inhabitants of Singapore are Malays, because all the natives of that part of Asia are; therefore, all the natives of Singapore are cruel.'

CHAPTER VII.

FUNCTIONS OF THE SYLLOGISM.

138. Universal Element in Deductive Reasoning.

Philosophers of the Empiricist School—as Locke and Mill—have asserted that all inference is from particulars to particulars;

The essential feature of syllogistic reasoning is the subsumption of a particular case under a general rule; in other words, every deductive inference must rest on a universal element [*cf.* §§ 107, 115 (i.)]. This necessity has been denied by philosophers of the empiricist school, who hold that all knowledge is derived from experience. Thus, Locke (*Essay on the Human Understanding*, Bk. IV, Ch. xvii, § 8) says: "It is fit . . . to take notice of one manifest mistake in the "rules of syllogism, viz., that no syllogistic reasoning can be "right and conclusive, but what has at least one general "proposition in it. As if we could not reason, and have "knowledge about particulars; whereas, in truth, the matter "rightly considered, the immediate object of all our reason-"ing and knowledge is nothing but particulars." Mill, in his chapter on "The Functions and Logical Value of the Syllogism" (*Logic*, Bk. II, Ch. III), adopts and expands the same view. He says: "All inference is from particulars to "particulars: General propositions are merely registers of "such inferences already made, and short formulæ for "making more: The major premise of a syllogism, con-"sequently, is a formula of this description: and the "conclusion is not an inference drawn *from* the formula, but "an inference drawn *according to* the formula: the real "logical antecedent, or premise, being the particular facts "from which the general proposition was collected by

"induction" (§ 4). "Though there is always a process of "reasoning or inference where a syllogism is used, the "syllogism is not a correct analysis of that process of "reasoning or inference; which is, on the contrary (when "not a mere inference from testimony), an inference from "particulars to particulars; authorized by a previous in-"ference from particulars to generals, and substantially the "same with it; of the nature, therefore, of Induction. But "while these conclusions appear to me undeniable, I must "yet enter a protest . . . . . against the doctrine that the "syllogistic art is useless for the purposes of reasoning. "The reasoning lies in the act of generalization, not in "interpreting the record of that act; but the syllogistic "form is an indispensable collateral security for the correct-"ness of the generalization itself. . . . The value . . . of "the syllogistic form, and of the rules for using it correctly, "does not consist in their being the form and rules according "to which our reasonings are necessarily, or even usually "made; but in their furnishing us with a mode in which "those reasonings may always be represented, and which is "admirably calculated, if they are inconclusive, to bring "their inconclusiveness to light" (*ibid.*, § 5). The "universal "type of the reasoning process" is "resolvable in all cases "into the following elements: Certain individuals have a "given attribute; an individual or individuals resemble the "former in certain other attributes; therefore they resemble "them also in the given attribute" (*ibid.*, § 7).

and Mill regards the syllogism as valuable simply as a test of reasoning.

This view of the syllogism was accepted by Sir J. Herschel, Dr. Whewell, Mr. Bailey, Professor Bain, and other logicians; but it has been strongly opposed by Mansel, De Morgan, Dr. J. Martineau, Dr. Ray, Professor Bowen, and Sir W. Hamilton, and is in conflict with the traditional logical doctrine.

In examining this doctrine we will pass over the obvious contradiction involved in saying that "*all* inference is from "particulars to particulars" and yet asserting the possibility of an "inference from particulars to generals," and ask at once whether we do really reason from particulars to par-

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ticulars at all. The possibility of this has been strenuously denied by Mr. Bradley, though he is by no means an upholder of the syllogism. He says: "The thesis *to be* "proved is that an inference is made direct from par-"ticulars, as such, to other particulars. The conclusion "which *is* proved is that from experience of particulars "we somehow get a particular conclusion" (*Principles of Logic*, pp. 323-4). This conclusion may be granted at once. We frequently do reason by analogy from our experience of particulars to another particular instance, and such reasoning is fairly described in the last sentence quoted above from Mill, though we must demur to the claim that it is the "universal type of the reasoning process." Such arguments are often fallacious, but even when they are valid, on what do they really rest? Surely on a generalization. They are, as Mill says, arguments from *resemblance*.

But the inference is really made from a universal element in the particular.

But, "whenever we reason from resemblance we reason from "identity, from that which is the same in several particulars "and is itself not a particular. And is it not obvious that, "in arguing from particular cases, we leave out some of the "differences, and that we could not argue if we did not leave "them out? Is it not then palpable that, when the differ-"ences are disregarded, the residue is a universal? Is it "not once more clear that, in vicious inferences by analogy, "the fault can be found in a wrong gen ralization?" (Bradley, *ibid.*, p. 326). Mill's error springs from a too material view of Logic [*cf.* § 8 (ii) (*c*)]. He fixes his attention upon the whole concrete instance on which the generalization is founded, and overlooks the fact that it is not from this instance as a whole—*i.e.*, as *particular*—that the conclusion is drawn, but from only some elements of it, and that these are made the basis of the inference simply because they are regarded as common to all similar cases—*i.e.*, are *universal*. This is evident upon a careful examination of an example he gives (*ibid.*, § 3): "It is not only the village matron, who, when "called to a consultation upon the case of a neighbour's child, "pronounces on the evil and its remedy simply on the recol-"lection and authority of what she accounts the similar case

"of her Lucy." But why does she account it "a similar "case"? Is it not because she regards the symptoms observed in both cases as marks of the presence of a certain disease? But if so, she is reasoning, not from her Lucy as an individual but, from the universal connexion between a certain disease and the symptoms Lucy exhibited in her sickness; and thence she infers that the remedies which proved efficacious in that case will prove equally beneficial, not only in this new case of the neighbour's child but, in all similar cases which may be brought under her notice.

Thus, even in cases where the inference is apparently founded on one or more particular experiences, it is really based on the universal element in which they agree; and this may be expressed in a general proposition which forms the major premise of a syllogism.

* 139. Validity of Syllogistic Reasoning.

The syllogism has been asserted to involve a *petitio principii*.

Not only has the syllogistic process been asserted to be valueless, but its very validity has been frequently denied, on the ground that it involves the fallacy of *petitio principii*. Strictly speaking, this should mean that the conclusion of every syllogism is itself assumed as one of the premises; but, more loosely, it is held to imply that the premises presuppose the truth of the conclusion, and cannot, therefore, be used to establish it. This argument was advanced, in the third century, by Sextus Empiricus, who said that the major premise must result from a complete testing of every instance which can come under it, and that, therefore, to deduce an individual fact from a general principle is to argue in a circle. The same argument has been adopted by the Empiricist school generally. Thus, Mill says: "It must "be granted that in every syllogism, considered as an argu- "ment to prove the conclusion, there is a *petitio principii*; " . . . that no reasoning from generals to particulars can, "as such, prove anything: since from a general principle we "cannot infer any particulars, but those which the principle

Mill supports this view.

Book IV. Ch. VII.

"itself assumes as known" (*Logic*, Bk. II, Ch. iii, § 2). Mill then proceeds to argue that the real inference is from particulars to particulars, and that the syllogism is merely a guarantee of the validity of those inferences (*cf.* § 138).

Answers—1. The major premise is not a mere summation of instances,

If a universal proposition be regarded as a mere 'universal of fact,' or summary of examined particulars, the cogency of this objection to the syllogism must be granted. Very little reflexion is, however, needed to show that the vast majority of universal propositions are made on the strength of the examination of a small number of instances; indeed, no writer has insisted on this more strongly than Mill himself. The justification of such general propositions will be the subject of the next book, in which we shall deal with Induction; it is sufficient here to say that their force depends upon a recognition of the fact that they are 'universals of reason,' or expressions of necessary connexions of attributes. The truth of such a proposition is recognized, and even held to be necessary, before the totality of instances which come under it have been examined, or are, indeed, known. For instance, the laws of Kepler are syllogistically applied to all newly discovered planets and satellites without a doubt of the accuracy of the conclusion. Similarly, the universal validity of the law of gravitation was held to be so certain, that when the observed orbit of the planet Uranus appeared to violate it, the existence of a disturbing cause was inferred —an inference which led to the discovery of the planet Neptune.

but generally expresses a necessary connexion of attributes.

2. A minor as well as a major premise is an essential part of every syllogism.

Again, a syllogistic inference requires the combination of both premises, but the objection we are considering involves the tacit assumption that the minor is unnecessary. "When "you admitted the major premise," says Mill, "you asserted "the conclusion" (*ibid*). But, if so, surely the minor premise is superfluous. Mill, indeed, denies this, on the ground that the major premise does not individually specify all it includes, but only indicates them by marks, and that the office of the minor premise is to compare any new individual with the marks. "But since, by supposition, the new in-"dividual has the marks, whether we have ascertained him

"to have them or not; if we have affirmed the major "premise, we have asserted him" to have them (Mill, *ibid.*, § 8, note). As, however, this assertion was evidently not foreseen when the reasoner affirmed the major premise, Mill has to introduce a novel doctrine of 'unconscious assertion.' But, as assertion is an act of judgment, it cannot be unconscious; and to say, as Mill does (*ibid.*, § 2, note), that a person can assert a fact which he does not *know* is not only to talk very bad psychology but to fall into an absolute contradiction in terms. The necessity to a syllogistic inference of the minor premise is, then, a proof that such an inference is not a *petitio principii.*

3. Knowledge can be increased by syllogistic argument.

If the syllogism were really open to the charge of *petitio principii*, it would, of course, follow that no advance could be made in knowledge by its means. But the objection springs from a too objective view of Logic; from neglecting to remember the difference between what *is* in the facts of the external world, and what *we know to be* in them. Inference cannot, of course, give us more than already exists in the world, but it may help us to see and understand more. It is, indeed, our imperfect knowledge which makes inference of new truths possible. Were our knowledge complete, all truths would lie open before us, and such inference would be both unnecessary and impossible. For the truth of the conclusion is, in fact, concomitant with that of the premises from which we deduce it; it does not succeed them, though our perception of it may follow our perception of them. For, though the objectors to the syllogism deny the fact, it is certainly possible to accept the premises without deducing the conclusion. The shortness of the syllogistic process, and the triteness of the examples of it commonly given in treatises on Logic, disguise this possibility, and give plausibility to the assertion that no advance in knowledge is really made by syllogism. But, because, as statements of fact, the premises contain the conclusion, it by no means follows that "in studying how to draw the conclusion, we [are] studying "to know what we knew before. All the propositions of "pure geometry, which multiply so fast that it is only a

This charge against the syllogism is due to a too objective view of Logic.

Imperfect knowledge makes inference possible.

It is possible to accept the premises and not draw the conclusion.

This possibility is exemplified in mathematics.

"small and isolated class even among mathematicians who "know all that has been done in that science, are certainly "contained in, that is necessarily deducible from, a very "few simple notions. But *to be known from* these premises "is very different from being *known with* them. Another "form of the assertion is that consequences are *virtually* "contained in the premises, or (I suppose) *as good as* con- "tained in the premises. Persons not spoiled by sophistry "will smile when they are told that knowing two straight "lines cannot enclose a space, the whole is greater than its "part, etc.—they as good as know that the three intersections "of opposite sides of a hexagon inscribed in a circle must be "in the same straight line. Many of my readers will learn "this now for the first time; it will comfort them much to "be assured, on many high authorities, that they virtually "knew it ever since their childhood. They can now ponder "upon the distinction, as to the state of their own minds, "between virtual knowledge and absolute ignorance" (De Morgan, *Formal Logic*, pp. 44-5).

4. Proof does not depend on novelty.

Nor, indeed, even were this objection true would it be to the point. It is a psychological, not a logical, objection. A proof does not cease to be a proof because it is thoroughly familiar to any individual mind. The conclusions of geometry, for example, do not cease to be inferences from mathematical axioms and definitions because the process of reasoning by which they are reached is understood and remembered. We may, indeed, look upon a formally stated syllogism as an analysis of the mode of deductive inference, and as such an analysis it makes explicit elements which, in the actual drawing of the inference, may be implicit, and so escape superficial observation. But, as was shown in the last section, in all deductive inference there must be an application of a universal judgment to a particular case; in other words, the elements of syllogism must be present though each may not separately engage attention.

Summary.

We may, then, sum up our answer to the charge of *petitio principii* brought against the syllogism under four heads:— the major is essentially not a mere summation of observed

instances; the minor is a necessary part of every syllogism; it is possible to accept the premises without drawing the conclusion, and hence to make progress in knowledge by means of syllogism; and the fact of inference depends on the rigidity of the proof, not on its novelty.

140. Limitations of Syllogistic Reasoning.

Many logicians have claimed that the syllogism is the [illegible] of valid reasoning.

Having shown the validity and value of the syllogism, we have now to enquire whether it is the *only* type of valid mediate inference. This has been strongly asserted by many logicians. Thus Whately claims that the syllogism is "the "form to which *all* correct reasoning may be ultimately "reduced" (*Elements of Logic*, 5th Ed., pp. 14-5); Professor Bowen asserts that "Reasoning, as such, must always be "syllogistic" (*Logic*, p. 353); and Dr. Ray says: "The "syllogism is the type of all valid reasoning; for no reason-"ing will be valid . . . unless it can be thrown into the "form of a syllogism" (*Ded. Log.*, p. 254).

But inferences from relations other than that of subject and attribute can not be expressed syllogistically.

In opposition to these claims it has been pointed out that the syllogism deals only with propositions which express the relation of subject and attribute, and that inferences from other relations, though they may be perfectly valid, not only are not made syllogistically but, cannot be satisfactorily expressed in that form. Such, for example, is the *argumentum à fortiori—A is greater than B, B is greater than C; therefore, A is greater than C.* Various attempts have been made to express such arguments syllogistically, the most successful of which is Mansel's (*Art. Log. Rud.*, 3rd. Ed., p. 198)—

> "*Whatever is greater than a greater than C is greater than C;*
> "*A is greater than a greater than C,*
> "*Therefore, A is greater than C.*"

But the whole argument is really assumed in the major premise, and the inference is, therefore, invalidated by a *petitio principii;* moreover, *B* does not appear in the premises, which cannot, therefore, express the whole argument.

A Logic of Relatives would deal with all relations, but will probably never be worked out,

If, then, account is to be taken of all valid inferences, we need a *Logic of Relatives* which "shall take account of "relations generally, instead of those merely which are "indicated by the ordinary logical copula 'is.'" (Venn, *Symbolic Logic*, p. 400.) Such a logic has never been worked out, and, perhaps, never will be; for, as Dr. Venn says (*ibid.*, p. 403): "the attempt to construct a Logic of Relatives "seems . . . altogether hopeless owing to the extreme "vagueness and generality of this conception of a Relation." Some attempt to classify copulas of relation was, indeed, made by De Morgan (*Syllabus*, pp. 30, 31), who divided them into *convertible* "in which the copular relation exists between "two names *both ways*," and *inconvertible*, in which it does not; and into *transitive* "in which the copular relation joins *X* "with *Z* whenever it joins *X* with *Y* and *Y* with *Z*," and *intransitive*, in which it does not. As an example of a copula which is both convertible and transitive, De Morgan gives 'is fastened to.' But the great majority of relations would be both inconvertible and intransitive, and the classification cannot be said to have much value. Mr. Bradley (*Logic*, pp. 243-4) gives a list of relations, which though it "does not pretend to be complete" is yet, probably, the best classification which has yet been put forward. He calls them "principles of inference," and enumerates five:—

though attempts to classify relations have been made, of which Mr. Bradley's is the most successful.

(1) *Synthesis of Subject and Attribute.* Under this all syllogistic inferences can be brought.

"(2) *Synthesis of Identity.* Where one term has one "and the same point in common with two or "more terms, there these others have the same "point in common . . . . . as 'If *A* is the brother "of *B*, and *B* of *C*, and *C* is the sister of *D*, then *A* "is the brother of *D*.'

"(3) *Synthesis of Degree.* When one term does, by virtue "of one and the same point in it, stand in a "relation of degree with two or more other "terms, then these others are also related in "degree . . as '*A* is hotter than *B* and *B* than *C*,

"therefore *A* than *C*.'" The *argumentum à fortiori* comes under this head.

"(4 and 5) *Syntheses of Time and Space.* When one and "the same term stands to two or more other "terms in any relation of time or space, there we "must have a relation of time or space between "these others. *Examples:* '*A* is north of *B* and "*B* west of *C*, therefore *C* south-east of *A*'; '*A* is a "day before *B*, *B* contemporary with *C*, therefore "*C* a day after *A*.'"

Arguments from relations other than subject and attribute are not deductive.

The validity of the arguments in classes (2) to (5) may be granted at once, as may the fact that they are not syllogistic. But it must be pointed out that neither are they deductive; for in them is no subordination of a special case under a general principle, but an inference of co-ordination from particular to particular. No doubt, the validity of the inferences rests upon material considerations of degree, time, space, etc., which are universally applicable; but these considerations stand in the same relation to the special arguments as the *dicta* of the four figures do to the syllogisms in those figures; and are not, therefore, the implied major premises of the arguments. The syllogism remains, then, as the one type of deductive reasoning, and should not be discarded on account of the existence of these other valid inferences, whose scope is not very great, and whose want of generality must always make them of but little importance. On the contrary, as Leibniz says (*Nouv. Ess.*, iv, 17, § 4): "The "discovery of the syllogism is one of the most beautiful and "greatest ever made by the human mind; it is a kind of "universal mathematic whose importance is not sufficiently "known, and when we know and are able to use it well, we "may say that it has a kind of infallibility :—nothing can be "more important than the art of formal argumentation "according to true logic."

The syllogism is the one type of deductive inference.

END OF VOLUME I.

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